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Particle Swarms in Optimization and Control *

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Abstract: In the last decennium, particle swarms have received considerable attention in the fields of optimization and control. Inspired by swarms of social animals, such as birds, fish, and termites, simple behavior on the local level has been shown to result in useful complex behavior on the global level. Particle Swarm Optimization has proven to be a very powerful optimization heuristic, and swarm aggregation based on artificial potential fields enjoys a growing interest for controlling particles in a swarm. Especially the flexibility, scalability, and robustness to errors on a local level are intrinsic properties of swarms that have attracted the interest of researchers in applying swarm technology to various problems. In this contribution, we present an overview of the application of particle swarms for optimization and control of swarm aggregation.

1. INTRODUCTION

In recent years, the collective behavior of large numbers of moving cooperative agents, frequently called particles, has proven to be useful in the fields of optimization and control. The collection of these particles is called a *swarm* and its application is referred to as *swarm intelligence*. The power of swarm intelligence is that simple behavior on the local level can result in useful, often more complex, behavior on the global level. Even if the individual agents are too simple for the label 'intelligent', the swarm often does manifest intelligent behavior. The global behavior of the swarm is difficult to predict based on the local dynamics of the particles. A number of publications, such as (Clerc and Kennedy, 2002; Gazi, 2005; Gazi and Fidan, 2007; Jadbabaie et al., 2003), has however set the foundations for the analysis and application of swarm intelligence. This paper briefly reviews the major advances of swarm intelligence, analyzed from the viewpoint of optimization and control.

Swarming is a term from biology denoting the collective motion of a group of insects, bacteria, or groups of animals. Swarm intelligence is inspired by biological social animals, such as flocks of birds, schools of fish, and herds of running animals. Reynolds (1987) has demonstrated a visually attractive simulation of flocking by implementing only three basic local rules for each virtual bird, or *boid* (i.e., bird-oid). These rules were: flock centering, collision avoidance, and velocity matching. In a similar study, Vicsek et al. (1995) demonstrate the flocking of bird-like particles by averaging the flight direction of each particle with that of its neighbors. These results have been analyzed mathematically in (Jadbabaie et al., 2003). A quantitative study using a kinematic model of group motion is given by Okubo (1986). Grünbaum (1998) shows how schooling helps animals to find food sources in a noisy environment.

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Swarms have many advantages when compared to their individual members. In biological swarms, the group often offers protection from predators (school of fish, herd of antelopes), more efficient motion (flock of birds), or more effective food collection (ants, termites). In general, a swarm may complete tasks that are impossible to complete by the individuals alone. The characteristics of a swarm can be summarized as follows:

- A swarm consists of a set of cooperating autonomous individuals.
- The individuals have their own strategies they aim to satisfy and they are not aware of the global objective.
- Each individual locally interacts with the environment and communicates with its neighbors.
- There is neither a supervisor, nor a fixed and predefined hierarchical structure.

The swarms approach to distributed systems of moving agents is slowly finding a way to engineering applications, such as controlling platoons of vehicles on freeways (Liu et al., 2001), robot navigation (Feddema et al., 2002), and control of unmanned aerial vehicles (Deming and Perlovsky, 2007). The most important intrinsic advantages of swarms in engineering applications are:

- Simple homogenous individuals can be produced in series, resulting in lower production costs.
- The swarm is potentially robust to errors of the individuals, so the individuals can be made of relatively unreliable components. Malfunctioning individuals can easily be removed, or replaced.
- Scalability; individuals can easily be added, or removed.

Generally, in multi-agent systems, the coordination of the agents is achieved by complex strategies, in a fixed topology. Often a central controller is used to determine the optimal action for each of the agents. Such methods, however, scale poorly with the number of agents. Swarm intelligence aims at controlling a large number of cooperative autonomous agents, in a varying topology, with simple, local rules. The analysis of a swarm intelligence

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system typically focusses on the dynamics of the swarm as a whole, rather than on the dynamics of the individual agents.

In this paper, the application of swarm intelligence to optimization and control is considered. This overview is particularly useful as a concise review of the state of the art of these subjects. The most prevalent method regarding optimization is *Particle Swarm Optimization* (PSO). In PSO, the individual particles influence each other in their search for the optimum in a certain parameter space and in many cases, the swarm as a whole converges to this optimum. Regarding swarm intelligence for control, the most prevalent control problem is *Swarm Aggregation*, in which the particles have to form a cohesive swarm with certain characteristics, such as size, shape, and location in the environment. This paper discusses the method of artificial potential fields, which has received increased attention in the recent literature. In this method, attraction and repulsion forces are assigned to all objects in the environment. The particles move based on the nett force of the potential field. This method is a more general way of implementing the rules, defined by (Reynolds, 1987).

The rest of this paper is structured as follows. In Section 2, swarm intelligence for optimization is discussed. Section 3 discusses the modeling, control, and analysis of swarms of particles moving in two or three dimensions, and Section 4 concludes this paper.

2. PARTICLE SWARM OPTIMIZATION

In many optimization problems, the size of the search space rapidly increases with the number of variables and the domain of the values they can take. Finding an optimum in these search spaces quickly becomes an intractable problem, due to what is referred to as the *curse of dimensionality*. Rather than finding the global optimal solution, optimization heuristics have been developed to find sufficiently good solutions in polynomial time (Dorigo and Stützle, 2004). Swarm intelligence presents a class of such heuristics.

Particle Swarm Optimization (PSO) (Kennedy and Eberhart, 1995) has been developed to solve nonlinear multidimensional optimization problems. In analogy to flocks of birds, PSO casts the optimization problem in a parameter space, through which a set of particles flies. However, as opposed to birds, which fly in a three-dimensional space, the space of the particles can be of arbitrary dimension. The nonlinear function, the optimum of which needs to be found, maps the parameters to fitness values. The particles search on the local level and keep one another updated on the best solutions (associated with higher fitness values) found so far, so that they are biased towards more promising regions of the search space.

2.1 PSO Update Rule

In PSO, the problem state space is defined as $X \subseteq \mathbb{R}^n$, with $\mathbf{x}_i(k)$ the position of a particle i and $\mathbf{v}_i(k)$ its velocity at time k. After initialization, the particles update their state in each iteration of the algorithm, with the following rule:

$$\mathbf{v}_i(k+1) = w(k)\mathbf{v}_i(k) + c_1 r_1(k)[\mathbf{x}_{i,\text{pbest}}(k) - \mathbf{x}_i(k)] + c_2 r_2(k)[\mathbf{x}_{i,\text{lbest}}(k) - \mathbf{x}_i(k)] \quad (1)$$

$$\mathbf{x}_i(k+1) = \mathbf{x}_i(k) + \mathbf{v}_i(k), \quad (2)$$

where k is the current time step, $\mathbf{x}_{i,\text{pbest}}(k)$ is the *personal best* position, $\mathbf{x}_{i,\text{lbest}}(k)$ is the *local best* position, $w(k)$ is the inertia weight, $r_{1,2}(k)$ are random variables, and $c_{1,2}$ are positive acceleration constants.

In each iteration, a fitness function $F : X \rightarrow \mathbb{R}$ is evaluated for the values of $\mathbf{x}_i(k)$ and compared to the personal best values $\mathbf{x}_{i,\text{pbest}}(k)$. If a better value, corresponding to a higher fitness, has been found for a particle i, its personal best value is replaced by $\mathbf{x}_i(k)$. If the maximum of $\mathbf{x}_i(k)$ over all i in some neighborhood is higher than the current $\mathbf{x}_{i,\text{lbest}}(k)$, the latter value is replaced by that value. Sometimes, the neighborhood is considered to cover the complete swarm. In that case, the local best is called the *global best* position of a particle, $\mathbf{x}_{i,\text{gbest}}(k)$. Each particle in the swarm is attracted towards its personal best solution and its local best solution. In this way, it learns to find the optimum of the fitness function, not only by its own experience, but from other members of the swarm as well. The values of the inertia weight $w(k)$ and the range of the random variables $r_{1,2}(k)$ influence the convergence properties of the particle swarm. The positive acceleration constants $c_{1,2}$ trade off exploration and exploitation. The convergence of PSO has been proven in (Clerc and Kennedy, 2002) and in (Kadirkamanathan et al., 2006).

2.2 Variations on the Basic Update Rule

As PSO is a very active field of research, new variations are proposed continuously. Some include special rules of selecting the local best position, moving particles to random places in the environment, and information exchange between multiple cooperating swarms (Baskar and Suganthan, 2004; El-Abd and Kamel, 2005; Kennedy and Eberhart, 2001). Other authors propose new PSO-like methods inspired by other biological counterparts, such as the immune system (Afshinmanesh et al., 2005).

Apart from PSO, another interesting swarm intelligence optimization heuristic is *Ant Colony Optimization* (Dorigo and Stützle, 2004; Dorigo and Blum, 2005). Ant Colony Optimization has proven to be successful at finding good solutions in polynomial time to many NP-complete combinatorial optimization problems. Agents are modeled as ants traversing the nodes of a graph, representing the optimization problem. Traversing a node corresponds to adding it to a partial solution. In ACO, the agents communicate by *pheromones*, or values assigned to the connections between the nodes, which influence the probability for other agents to traverse these connections. It is proven that this mechanism results in a convergence of the solutions constructed by all the agents to the optimal combination of nodes and connections (Dorigo and Stützle, 2004; Dorigo and Blum, 2005).

2.3 Applications of PSO

PSO has successfully been applied to many optimization problems. Chen et al. (2006) and Gaing (2004) apply PSO to the tuning of PID controllers. Training of feedforward neural networks is described in (Kennedy and Eberhart, 2001). In the field of control of power plants, PSO has also been applied in (Heo et al., 2006; Gaing, 2004). Several other applications of PSO include the synthesis of antenna arrays (Chen et al., 2005), clustering in mobile ad hoc networks (Ji et al., 2004), and traffic incident detection (Srinivasan et al., 2003).

For a more detailed description of the basic update rule, its variations, and application areas, please refer to (Kennedy and Eberhart, 2001) and the references therein. That paper also contains a nice overview of the relation between swarm intelligence, biological flocking, and the field of evolutionary computation.

2.4 Concluding Remarks

The previous discussion has briefly presented the applicability of swarms for optimization. Rather than solving the problem analytically, or in an enumerative way, the particles in the swarm influence each other such that they are driven towards more promising regions of the search space, in which they eventually converge to the optimum. The next section describes methods for controlling swarm aggregation. In contrast to optimization, the control objective is not anymore for the particles to converge to a single point in a parameter space, but to form a cohesive swarm with some specific characteristics, such as size, shape, and location.

3. CONTROLLING A PARTICLE SWARM

The control of a swarm of moving agents has mainly focussed on three control problems: rendezvous, formation control, and swarm aggregation. An overview of these problems and associated controller design can be found in (Martinez et al., 2007).

The *rendezvous* problem is to find control strategies under which all the agents in the swarm group eventually agree on and reach a single common location. This problem is analyzed for synchronous communication, where the agents communicate synchronized with a common clock, as well as the asynchronous case in (Lin et al., 2003, 2004). The rendezvous problem also relates to the distributed consensus problem, which is clearly covered in (Olfati-Saber et al., 2007). This paper presents stability and performance conditions of the swarm based on the properties of the graph Laplacian, describing the connectivity of the individuals.

Formation control refers to the control problem where the agents need to control their distance to other agents, as specified in a formation graph. Typically, the formation needs to be maintained while the agents follow a trajectory and may encounter obstacles. Formation control is studied in, e.g., (Gazi and Fidan, 2007), in which rigid graph theory is used to analyze the properties of the formation in various conditions. This study also includes an overview of the control problems related to formation control, such as splitting and merging of formations.

Swarm aggregation deals with the control problem in which the agents have to aggregate to form a cohesive swarm.

Controlling a swarm is a difficult task. The large number of particles and interactions makes explicit analysis of the motion of all the particles rapidly infeasible. This paper focuses on swarm aggregation as the control problem of interest. The main method for controlling the motion of the particles uses artificial potential fields. This method is considered in, e.g., (Barnes et al., 2006; Chu et al., 2006; Gazi and Passino, 2003, 2004a,b, 2005; Gazi, 2005; Kim et al., 2006; Liu et al., 2003a,b; Liu and Passino, 2004).

3.1 Particle Dynamics

Swarm aggregation is the convergence of a set of particles to a cohesive swarm in an environment defined by a state $\mathbf{x} \in X \subseteq \mathbb{R}^n$, with X the state space and n the dimension of the state space. When analyzing large distributed systems, it is convenient to start with a simple model of the particle dynamics and their interaction. The particles are modeled by a simple kinematic model:

$$\dot{\mathbf{x}}_i(t) = \mathbf{u}_i(t), \tag{3}$$

where the position of a particle i at time t is denoted by $\mathbf{x}_i(t)$ and its corresponding input by $\mathbf{u}_i(t)$. This model allows proof-of-concept design of swarm systems, where at a later stage, the particle dynamics (3) can be replaced by a more realistic model, like a point-mass model or full-actuator model. The full actuator model has been considered in (Gazi and Passino, 2005). In (Gazi and Fidan, 2007), more models are described, such as the non-holonomic unicycle model, Dubins' vehicle model, and the self-propelled particle model. The interested reader is referred to that paper and the references therein.

3.2 Artificial Potential Fields

In the environment, all the objects, such as the particles and obstacles are assigned a certain *potential function*, which defines the force acting upon a particle at a certain distance. The value of the artificial potential field is the sum of the values of all the potential functions. The input to the particle dynamics (3) is the local value of the artificial potential field. For N particles present in an obstacle-free environment, (3) becomes:

$$\dot{\mathbf{x}}_i(t) = \sum_{j=1, j\neq i}^{N} g_i(\mathbf{x}_i(t) - \mathbf{x}_j(t)), \quad \text{for } i = 1, \ldots, N \tag{4}$$

where $g_i(\mathbf{x}_i(t) - \mathbf{x}_j(t))$ represents the potential function assigned to particle i that governs the attraction and repulsion of the particles in the swarm. Note that each particle may have its own potential function. This means that particles may have potential functions that are different in structure, or have the same structure, but different parameter values. A swarm in which at least one particle has a different potential function from the other particles, is called a heterogeneous swarm. When all the particles have the same potential functions, the swarm is said to be homogeneous. Although this framework allows for heterogeneous swarms, almost all the research published is on the analysis and control of homogeneous swarms. The main reason for this is that proving convergence for heterogeneous swarms is much more involving than for homogeneous swarms. The general class of attraction/repulsion functions for which convergence is proven in (Gazi and Passino, 2004b) is of the type:

$$g(\mathbf{y}) = -\mathbf{y}[g_a(||\mathbf{y}||) - g_r(||\mathbf{y}||)], \tag{5}$$

where $g_a : \mathbb{R}^+ \to \mathbb{R}^+$ and $g_r : \mathbb{R}^+ \to \mathbb{R}^+$ represent the magnitude of the attraction and repulsion term respectively and $||\mathbf{y}|| = \sqrt{\mathbf{y}^T\mathbf{y}}$ is the Euclidean norm. Here, the vector $\mathbf{y}$ represents the distance between two particles $\mathbf{y} = \mathbf{x}_i - \mathbf{x}_j$, for $i, j \in \{1, \ldots, M\}$. The explicit dependence on time is left out for the sake of brevity. One particular function that is frequently used to model the particles is:

$$g(\mathbf{y}) = -\mathbf{y}\left(a - b\exp\left(-\frac{||\mathbf{y}||^2}{c}\right)\right), \tag{6}$$

with a, b, and c positive constants such that $b > a$. For large distances, a dominates and the function is attractive. For small distances, the term $b\exp\left(-\frac{||\mathbf{y}||^2}{c}\right)$ dominates and the function is repulsive. The function (6) is plotted in Fig. 1 for a scalar $\mathbf{y}$ and for several values of the parameters.

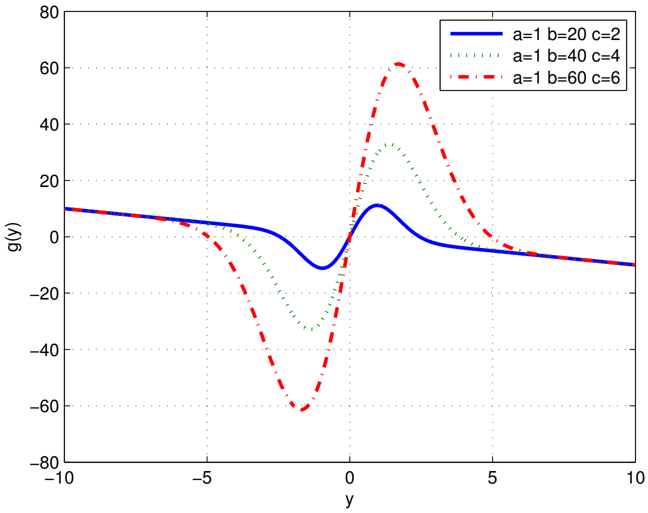

Fig. 1. The particle potential function (6) for several values of the parameters.

We can deduce from the motion equation (4) and the potential function (6) that the sign of the velocity of the particles changes at the set of points $\mathcal{Y} = \{g(\mathbf{y}) = 0\} = \{\mathbf{y} = 0 \text{ or } ||\mathbf{y}|| = \delta = \sqrt{c\ln(b/a)}\}$, i.e., when the particles cross each other, or when they are at a distance δ known as the *comfortable distance*. The equilibrium $\mathbf{y} = 0$ is unstable, so the particles will move away from each other when crossing. The set of equilibrium points $||\mathbf{y}|| = \delta$ is stable because the force from the potential function drives the state of the particles to this equilibrium. The swarm will thus converge to the state where each individual reaches its comfortable distance with respect to all the other individuals. Note that in this analysis, the particles are allowed to move through each other, and collisions are not considered. Collision avoidance based on potential functions is addressed in, e.g., (Barnes et al., 2006) and (Kim et al., 2006).

Obstacles can also be modeled by potential functions. A typical potential function for an obstacle is:

$$g(\mathbf{y}) = -\text{sign}(\mathbf{y})\left(-b\exp\left(-\frac{||\mathbf{y}||^2}{c}\right)\right), \tag{7}$$

where $\text{sign}(\mathbf{y}) = 1$ for $\mathbf{y} \geq 0$ and $\text{sign}(\mathbf{y}) = -1$ for $\mathbf{y} < 0$. The attraction term g_a of (5) is set to 0, b to a high value so the repulsion is large and collisions are avoided, and c set to a value determining the range of influence, which acts as a safety margin to the obstacle.

3.3 Aggregation in a Nutrient Profile

Gazi and Passino (2004a) consider an extension of the motion dynamics from (4) by including a term $\sigma(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ representing the nutrient profile. The term nutrient stems from its analogy with the environment in which insects forage their nutrients. The profile contains the attractant/repellent parts of the environment and the motion equation for each particle becomes:

$$\dot{\mathbf{x}}_i = -\nabla_{\mathbf{x}}\sigma(\mathbf{x}_i) + \sum_{j=1, j\neq i}^{N+O} g_i(\mathbf{x}_i - \mathbf{x}_j), \quad \text{for } i = 1, \ldots, N,$$

where the term $-\nabla_{\mathbf{x}}\sigma(\mathbf{x}_i)$ represents negative gradient of the nutrient profile as measured by a particle i. It results in the motion of the individual particles toward the attractive regions and away from the more repellent regions. The number of obstacles is denoted by O, with the obstacles indexed from $N + 1$ to $N + O$.

The nutrient profile presents a way to control the position of the swarm in the environment. A changing nutrient profile can represent a moving setpoint for trajectory tracking. The introduction of a nutrient profile shows similarities with PSO, in which the particles are controlled to the global optimum in a parameter space. However, the nutrient profiles studied by Gazi and Passino (2004a); Liu et al. (2007) are smooth functions, without local optima, whereas typical optimization problems solved by PSO contain many local optima (Kennedy and Eberhart, 2001).

3.4 Convergence and Stability

Convergence and stability analysis of swarm aggregation based on artificial potential fields is not trivial. The potential field contains many local optima and changes continuously as the particles move and have a varying distance to the other objects while aggregating. Convergence and stability proofs are mainly published in (Gazi and Passino, 2003, 2004b; Gazi, 2005; Liu et al., 2003a).

Gazi and Passino (2003) have proved that the motion of the particles over time is towards the center of the swarm. As time progresses, all members of the swarm will converge to a hyperball

$$B_\epsilon(\overline{\mathbf{x}}) = \{\mathbf{x}_i : ||\mathbf{x}_i - \overline{\mathbf{x}}|| \leq \epsilon\}, \quad \forall\, i = 1, \ldots, N,$$

where

$$\epsilon = \frac{b}{a}\sqrt{\frac{c}{2}}\exp(-\frac{1}{2})$$

is the swarm size and

$$\overline{\mathbf{x}} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_i,$$

the swarm center. Moreover, they prove that the convergence will occur in finite time. They also provide a bound on the swarm size for finite number of agents ϵ_N, as

$$\epsilon_N = \frac{b(N-1)}{aN}\sqrt{\frac{c}{2}}\exp\left(-\frac{1}{2}\right).$$

This quantity is always smaller than ϵ for finite values of N and when $N \to \infty$, this bound becomes equal to ϵ. In other words, ϵ is the largest possible bound on the swarm size, independent of the number of particles in the swarm.

This result is very conservative, in the sense that it tells nothing about other properties of the swarm, such as the shape, orientation, and distribution of the particles within this hyperball. A detailed analysis of the convergence of the swarm to such macroscopic characteristics has not been published yet.

In relation to formation control, the convergence to characteristics on a local level, such as the exact distance between all the particles is studied in (Gazi, 2005). He proved the convergence of the swarm when *formation constraints* are added to (4):

$$||\mathbf{x}_i - \mathbf{x}_j|| = d_{i,j}, \quad \text{for } i,j = 1,\ldots,N,$$

with $d_{i,j}$ the desired distance between two particles i,j in the swarm.

3.5 Concluding Remarks

This section has presented the main results for the control of swarm aggregation using artificial potential fields for modeling the interaction between the particles and other objects in the environment. Artificial potential fields provide a general and convenient way of modeling the influence of all the objects in the environment on the particle motion. However, the resulting potential field generally contains many local optima and is varying in time due to the continuous motion of the particles while aggregating. For these reasons, the analysis and application of this method has been limited to simplified scenarios.

The same holds for the application of potential functions to control real swarm robotic systems. This approach can be used when the robots are capable of determining the distance between them and the other objects in the environment, and when they know the potential functions associated to them. Most of the research on particle swarms has been done on an analysis and simulation basis. Results on swarm aggregation are almost always regarded as proofs of concept, because the motion dynamics of the particles do not correspond to the dynamics of realistic agents.

4. CONCLUSIONS

This paper has presented an overview of the modeling and control of particle swarms applied to global non-linear optimization and swarm aggregation. Particle Swarm Optimization has been reviewed as an optimization heuristic in which virtual particles often converge to the global optimal value in the search space, even in the presence of local optima. Regarding the control of real moving agent systems, swarm aggregation has been reviewed as a method in which the agents, modeled as particles, are controlled to form a cohesive swarm. The concept of controlling the particles in the swarm by artificial potential fields, in which the interaction between the particles and all the objects in the environment is modeled by potential functions has been discussed.

Swarm intelligence for controlling large numbers of agents for real-world applications is still far from being realized. The analysis has only proven that homogenous swarms, for a certain class of potential functions and in an obstacle-free environment, converge within a certain hyperball. Analysis of swarms in more realistic circumstances and the development of systematic control design is necessary before swarm intelligence can be applied to practical relevant control problems.

REFERENCES

F. Afshinmanesh, A. Marandi, and A. Rahimi-Kian. A novel binary particle swarm optimization method using artificial immune system. In *Proceedings of the International Conference on Computer as a Tool (EUROCON 2005)*, volume 1, pages 217–220, Belgrade, Serbia & Montenegro, November 2005.

L. Barnes, W. Alvis, M. Fields, K. Valavanis, and W. Moreno. Swarm formation control with potential fields formed by bivariate normal functions. In *Proceedings of the Mediterranean Conference on Control and Automation (MED 2006)*, pages 1–7, Ancona, Italy, June 2006.

S. Baskar and P. N. Suganthan. A novel concurrent particle swarm optimization. In *Proceedings of the 2004 Congress on Evolutionary Computation (CEC 2004)*, volume 1, pages 792–796, Portland, OR, USA, June 2004.

J. Chen, Z. Ren, and X. Fan. Particle swarm optimization with adaptive mutation and its application research in tuning of PID parameters. In *Proceedings of the International Symposium on Systems and Control in Aerospace and Astronautics (ISSCAA 2006)*, pages 526–530, Harbin, China, January 2006.

T. B. Chen, Y. B. Chen, Y. C. Jiao, and E. S. Zhang. Synthesis of antenna array using particle swarm optimization. In *Proceedings of the Asia-Pacific Conference on Microwave (APMC 2005)*, volume 3, Suzhou, China, December 2005.

T. Chu, L. Wang, T. Chen, and S. Mu. Complex emergent dynamics of anisotropic swarms: convergence vs oscillation. *Chaos, Solitons and Fractals*, 30(4):875–885, November 2006.

M. Clerc and J. Kennedy. The particle swarm - explosion, stability, and convergence in a multidimensional complex space. *IEEE Transactions on Evolutionary Computation*, 6(1):58–73, February 2002.

R. W. Deming and L. I. Perlovsky. Concurrent multi-target localization, data association, and navigation for a swarm of flying sensors. *Information Fusion*, 8(3): 316–330, July 2007.

M. Dorigo and C. Blum. Ant colony optimization theory: a survey. *Theoretical Computer Science*, 344(2-3):243–278, November 2005.

M. Dorigo and T. Stützle. *Ant Colony Optimization*. The MIT Press, Cambridge, MA, USA, 2004.

M. El-Abd and M. Kamel. Information exchange in multiple cooperating swarms. In *Proceedings of the 2005 IEEE Swarm Intelligence Symposium (SIS 2005)*, pages 138–142, Pasadena, CA, USA, June 2005.

J. T. Feddema, C. Lewis, and D. A. Schoenwald. Decentralized control of cooperative robotic vehicles: theory and application. *IEEE Transactions on Robotics and Automation*, 18(5):852–864, October 2002.

Z. Gaing. A particle swarm optimization approach for optimum design of PID controller in AVR system. *IEEE Transactions on Energy Conversion*, 19(2):384–391, June 2004.

V. Gazi. Swarm aggregations using artificial potentials and sliding-mode control. *IEEE Transactions on Robotics*, 21(6):1208–1214, December 2005.

V. Gazi and B. Fidan. *Coordination and control of multi-agent dynamic systems: Models and approaches*, volume 4433 of *Lecture Notes in Computer Science*, chapter Swarm Robotics: SAB 2006, pages 71–102. Springer-Verlag, 2007.

V. Gazi and K. M. Passino. Stability analysis of swarms. *IEEE Transactions on Automatic Control*, 48(4):692–697, April 2003.

V. Gazi and K. M. Passino. Stability analysis of social foraging swarms. *IEEE Transactions on Systems, Man and Cybernetics, Part B*, 34(1):539–557, February 2004a.

V. Gazi and K. M. Passino. A class of attractions/ repulsion functions for stable swarm aggregations. *International Journal of Control*, 77(18):1567–1579, December 2004b.

V. Gazi and K. M. Passino. Stability of a one-dimensional discrete-time asynchronous swarm. *IEEE Transactions on Systems, Man and Cybernetics, Part B*, 35(4):834–841, August 2005.

D. Grünbaum. Schooling as a strategy for taxis in a noisy environment. *Evolutionary Ecology*, 12(5):503–522, July 1998.

J. S. Heo, K. Y. Lee, and R. Garduno-Ramirez. Multi-objective control of power plants using particle swarm optimization techniques. *IEEE Transactions on Energy Conversion*, 21(2):552–561, June 2006.

A. Jadbabaie, J. Lin, and A. S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control*, 48(6):988–1001, June 2003.

C. Ji, Y. Zhang, S. Gao, P. Yuan, and Z. Li. Particle swarm optimization for mobile ad hoc networks clustering. In *Proceedings of the IEEE International Conference on Networking, Sensing and Control*, volume 1, pages 372–375, Taipei, Taiwan, March 2004.

V. Kadirkamanathan, K. Selvarajah, and P. J. Fleming. Stability analysis of the particle dynamics in particle swarm optimizer. *IEEE Transactions on Evolutionary Computation*, 10(3):245–255, June 2006.

J. Kennedy and R. C. Eberhart. Particle swarm optimization. In *Proceedings of the IEEE International Conference on Neural Networks*, pages 1942–1948, Perth, Western Australia, November 1995.

J. Kennedy and R. C. Eberhart. *Swarm Intelligence*. Morgan Kaufmann Publishers Inc., San Francisco, CA, USA, 2001.

D. H. Kim, H. O. Wang, G. Ye, and S. Shin. Decentralized control of autonomous swarm systems using artificial potential functions: analytical design guidelines. *Journal of Intelligent and Robotic Systems*, 45(4):369–394, April 2006.

J. Lin, A. S. Morse, and B. D. O. Anderson. The multi-agent rendezvous problem. In *Proceedings of the 42nd IEEE Conference on Decision and Control (CDC 2003)*, volume 2, pages 1508–1513, Maui, HI, USA, December 2003.

J. Lin, A. S. Morse, and B. D. O. Anderson. The multi-agent rendezvous problem - the asynchronous case. In *Proceedings of the 43rd IEEE Conference on Decision and Control (CDC 2004)*, volume 2, pages 1926–1931, Paradise Island, Bahamas, December 2004.

B. Liu, T. Chu, L. Wang, and Z. Wang. Collective motion of a class of social foraging swarms. *Chaos, Solitons and Fractals*, 2007.

X. Liu, A. Goldsmith, S. S. Mahal, and J. K. Hedrick. Effects of communication delay on string stability in vehicle platoons. In *Proceedings of the IEEE Conference on Intelligent Transportation Systems*, pages 625–630, Oakland, CA, USA, August 2001.

Y. Liu and K. M. Passino. Stable social foraging swarms in a noisy environment. *IEEE Transactions on Automatic Control*, 49(1):30–44, January 2004.

Y. Liu, K. M. Passino, and M. M. Polycarpou. Stability analysis of one-dimensional asynchronous swarms. *IEEE Transactions on Automatic Control*, 48(10):1848–1854, October 2003a.

Y. Liu, K. M. Passino, and M. M. Polycarpou. Stability analysis of m-dimensional asynchronous swarms with a fixed communication topology. *IEEE Transactions on Automatic Control*, 48(1):76–95, January 2003b.

S. Martinez, J. Cortes, and F. Bullo. Motion coordination with distributed information. *IEEE Control Systems Magazine*, 27(4):75–88, August 2007.

A. Okubo. Dynamical aspects of animal grouping: swarms, schools, flocks, and herds. *Advances in Biophysics*, 22: 1–94, 1986.

R. Olfati-Saber, J. A. Fax, and R. M. Murray. Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE*, 95(1):215–233, 2007.

C. W. Reynolds. Flocks, herds and schools: a distributed behavioral model. *Computer Graphics*, 21(4):25–34, 1987.

D. Srinivasan, W. H. Loo, and R. L. Cheu. Traffic incident detection using particle swarm optimization. In *Proceedings of the 2003 IEEE Swarm Intelligence Symposium (SIS 2003)*, pages 144–151, Indianapolis, IN, USA, April 2003.

T. Vicsek, A. Czirok, E. Ben-Jacob, I. Cohen, and O. Schochet. Novel type of phase transition in a system of self-driven particles. *Physical Review Letters*, 75(6):1226–1229, August 1995.