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# A new Control Strategy for Trajectory Tracking of Fire–Rescue Turntable Ladders

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**Abstract:** Modern fire-rescue turntable ladders are constructed in a lightweight mode to increase their maximum operation velocities, maximum length, and outreach respectively. Hence, the ladder has a limited stiffness and will be more and more subject to oscillations of deflection along with dominant overtones. This paper deals with the active oscillation damping of such ladders. A new feedforward and feedback control strategy is applied. The feedforward control is calculated through system inversion of a multi-body system utilizing its differentially flatness. The design of the feedback is based on partial differential equations (PDE) describing a Euler–Bernoulli model of a beam with a concentrated point mass at the end. The modal representation of the system is constructed based on the analytical form of the eigenfunctions. For active oscillation damping by feedback without a dynamical observer the ladder was equipped with a gyroscope additionally to strain gauges. Due to computational efforts and measurement noise a reduced state vector is disposed for stabilization. The proposed control approach allows damping the fundamental oscillation as well as the first dominant overtone and asymptotically stabilizing the system around the reference trajectory. Measurement results from the IVECO DLK 55 CS fire-rescue turntable ladder validate the good performance of the control.

Keywords: differentially flatness; system inversion; modal transform; distributed-parameter system; oscillation damping.

## 1. INTRODUCTION

In this paper a new control strategy for active damping of a fire-rescue turntable ladder is presented. Lightweight construction is applied to modern turntable ladders such as the IVECO DLK 55 CS (Fig. 1) in order to increase its maximum operation speeds, maximum ladder's length, and its horizontal outreach. Hence the ladder has a very limited stiffness and will be more and more subject to oscillations along with dominant overtones. The IVECO DLK 55 CS turntable ladder is characterized by a maximum extension of the ladder set extension of $L = 53.2\,\mathrm{m}$ and a maximum outreach of $22\,\mathrm{m}$ (at min. $\varphi_A = 68^\circ$) respectively. The erection angle covers a space of $\varphi_A = [-12^\circ \ldots 75^\circ]$. In the vertical plane the ladder is driven by hydraulic cylinders, which provide a maximum angular velocity of the erection angle $\dot{\varphi}_A = 3\,^\circ/\mathrm{s}$. The cage has a maximal loading of 300 kg, which corresponds to three fire fighters with full equipment.

In the proposed paper we shell use different mathematical models for feedforward and feedback design. For the feedforward loop the differential equations of motion are derived by applying the Lagrangian formalism (Banerjee [2005], deWit:96)). The dynamics of the hydraulic actuators are approximated by a 1st–order transfer function. The design of the feedforward loop is formulated based on simplifications of this dynamical model. An Euler-Bernoulli model of a beam with a point mass at the end is considered by Kharitonov et al. [2007]. Based on the analytical eigenfunctions the modal description of the plant is constructed, but considering only the first and the second mode. On this basis a feedback law is derived. This Ansatz is enhanced to different ladder's lengths. The plant is equipped with gyroscopes in addition to the strain gauges. Hence an observer is unnecessary because all states are determined by solving a system of algebraic equations

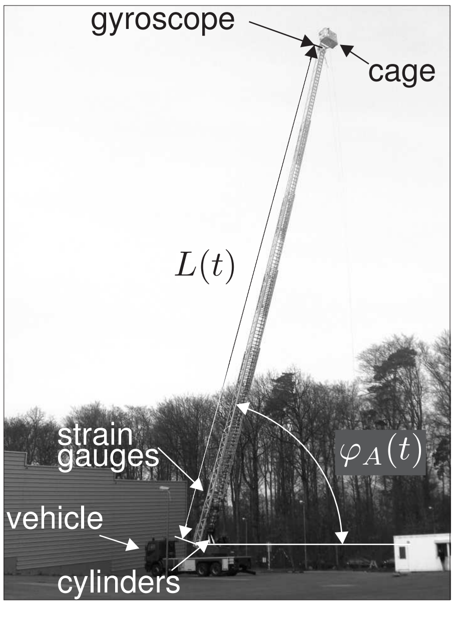


Fig. 1. turntable ladder: IVECO Magirus DLK 55 CS (approach angle $\varphi_A \approx 68^\circ$, ladder length $L = 53.2\,\mathrm{m}$)

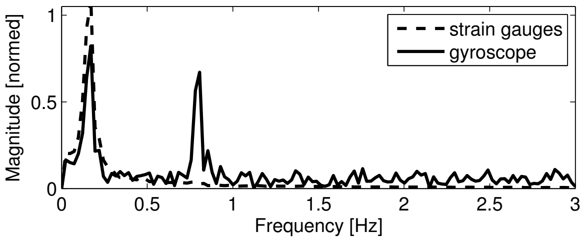


Fig. 2. Discrete fast fourier transform of a step response with a length of $L = 53.2\,\mathrm{m}$

of the two measurement systems. The control concept has been applied to a IVECO DLK 55 CS fire–rescue turntable ladder. The ladder is driven with a micro controller working with fix-point arithmetic. Though computational costs have to be taken into account during the development of the control concept.

In recent years the task of active oscillation damping of turntable ladders with the length up to 30 m was considered in some works. In Lambeck and Sawodny [2007], Lambeck et al. [2006], Sawodny et al. [2002] a trajectory tracking control based on a dynamic model of the ladder and decentralized control strategy has been developed. In these works the information about the system state is limited to the erecting angle and the strain gauges at a point close to the hub. By using these signals, a feedback was designed within framework of a multi-body system model. The designed controller is able to reduce effectively the swaying concerning the fundamental oscillation, but it cannot damp undesirable high–frequency oscillations (overtones), which become large especially for large lengths of the ladder.

In Zuyev and Sawodny [2005] the turntable ladder was considered as a flexible manipulator model with passive joints based on the Euler–Bernoulli beam concept. The feedback design is based on the Galerkin approximation. In Zuyev and Sawodny [2006, 2007] a similar approach is applied on a Timoshenko model of a beam. For the realization of the designed feedback law it is proposed to use the dynamical observer needing more computational power of a micro controller.

In the paper an overview on the control structure is given in section 2. Section 2.1 and 2.2 deal with the details of the control design. In section 2.1 a mathematical model of the ladder is derived and the feedforward control is formulated based on ordinary differential equations (ODE). The calculation of the feedback by using partial differential equations (PDE), which describe a Euler–Bernoulli model of a beam, is presented in section 2.2. Measurement results of the IVECO DLK 55 CS fire–rescue turntable ladder are obtained and analyzed in section 3. In Section 4 concluding remarks are given and aspects of future work are discussed.

## 2. CONTROL STRATEGY

The control consists of a feedforward and a feedback loop. The structure is presented in Fig. 3. The feedforward control is designed within the framework of a multi-body system model utilizing a flatness based approach for system inversion. The reference input to the closed loop system is $\dot{y}_{ref}(t)$, which is the signal coming from the hand lever of

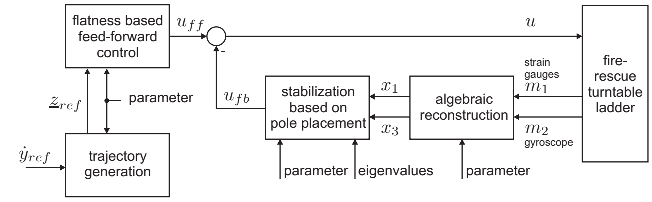


Fig. 3. Scheme of the control structure

the operator. Therefore a trajectory generator is needed to provide feasible reference trajectories $\left(\underline{z}_{ref}(t)\right)$. The stabilizing feedback is dimensioned with pole placement on the modal state space description of the Euler–Bernoulli model of a beam. The states $(x_1(t), x_3(t))$ of the system (amplitudes of the two modes) are constructed by solving a system of algebraic equations by utilizing the two measurements $(m_1(t), m_2(t))$.

Due to the fact that the ladder can be extended to different lengths the parameters of the ladder have to be updated in every time step (e. g. stiffness, eigenfrequencies and damping coefficients of the dominant modes, et cetera). Fortunately the ladder's length is changing very slowly, so it can be considered as a parameter. The time variance of the system can be neglected for the controller design, because all the parameters depend on the length of the ladder. But a gain scheduling depending on the ladder's length is necessary to obtain a good performance within the whole range.

Although the oscillation of deflection will be considered in the vertical plane, the action of gravity on the concentrated mass is neglected. Since the mathematical model is linear for small angles and the steady state solution taking into account the action of gravity and the prestressing of the ladder can be always subtracted for the task of stabilization.

### 2.1 Feedforward Control

*Multi-Body Model* The dynamic model of the turntable ladder can be derived by using the Lagrange formalism on a multi-body system with elasticity and damper-elements (Fig. 4), where the ladder set together with the cage and the vehicle are approximated by two equivalent masses ($m_h \ldots$ vehicle, $m_l \ldots$ ladder and cage) and the arm elasticity is approximated by spring-damper elements. Thus the ladder is considered as a massless leaf spring with the length $L$, the stiffness coefficient $c_v$, the damping coefficient $d_v$, and at the end of it the point mass $m_l$. The deflection at the end of the ladder is named $v_z$. The turntable ladder has some more degrees of freedom such as the turning motion of the hub ($\varphi_R$), but as mentioned before the focus of the paper is on the motion in the vertical plane.

$$
\begin{aligned}
0 = {} & m_l \ddot{v}_z(t) + m_l L \ddot{\varphi}_A(t) + d_v \dot{v}_z(t) + c_v v_z(t) + \\
& + \underbrace{\frac{\left(L\dot{\varphi}_R(t) + \dot{v}_y(t)\right)^2}{2L} \sin\left(2\varphi_A(t) + \frac{2}{L} v_z(t)\right)}_{p_R(t)}
\end{aligned}
\tag{1}
$$

Even though the Coriolis-force does not bother during the controller design, the effect of the rotary motion on

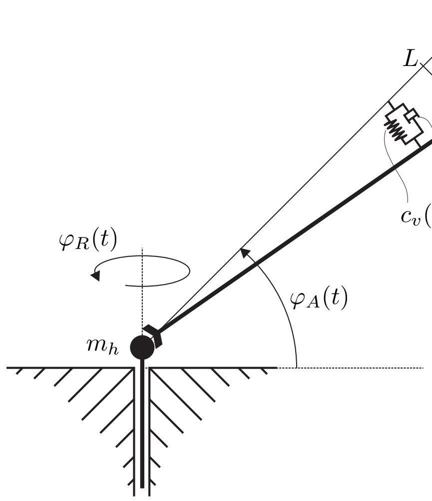


Fig. 4. Multi-body system model with elastic degree of freedom as model for feedforward control design

the erecting motion is neglected ($p_R(t) \equiv 0$), because the rotary motion is quiet slowly ($\dot{\varphi}_R \ll 1$) and this axis is actively damped by a separate controller ($\dot{v}_z \ll 1$) as well. For the sake of less computational costs this simplification is quiet convenient.

The dynamics of the actuators is approximated by 1st–order transfer function, because there is a subsidiary flow control of the cylinders' hydraulics. The nonlinear dc gain depending on the erecting angle $k_A(\varphi_A)$ is determined from the construction.

$$\ddot{\varphi}_A = -\frac{1}{\tau_A}\dot{\varphi}_A + \frac{k_A(\varphi_A)}{\tau_A}u \tag{2}$$

By choosing the state vector as

$$\underline{x}(t) = [\, \varphi_A(t) \;\; \dot{\varphi}_A(t) \;\; v_z(t) \;\; \dot{v}_z(t) \,]^T$$

the following state space is the result

$$\begin{aligned}\dot{\underline{x}} &= \underline{f}(\underline{x}) + \underline{g}(\underline{x})\,u \\ &= \begin{bmatrix} x_2 \\ -\dfrac{1}{\tau_A}x_2 \\ x_4 \\ \dfrac{L}{\tau_A}x_2 - \dfrac{c_v}{m_l}x_3 - \dfrac{d_v}{m_l}x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{k_A(x_1)}{\tau_A} \\ 0 \\ -\dfrac{Lk_A(x_1)}{\tau_A} \end{bmatrix} u \end{aligned} \tag{3a}$$

$$y = h(\underline{x}) = x_1 + \arctan\left(\frac{x_3}{L}\right) \tag{3b}$$

*Model Inversion* To invert the model mentioned in (3a) a differentially flat output with the relative degree $r = n$ has to be found. The relative degree $r$ is defined by the following conditions:

$$\begin{aligned} L_g L_f^i h(\underline{x}) &= 0 \quad \forall i = 0, \ldots, r-2 \\ L_g L_f^{r-1} h(\underline{x}) &\neq 0 \quad \forall \underline{x} \in R^n \end{aligned} \tag{4}$$

The differential operator $L_f$ represents the Lie derivative of the argument along the vector field $\underline{f}$ and $L_g$ along the vector field $\underline{g}$ respectively. The real output mentioned in (3b) has a relative degree of $r = 2$. Thus $y$ is not a differentially flat output, because the system is of 4th–order. But the deflection proportional to the ladder's length is very small. If we assume $v_z/L \ll 1$ a new output $\tilde{y} = \tilde{h}(\underline{x}) = x_1 + x_3/L$ with the relative degree of $r = 4$ is obtained. The difference between the control output and the differentially flat output are negligible. By using the Byrnes-Isidori normal form

$$\begin{aligned} \bar{S}: \quad & \tilde{y} = z_1, \; \dot{z}_1 = z_2, \; \ldots \;, \dot{z}_{r-1} = z_r \\ & \dot{z}_r = \left[L_f^r h + L_g L_f^{r-1} h u\right] \circ \phi^{-1}(\underline{z}) = \underline{a}(\underline{z}) + \underline{b}(\underline{z})\,u \end{aligned}$$

the system comes up in the form

$$z_1 = x_1 + L^{-1}x_3 \tag{5a}$$

$$z_2 = x_2 + L^{-1}x_4 \tag{5b}$$

$$z_3 = -\frac{c_v}{m_1 L}x_3 \tag{5c}$$

$$z_4 = -\frac{c_v}{m_1 L}x_4 \tag{5d}$$

$$\dot{z}_4 = -\frac{c_v}{\tau_A m_1^2 L}\left(m_1 L x_2 - \tau_A c_v x_3 - m_1 L k_A u\right). \tag{5e}$$

Via a diffeomorph state transform

$$\underline{z} = \underline{\phi}(\underline{x}), \quad z_i = \phi_i(\underline{x}) = L_{\underline{f}}^{i-1} h(\underline{x}) \quad i = 1, \ldots, r$$

the model can be inverted with respect to the differentially flat output. The new control input is defined as n-th derivative with respect to the time of the differentially flat output

$$\nu = \dot{z}_4 = \tilde{y}^{(IV)}$$

and so the control signal $u$ is determined by

$$u = \frac{-\underline{a}(\underline{z}) + \nu}{\underline{b}(\underline{z})}$$

$$u = \frac{1}{c_v \tilde{k}_A(z_1, z_3)}\left(c_v z_2 + c_v \tau_A z_3 + m_l z_4 + \tau_A m_l \nu\right). \tag{6}$$

By the model inversion we obtain a chain of four integrators (system's order) with the input $\nu$ and the differentially flat output $\tilde{y} = z_1$.

### 2.2 Feedback Control

Applying the feedforward control law (6) with feasible trajectories for $\nu$ and $\underline{z}$ accordingly the load sway will be much less compared to an uncontrolled motion. But for the reason of model mismatches and assumptions, which have been made in the design, the load sway is not eliminated totally. Perturbances such as wind and people moving or working on the platform of course cannot be compensated by feedforward control. So a feedback loop is needfully to stabilize the system around the reference trajectories to ensure a minimum of load sway.

*Euler–Bernoulli model* The mathematical model of the two-body system (ladder and cage) corresponds to a hybrid system consisting of a distributed as well as a lumped parameter system due to the concentrated mass at the end of the ladder (Fig. 5). It leads to the boundary conditions which have to obey the dynamics of this concentrated mass.

The ladder is driven by a control torque $M(t)$ at its end $z = 0$. $L$ is the length of the ladder, $\varphi_A(t)$ corresponds to the angle between the moving axis $Oz$ and the horizontal direction, $w(z,t)$ is the deflection from the center line of the beam describing the ladder. $M_p$ and $J_p$ are the mass

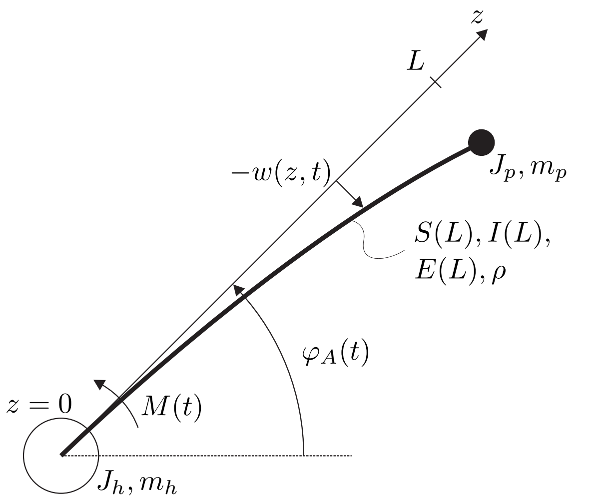


Fig. 5. Euler–Bernoulli beam with a point mass at the end as model for feedback control design

and moment of inertia of the cage with respect to its center of mass. Assuming that the angular velocity $\dot{\varphi_A}(t)$ is small and neglecting Coriolis–forces, the ladder concerning the distributed part of the plant can be described as the Euler–Bernoulli beam (Moallem et al. [2000], Wit et al. [1996]) by the following partial differential equation

$$EI\frac{\partial^4 w(z,t)}{\partial z^4} + \rho S\frac{\partial^2 \left[\varphi_A(t)z + w(z,t)\right]}{\partial t^2} = 0, \quad z \in (0,L),\ t > 0, \tag{7}$$

where $E$ is the Young's modulus, $I$ is the moment of inertia of the cross section, $\rho$ is defined as the density, and $S$ is the cross section area of the ladder. These parameters were evaluated through experimental identifications. The major effects could be reproduced by simulations of the ladder as a single flexible link. All except $\rho$ are varying with the length of the ladder. But the ladder's length is changing very slowly, therefore the feedback can be derived as if the ladder had a constant length. The calculations just need to be repeated for every change in the ladder's length. The boundary conditions at the end $z = 0$ of the beam have the following form (fixed end)

$$w(0,t) = 0,\ t > 0, \tag{8}$$

$$\frac{\partial w(0,t)}{\partial z} = 0,\ t > 0. \tag{9}$$

The boundary conditions at the end $z = L$ describing the connection between the distributed and lumped parts of the plant can be written as follows (Aoustin et al. [1997], Moallem et al. [2000])

$$-EI\frac{\partial^2 w(L,t)}{\partial z^2} = J_p\frac{d^2}{dt^2}\left(\varphi_A(t) + \frac{\partial w(L,t)}{\partial z}\right),\ t > 0, \tag{10}$$

$$EI\frac{\partial^3 w(L,t)}{\partial z^3} = M_p\frac{d^2}{dt^2}\Big(\varphi_A(t)L + w(L,t)\Big),\ t > 0. \tag{11}$$

The terms in the right part describe the moment and force provided by the moving concentrated mass, respectively. Introducing the new depending value

$$V(z,t) \triangleq \varphi_A(t)z + w(z,t),$$

the system equation (7) together with the boundary conditions (8)–(11) take the form

$$EI\frac{\partial^4 V(z,t)}{\partial z^4} + \rho S\frac{\partial^2 V(z,t)}{\partial t^2} = 0, \tag{12}$$

$$V(0,t) = 0, \tag{13}$$

$$\frac{\partial V(0,t)}{\partial z} = \varphi_A(t), \tag{14}$$

$$-EI\frac{\partial^2 V(L,t)}{\partial z^2} = J_p\frac{d^2}{dt^2}\left(\frac{\partial V(L,t)}{\partial z}\right), \tag{15}$$

$$EI\frac{\partial^3 V(L,t)}{\partial z^3} = M_p\frac{d^2 V(L,t)}{dt^2}. \tag{16}$$

For the simplicity, in this paper the control input $u(t)$ is introduced as follows

$$u(t) \triangleq \varphi_A(t) = \frac{\partial V(0,t)}{\partial z}. \tag{17}$$

The dynamics of the drive mechanism will not be taken into account here, although the actual hub torque at the end $z = 0$ of the beam is to be calculated by

$$M(t) = J_h\frac{d^2\varphi_A(t)}{dt^2} - EI\frac{\partial^2 w(0,t)}{\partial z^2}, \tag{18}$$

where $J_h$ is the hub inertia.

*Feedback Design* Via a modal transform of the system in (13)–(16), which will be presented in detail by Kharitonov et al. [2007], one obtains

$$\frac{d^2 V_k^*(t)}{dt^2} + \omega_k^2 V_k^*(t) = \left(\omega_k^2 f_k^* - \frac{f_k^{(IV)*}}{\eta}\right) u(t). \tag{19}$$

where $\eta \triangleq \rho S/EI$, $f_k^*$ and $f_k^{(IV)*}$ are series expansions in the basis of eigenfunctions $(Z_{k0})$, and $\omega_i$ are the circular frequencies corresponding to the eigenvalues $\lambda_i$. All these parameters depend on the ladder's length.

The task of stabilization will be considered. Further the equilibrium $V(z,t) \equiv 0 \quad \forall z \in [0,L]$ will be treated as the operating point without loss of generality. For the technical implementation of the feedback law one proposes to develop a feedback based only on the first two modes. Indeed, the limiting frequency provided by the hydraulic cylinders is 3 Hz but the frequency corresponding to the third mode is higher especially for shorter ladder's lengths. If we consider the first two dominant modes the state values will be introduced as follows

$$x_1(t) \triangleq V_1^*(t),\ x_2(t) \triangleq \dot{x}_1(t) = \dot{V}_1^*(t),$$
$$x_3(t) \triangleq V_2^*(t),\ x_4(t) \triangleq \dot{x}_3(t) = \dot{V}_2^*(t).$$

Even so internal damping is neglected in the Euler–Bernoulli model of beam, it can be observed in reality. Through experimental measurements the damping coefficients of the two first modes $(D_1, D_2)$ have been identified and (19) was augmented respectively. The system description in the state space takes the form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -{\omega_1}^2 & -2D_1\omega_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -{\omega_2}^2 & -2D_2\omega_1 \end{bmatrix}}_{\triangleq A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} +$$

$$+ \underbrace{\begin{bmatrix} 0 \\ {\omega_1}^2 f_1^* - \dfrac{f_1^{(IV)*}}{\eta} \\ 0 \\ {\omega_2}^2 f_2^* - \dfrac{f_2^{(IV)*}}{\eta} \end{bmatrix}}_{\triangleq B} u(t). \tag{20}$$

In order to be able to stabilize the plant state values must be available for a feedback. For this the necessary state values can be obtained by a Luenberger–type observer or measured by some sensors. The IVECO DLK 55 CS turntable ladder was equipped with strain gauges located at the point $z = z_1$ providing

$$m_1(t) = \frac{\partial^2 w(z_1, t)}{\partial z^2} = \frac{\partial^2 V(z_1, t)}{\partial z^2}$$

and a gyroscope located at $z = z_2$ sensing

$$m_2(t) = \frac{\partial^2 V(z_2, t)}{\partial z \partial t}.$$

If only two dominant modes are taken into account, the values to be measured can be represented as follows

$$m_1(t) = \frac{\partial^2 V(z_1, t)}{\partial z^2} = \hat{x}_1(t) Z_{10}''(z_1) + \hat{x}_3(t) Z_{20}''(z_1),$$

$$m_2(t) = \frac{\partial^2 V(z_2, t)}{\partial z \partial t} = \hat{x}_2(t) Z_{10}'(z_2) + \hat{x}_4(t) Z_{20}'(z_2).$$

In order to evaluate all state values one needs additional information. These can be obtained as follows (the gauge signal $m_1(t)$ is assumed to be sufficient smooth)

$$\frac{dm_1(t)}{dt} = \frac{\partial^3 V(z_1, t)}{\partial z^2 \partial t} = \hat{x}_2(t) Z_{10}''(z_1) + \hat{x}_4(t) Z_{20}''(z_1),$$

$$\int_0^t m_2(\tau) d\tau = \frac{\partial V(z_2, t)}{\partial z} = \hat{x}_1(t) Z_{10}'(z_2) + \hat{x}_3(t) Z_{20}'(z_2).$$

The estimates of the state values can be obtained in this case as a solution of the following system of linear algebraic equation

$$\begin{bmatrix} Z_{10}''(z_1) & 0 & Z_{20}''(z_1) & 0 \\ 0 & Z_{10}'(z_2) & 0 & Z_{20}'(z_2) \\ 0 & Z_{10}''(z_1) & 0 & Z_{20}''(z_1) \\ Z_{10}'(z_2) & 0 & Z_{20}'(z_2) & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \\ \hat{x}_3(t) \\ \hat{x}_4(t) \end{bmatrix} =$$

$$= \begin{bmatrix} m_1(t) \\ m_2(t) \\ dm_1(t)/dt \\ \int_0^t m_2(\tau) d\tau \end{bmatrix}. \tag{21}$$

From the numerical point of view it is evident, the points for measurements $z_1$ and $z_2$ must be chosen in such a way, that the condition number with respect to inversion of the system matrix (21) becomes small. Obviously in the system matrix (21) the states $\hat{x}_1$ and $\hat{x}_3$ are not interdependent with the $\hat{x}_2$ and $\hat{x}_4$. For computational efforts and measurement noise a reduced state vector is introduced

$$\underline{x}(t) = [\, \hat{x}_1(t) \;\; \hat{x}_3(t) \,]^T .$$

So a system of linear algebraic equation of 2nd–order follows. The states can be calculated by

$$\begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_3(t) \end{bmatrix} = \begin{bmatrix} Z_{10}''(z_1) & Z_{20}''(z_1) \\ Z_{10}'(z_2) & Z_{20}'(z_2) \end{bmatrix}^{-1} \begin{bmatrix} m_1(t) \\ \int_0^t m_2(\tau) d\tau \end{bmatrix}. \tag{22}$$

Using only two estimates of the state values the state feedback can be realized as

$$u_{fb}(t) = \underline{K} \, [\, \hat{x}_1(t) \;\; \hat{x}_2(t) \;\; \hat{x}_3(t) \;\; \hat{x}_4(t) \,]^T ,$$
$$\text{with} \quad \underline{K} = [\, k_1(t) \;\; 0 \;\; k_3(t) \;\; 0 \,]$$

where the controller gain matrix $\underline{K}$ can be constructed by standard means, e. g. by means of pole placement along the root locus of the matrix $(A - BK)$. The asymptotic stability of the equilibrium is assured, since the Kalman controllability criterion for the system (20) with its parameters is satisfied through all lengths of the ladder, i. e.

$$\det \left[\, B \;\; AB \;\; A^2B \;\; A^3B \,\right]$$
$$= b_2^2 b_4^2 \begin{bmatrix} \omega_1^4 + \omega_2^4 + \left(4D_1^2 + 4D_2^2 - 2\right)\omega_1^2\omega_2^2 \ldots \\ \ldots - 4D_1D_2\left(\omega_1^3\omega_2 + \omega_1\omega_2^3\right) \end{bmatrix}$$
$$\neq 0 \;\forall\; L.$$

## 3. MEASUREMENT RESULTS

In this section we present experimental results, which could be achieved with the proposed control strategy. In the experiment the ladder has been erected from $66.5\,^\circ$ to $71\,^\circ$ and lowered again without any controller in action. Afterward ($t > 119\,\mathrm{s}$) the same motion was repeated with the proposed active oscillation damping. The ladder has a length of 53.2 m.

Figure 6 shows the angular position of the ladder's tip. The solid line is the reference trajectory of the differentially flat output $(z_{1,ref} = \tilde{y}_{ref} \approx y_{ref})$ and dash–dotted line is the sensed position $\big(z_1 = \varphi_A + v_z L^{-1},\, v_z(t) \equiv w(L,t)\big)$. So, the ladder is damped within half the periodic time of the first mode. The overshoot is about 0.3 m of deflection at the end of the ladder (see Fig. 7). In general there is a reduction of approximately 40 % compared to the uncontrolled motion. The oscillations are damped during motion and very low residual load sway is achieved as well.

The time response of the two dominant modes presented in Fig. 8 reveals that the amplitudes of the second mode can be slightly higher with the controller in action. This is because the feedforward control excites higher amplitudes, which is due to the fact that multiple modes were not taken into account during the design. But obviously, both modes are damped actively by the stabilizing feedback.

## 4. CONCLUSION

In this paper a control approach for erecting motion of a fire–rescue turntable ladder was presented. A model

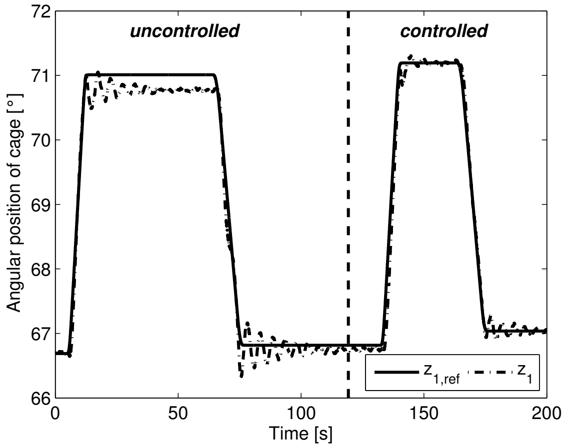


Fig. 6. The angular position of the cage at the end of the ladder in degrees

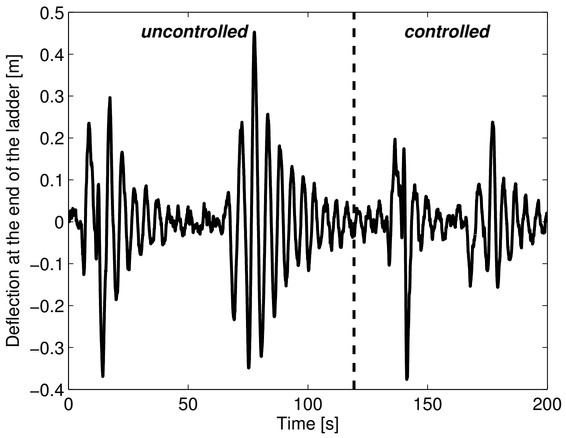


Fig. 7. The deflection at the end of the ladder in m

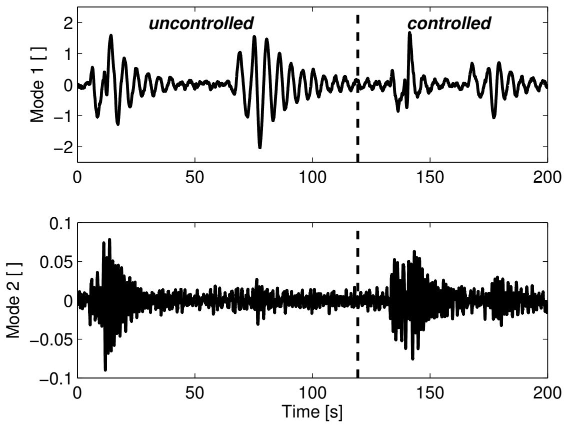


Fig. 8. Estimated states $(\hat{x}_1, \hat{x}_3)$ of the two dominant modes

inversion based on a differentially flat output for feedforward control is applied to a simple linear model of the ladder. The feedback loop is designed on Euler–Bernoulli model of a beam with a concentrated mass at the end. Based on the analytical form of the eigenfunctions the modal description of the plant was derived. The control law is realized on a micro controller system with fix-point arithmetic and limited computational power. Therefore only two state values were constructed from a algebraic equation taking the two measurements into account. The controller gain was calculated by the means of pole placement. The properties of the ladder depending on its length are taken into account as parameters and are updated in every time step of the micro controller system. In contrast to earlier works the proposed approach can be used for ladders with length over 30 m and it is adaptive to varying ladder lengths without additional requirements on a micro controller. At the present moment the proposed approach is verified at the IVECO DLK 55 CS turntable ladder. For future work the non-linear dynamics of the hydraulic cylinders will be taken into account more precisely and the feedforward control will be derived within the framework of the Euler–Bernoulli model of a beam.

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