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Eigenstructure Assignment for Helicopter Hover Control ⋆

Andrew Pomfret * **Stuart Griffin** ** **Tim Clarke** **

* *Department of Electronics, University of York, YO10 5DD, UK. (Tel: +44 1904 432319; e-mail: ajp109@ohm.york.ac.uk)*
** *Department of Electronics, University of York, YO10 5DD, UK.*

Abstract: In this paper, recent Eigenstructure Assignment (EA) algorithms are implemented to provide dynamic control of a Lynx helicopter in the hover, using the ideal eigenstructure derived by Clarke et al. [2003b]. First, a state-feedback controller is presented to demonstrate that the target eigenstructure is consistent with the kinematics of a helicopter, and that the closed-loop system meets the UK Ministry of Defence Defence Standard 00-970 requirements for Level 1 handling qualities. The EA algorithm for semi-proper systems presented by Pomfret et al. [2005] is then employed to demonstrate its efficacy and to allow comparison with the state-feedback case. Finally, the gain suppression methods from Pomfret and Clarke [2005] are used to introduce structure to a controller without affecting its performance.

Keywords: Helicopter control; Eigenstructure assignment

1. INTRODUCTION

Despite much recent work into nonlinear control design methodologies, these have not enjoyed widespread adoption in many areas of industry. Indeed the design of helicopter flight control systems often still relies on iterating single-input, single-output 'loop-at-a-time' linear control design techniques [Pomfret, 2006]. There remains, therefore, a clear need for the development of linear techniques which can provide an efficient replacement for such practices without requiring a radical change in the way in which the control problem is addressed.

Eigenstructure assignment is ideally suited to helicopter flight control because the handling qualities requirements of the UK UK Ministry of Defence Defence Standard 00-970 (Def.Stan.00-970) [Pitkin, 1989], and to a lesser extent the US ADS 33, are readily converted into restrictions on pole locations and modal coupling [Clarke et al., 2003b]. Additionally, recent algorithmic developments [Griffin, 1997, Clarke et al., 2003a, Pomfret et al., 2005, Pomfret and Clarke, 2005] have resulted in improved 'visibility' during the EA process, leading to a more intuitive connection between the various design parameters and the performance of the closed-loop system; such visibility is an essential part of any algorithm designed to replace 'loop-at-a-time' techniques.

The aim of this paper is to demonstrate, by means of design examples, the power and practicality of the ideal eigenstructure and EA algorithms mentioned above. Firstly, the state-feedback results of Griffin [1997] are replicated for a Lynx helicopter in the hover. However, state feedback is not a practical proposition for the control of rotorcraft, because velocity information is very difficult to measure directly to a suitable degree of accuracy, especially at low speed. Instead, accelerometers are often employed as part of an Inertial Measurement Unit (IMU). These provide information about linear movements in the body frame, but effectively measure not states but state derivatives.

It is therefore demonstrated next that state-feedback performance is achievable using a controller which does not have access directly to state information, but instead has access to a combination of states and state derivatives. This information takes the form of two sensed body accelerations and an earth-relative vertical velocity signal from a radar altimeter, and the methods developed by Pomfret et al. [2005] are employed to assign the desired eigenstructure.

Finally, a controller is synthesised that has access to all three IMU accelerometer signals *and* the earth-relative vertical velocity signal. This controller has more available Degrees of Freedom (DoF) than are required for complete eigenvalue and right-eigenvector assignment, and this extra design freedom is exploited by using the methods presented by Pomfret and Clarke [2005] to introduce structure to the controller. Once again it will be shown that the performance of the structured controller is identical to that of the original state-feedback example.

It should be noted that although EA gives no explicit robustness guarantees, several methods exist for trading off the desired eigenstructure for robustness improvements prior to assignment exist [e.g. Ensor and Davies, 2000, Griffin, 1997].

2. NOMENCLATURE

$\dot{h}$	Earth-relative vertical velocity
λ_n, $\mathbf{v}_n$	Eigenvalue and associated right eigenvector
$\mathbf{\Lambda}_d$, $\mathbf{\Lambda}_a$	Diagonal sets of desired and achieved eigenvalues
$\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$	State-variable system, input, output and direct transmission matrices

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$\mathbf{V}_d$, $\mathbf{V}_a$	Sets of desired and achieved right eigenvectors, ordered to be consistent with $\mathbf{\Lambda}_d$ and $\mathbf{\Lambda}_a$
ϕ, θ, ψ	Body angles (roll, pitch and yaw)
θ_0, θ_t	Collective pitch and tail rotor pitch command inputs
$\mathbf{x}$, $\mathbf{y}$	State vector and output vector
A_1, B_1	Lateral and longitudinal cyclic pitch command inputs
p, q, r	Body angular rates (roll, pitch and yaw)
u, v, w	Body-relative perturbation velocities (longitudinal, lateral and vertical/heave)

3. A STATE FEEDBACK CONTROL LAW

Griffin [1997] developed a state-feedback control law for an 8^{th}-order (reduced) model of a Lynx helicopter in hover. Information about the linear model used can be found in Appendix A. The results, reproduced here as Figures 1 to 3, are exactly those obtained by Griffin and are included in order to demonstrate the compliance of the ideal eigenstructure with the Def.Stan.00-970 and to provide a comparison with the results obtained using other algorithms.

The target eigenstructure is that presented by Clarke et al. [2003b], and the reader is referred there for more details of its derivation. The eigenvalues assigned to the system are listed in Table 1, with the subscripts on the eigenvalue names indicating the states which their associated modes should predominantly be coupled.

λ_p	λ_v	λ_q	λ_u	λ_w	λ_r
$-1.5 \pm j1.6$	-0.004	$-1.5 \pm j1.6$	-0.002	-0.33	-1.75

Table 1. Desired Eigenvalue Locations

The desired eigenvectors (rounded to $2dp.$ for compactness) are given in Box 1.

$$\mathbf{x} = [v\ p\ \phi\ u\ q\ \theta\ w\ r]^T$$

$$\mathbf{\Lambda}_d = \mathrm{diag}\left(\left[\lambda_p\ \bar{\lambda}_p\ \lambda_v\ \lambda_q\ \bar{\lambda}_q\ \lambda_u\ \lambda_w\ \lambda_r\right]\right)$$

$$\mathbf{V}_d = \begin{bmatrix} -0.31+0.33j & -0.31-0.33j & -1 \\ -0.31-0.33j & -0.31+0.33j & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdots$$

$$\cdots \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.31-0.33j & 0.31+0.33j & 1 & 0 & 0 \\ -0.31-0.33j & -0.31+0.33j & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Box 1: Desired Eigenvectors for Level 1 Def.Stan.00-970 Handling Qualities in Hover [from Clarke et al., 2003b]

The gain matrix, generated using standard state-feedback EA techniques [see Moore, 1976], is:

$$\mathbf{y} = [u\ v\ w\ p\ q\ r\ \phi\ \theta]^T$$

$$\begin{bmatrix} A_1 \\ B_1 \\ \theta_0 \\ \theta_t \end{bmatrix} = \begin{bmatrix} -0.0013 & 0.0004 & -0.0001 & 0.0525 \\ 0.0008 & 0 & -0.0001 & -0.0398 \\ 0.0001 & 0 & 0 & 0 \\ 0.0001 & 0.0011 & 0.0004 & -0.0576 \end{bmatrix} \cdots$$

$$\cdots \begin{bmatrix} -0.0361 & -0.0002 & -0.0383 & -0.0034 \\ 0.0743 & 0.0058 & -0.0015 & 0.2450 \\ 0.0007 & -0.0002 & 0.0062 & -0.0052 \\ 0.0154 & 0.1911 & -0.0807 & 0.0103 \end{bmatrix} \mathbf{y}$$

It can be seen that all the gains in this controller are small, with the largest being less than 0.25.

The achieved eigenvectors are given below, and have the same state and mode order as the ideal eigenvectors in Box 1 for ease of comparison. Again the entries have been rounded to $2dp.$ for compactness.

$$\mathbf{V}_a = \begin{bmatrix} -1.09+0.98j & -1.09-0.98j & -1 \\ -1.50-1.60j & -1.50+1.60j & 0 \\ 1 & 1 & 0 \\ -0.15+0.00j & -0.15-0.00j & 0 \\ -0.01-0.07j & -0.01+0.07j & 0 \\ 0.03+0.02j & 0.03-0.02j & 0 \\ -0.01+0.00j & -0.01-0.00j & 0 \\ -0.06-0.04j & -0.06+0.04j & 0 \end{bmatrix} \cdots$$

$$\cdots \begin{bmatrix} -0.14+0.01j & -0.14-0.01j & 0 & 0 & -0.06 \\ 0.00+0.05j & 0.00-0.05j & 0 & 0 & 0.04 \\ -0.02-0.02j & -0.02+0.02j & 0 & 0 & -0.06 \\ 1.29-0.76j & 1.29+0.76j & 1 & 0 & -0.02 \\ -1.50-1.60j & -1.50+1.60j & 0 & 0 & -0.03 \\ 1 & 1 & 0 & 0 & -0.02 \\ 0.01+0.01j & 0.01-0.01j & 0 & 1 & 0 \\ -0.06-0.03j & -0.06+0.03j & 0 & 0 & 1 \end{bmatrix}$$

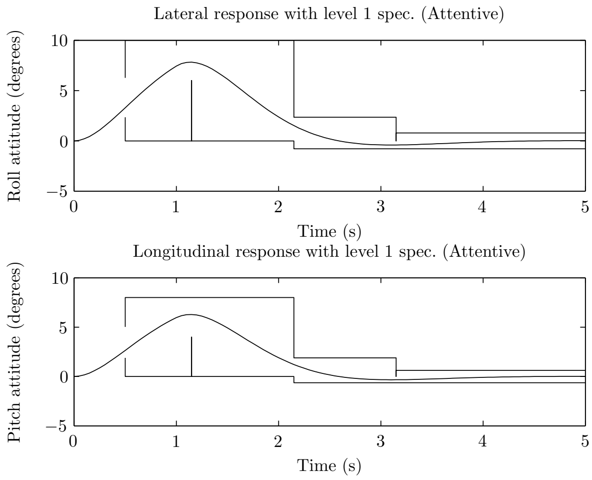

Fig. 1. Longitudinal and lateral responses of the state feedback controller [from Griffin, 1997]

Figure 1 shows the response of the helicopter's roll attitude ϕ to a one-second lateral pulse input on the cyclic pitch stick, and of its pitch attitude θ to a one-second longitudinal pulse input on the cyclic pitch stick. In each case the response is superimposed upon a template representing the Def.Stan.00-970 requirements for Level 1 handling qualities in the attentive flight phase [derived from Clarke et al., 2003b, Figure 1].

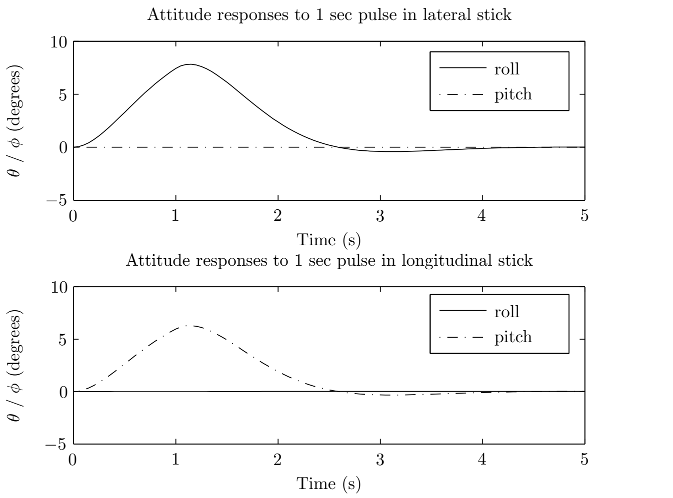

Fig. 2. On- and off-axis attitude responses to lateral and longitudinal stick [from Griffin, 1997]

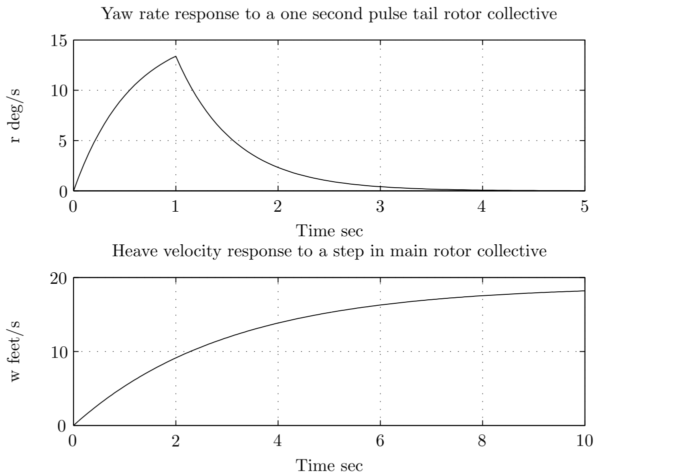

Fig. 3. Yaw and heave responses for the state feedback controller [from Griffin, 1997]

Figure 2 shows the cross-coupling between the roll and pitch attitudes when performing the same manoeuvres. The level of cross-coupling is negligible, due to the assignment of all right-eigenvectors and the close alignment of the allowable subspaces with the desired eigenvectors (indicating that the design requirements closely match the kinematics of the plant).

Finally, Figure 3 shows the yaw rate response (r) to the application of a one-second pulse applied to the tail rotor pitch, and the heave velocity response (w) to a step in the main rotor collective pitch.

4. A PSEUDO-STATE FEEDBACK EQUIVALENT

The states available in the original model [Griffin, 1997] are the longitudinal, lateral and vertical velocities u, v and w; the roll, pitch and yaw angular rates p, q and r; and the pitch and roll angles ϕ and θ. Direct measurement of velocities to a sufficient degree of accuracy is not practical though, and as a consequence either output-feedback techniques (which display inferior performance due to reduced DoF), or state reconstruction using observer-based techniques (which reduces design visibility and inevitably increases the system complexity) are traditionally required.

The alternative 'pseudo-state feedback' technique of Pomfret et al. [2005] instead uses measurements of state derivatives, in this case accelerations, to re-formulate the system into a semi-proper one (ie. one with a nonzero direct transmission matrix) with as many outputs as states. This allows assignment of a complete set of right-eigenvectors from the same allowable subspaces as in the state feedback case, and hence retains the performance of the state feedback controller without reducing visibility or adding complexity.

For his output-feedback control law examples, Griffin assumed that the velocity states were unmeasurable, but that vertical speed in the inertial frame ($\dot{h}$) could be measured. He also showed that $\dot{h}$ was equivalent in the hover to

$$\dot{h} \approx [0.057\ 0.057\ -1] \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Although there is no need to do so, for the purposes of demonstrating the flexibility of the pseudo-state feedback technique it will be assumed that $\dot{h}$ is measured as above, and also that the longitudinal and lateral accelerations $\dot{u}$ and $\dot{v}$ are measured.

Since $\dot{u}$ and $\dot{v}$ are state derivatives, it is necessary to reform the output matrix and the direct transmission matrix such that if

$$\mathbf{y} = \left[\dot{h}\ \dot{u}\ \dot{v}\ p\ q\ r\ \phi\ \theta\right]^T$$
$$\mathbf{x} = [u\ v\ w\ p\ q\ r\ \phi\ \theta]^T$$
$$\mathbf{u} = [A_1\ B_1\ \theta_0\ \theta_t]^T$$

then

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

which is achieved by setting

$$\mathbf{C} = \begin{bmatrix} 0.057 & 0.057 & -1 & 0 & 0 & 0 & 0 & 0 \\ a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} & a_{1,6} & a_{1,7} & a_{1,8} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} & a_{2,6} & a_{2,7} & a_{2,8} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ b_{1,1} & b_{1,2} & b_{1,3} & b_{1,4} \\ b_{2,1} & b_{2,2} & b_{2,3} & b_{2,4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where $a_{i,j}$ and $b_{i,j}$ are the corresponding elements of the system matrix $\mathbf{A}$ and input matrix $\mathbf{B}$, respectively.

The gain matrix, produced using the techniques from Pomfret et al. [2005] and the same desired eigenstructure as the state-feedback case above, is

$$\mathbf{K} = \begin{bmatrix} 0.0005 & 0.0237 & -0.0123 & 0.0536 \\ -0.0002 & -0.0198 & -0.0005 & -0.0336 \\ -0.0001 & -0.0027 & -0.0001 & 0.0009 \\ -0.0008 & -0.0308 & -0.0344 & -0.0255 \end{bmatrix} \ldots$$
$$\ldots \begin{bmatrix} -0.0356 & -0.0088 & -0.0001 & 0.1199 \\ 0.0673 & 0.0045 & -0.0001 & 0.1411 \\ -0.0003 & -0.0004 & 0.0066 & -0.0191 \\ -0.0161 & 0.1623 & 0.0252 & -0.1548 \end{bmatrix}$$

and it can be seen that the gains are of the same order of magnitude as in the state-feedback case, and are still acceptably small.

Figures 4 to 6 show the responses of the helicopter and pseudo-state feedback controller, and mirror directly Figures 1 to 3.

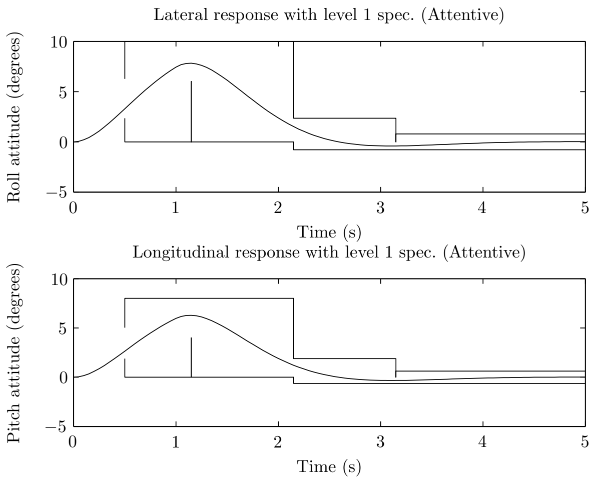

Fig. 4. Longitudinal and lateral responses of the pseudo-state feedback controller

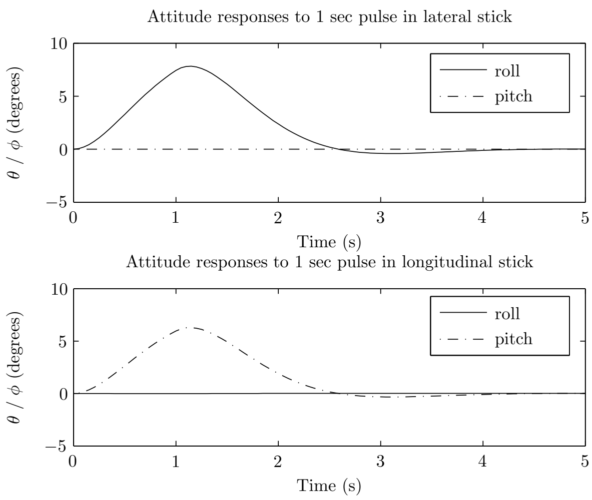

Fig. 5. On- and off-axis attitude responses to lateral and longitudinal stick

The achieved eigenvector set is shown below, and can be seen to be identical to that obtained using the state-feedback approach above.

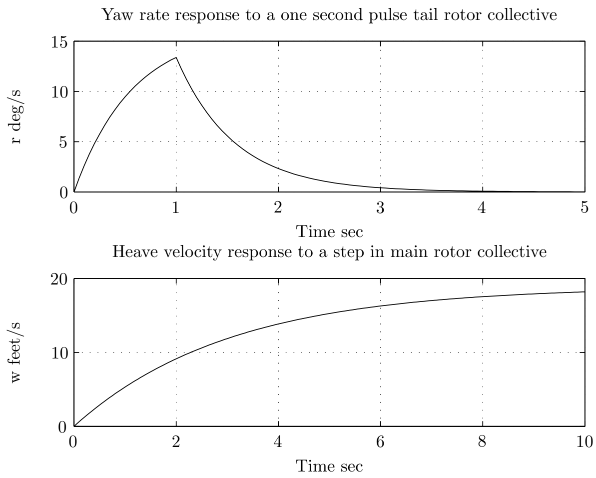

Fig. 6. Yaw and heave responses for the pseudo-state feedback controller

$$\mathbf{V}_a = \begin{bmatrix} -1.09+0.98j & -1.09-0.98j & -1 \\ -1.50-1.60j & -1.50+1.60j & 0 \\ 1 & 1 & 0 \\ -0.15+0.00j & -0.15-0.00j & 0 \\ -0.01-0.07j & -0.01+0.07j & 0 \\ 0.03+0.02j & 0.03-0.02j & 0 \\ -0.01+0.00j & -0.01-0.00j & 0 \\ -0.06-0.04j & -0.06+0.04j & 0 \end{bmatrix} \begin{matrix} \\ \\ \cdots \\ \\ \\ \cdots \\ \\ \\ \end{matrix}$$
$$\begin{matrix} \\ \\ \cdots \\ \\ \\ \cdots \\ \\ \\ \end{matrix} \begin{bmatrix} -0.14+0.01j & -0.14-0.01j & 0 & 0 & -0.06 \\ 0.00+0.05j & 0.00-0.05j & 0 & 0 & 0.04 \\ -0.02-0.02j & -0.02+0.02j & 0 & 0 & -0.06 \\ 1.29-0.76j & 1.29+0.76j & 1 & 0 & -0.02 \\ -1.50-1.60j & -1.50+1.60j & 0 & 0 & -0.03 \\ 1 & 1 & 0 & 0 & -0.02 \\ 0.01+0.01j & 0.01-0.01j & 0 & 1 & 0 \\ -0.06-0.03j & -0.06+0.03j & 0 & 0 & 1 \end{bmatrix}$$

The power of the pseudo-state feedback technique is clearly demonstrated. It may be seen that the performance of the state-feedback controller is retained by a controller which is practicable and uses only measurable quantities.

5. A STRUCTURED CONTROLLER

If a controller is synthesised using an output vector that consists of all three body accelerations in addition to a radar altimeter signal, the resulting gain matrix will possess more DoF than are required for complete pole assignment and the assignment of the full set of right eigenvectors.

Such a controller is easily generated using the same algorithmic approach as used above. The general formula for the gain matrix [Pomfret et al., 2005] is

$$\mathbf{K} = \mathbf{S}\mathbf{V}^{-1}\mathbf{C}^{\dagger}\left(\mathbf{I}+\mathbf{D}\mathbf{N}\right)^{-1} + \mathbf{Y}\big(\mathbf{I}-\mathbf{C}\mathbf{C}^{\dagger}\big)\left(\mathbf{I}+\mathbf{D}\mathbf{N}\right)^{-1}$$

where the matrix $\mathbf{V}$ is the set of achieved right eigenvectors, the matrices $\mathbf{S}$ and $\mathbf{N}$ are generated as part of the assignment process, and $\mathbf{Y}$ is a matrix of free parameters.

The minimum Frobenius norm controller, generated by setting $\mathbf{Y} = \mathbf{0}$, is:

$$\mathbf{y} = \left[\dot{h}\ \dot{u}\ \dot{v}\ \dot{w}\ p\ q\ r\ \phi\ \theta\right]^T$$

$$\mathbf{u} = \begin{bmatrix} 0.0028 & 0.0245 & -0.0086 & -0.0069 & 0.0512 \\ -0.0052 & -0.0216 & -0.0088 & 0.0154 & -0.0280 \\ 0.0010 & -0.0022 & 0.0017 & -0.0034 & -0.0003 \\ -0.0009 & -0.0308 & -0.0346 & 0.0003 & -0.0254 \end{bmatrix} \ldots$$

$$\ldots \begin{bmatrix} -0.0331 & -0.0057 & -0.0088 & 0.1208 \\ 0.0618 & -0.0023 & 0.0193 & 0.1391 \\ 0.0010 & 0.0011 & 0.0023 & -0.0187 \\ -0.0162 & 0.1622 & 0.0256 & -0.1549 \end{bmatrix} \mathbf{y}$$

It can be seen that the gains of this controller are extremely small.

Using the gain suppression techniques of Pomfret and Clarke [2005], it is theoretically possible now to reduce up to four of the gain matrix entries above to zero, without affecting the assigned eigenstructure. For the purposes of illustration, three of the four gains linking the radar altimeter to the inputs will be suppressed, leaving only the link between the altimeter and the collective pitch input. This involves finding a gain matrix $\mathbf{K}$ subject to the constraint that

$$\mathbf{U}\,\mathrm{vec}\,\mathbf{K} = \mathbf{0}$$

where the vec operator converts a matrix into a vector by stacking its columns. The permutation matrix $\mathbf{U}$ has exactly one nonzero element per row, the location of which corresponds to an entry in the gain vector that is to be suppressed. In this case then, $\mathbf{U}$ can be formulated as

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 & 0 & 0 \end{bmatrix}$$

Applying this permutation matrix with the algorithm from Pomfret and Clarke [2005] yields the following gain matrix, whose entries can once again be seen to be very small:

$$\mathbf{K} = \begin{bmatrix} 0 & 0.0235 & -0.0132 & 0.0016 & 0.0542 \\ 0 & -0.0197 & -0.0002 & -0.0006 & -0.0338 \\ 0.0037 & -0.0012 & 0.0062 & -0.0117 & -0.0033 \\ 0 & -0.0305 & -0.0331 & -0.0025 & -0.0264 \end{bmatrix} \ldots$$

$$\ldots \begin{bmatrix} -0.0361 & -0.0095 & 0.0019 & 0.1197 \\ 0.0675 & 0.0047 & -0.0009 & 0.1412 \\ 0.0039 & 0.0048 & -0.0082 & -0.0176 \\ -0.0152 & 0.1634 & 0.0221 & -0.1545 \end{bmatrix}$$

It can clearly be seen that the only coupling from the radar altimeter output (the first column of the gain matrix) is to the collective pitch input. There is still one further degree of freedom remaining after this assignment, so one more gain matrix entry could be suppressed if required.

The set of achieved eigenvectors is once more identical to that of the state-feedback controller, and the performance is consequently also the same. In order to avoid excessive repetition, evidence of this is presented only in the form of Figure 7, which may be compared with Figures 1 and 4.

6. CONCLUSIONS

It has been seen that the algorithm presented by Pomfret et al. [2005] provides a suitable mechanism for the design of controllers for helicopters. In particular, by feeding back state derivatives via accelerometers, a helicopter has been controlled in the hover using pseudo-state feedback techniques; without this acceleration feedback, Griffin [1997] resorted to output feedback, with inevitably inferior results.

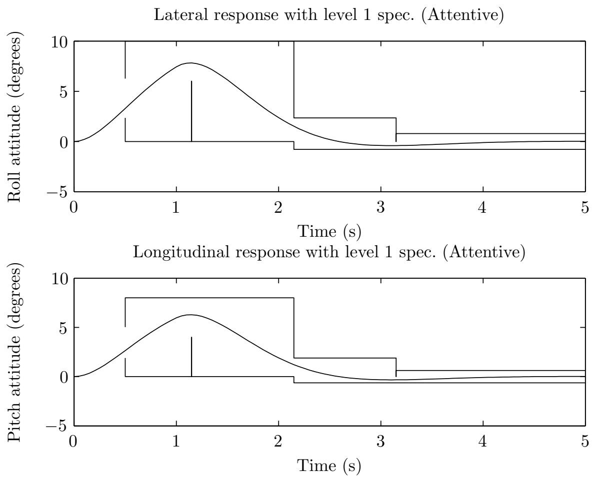

Fig. 7. Longitudinal and lateral responses of the structured controller

A further pseudo-state feedback design, having more DoF than are required for complete pole placement, has been subjected to the structural imposition techniques of Pomfret and Clarke [2005]. These techniques have been found to perform well.

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Appendix A. HELICOPTER MODEL

The helicopter model used in this paper was generated from an 11-state nonlinear simulation of a Lynx helicopter, trimmed in the hover and linearised. The resulting linear model underwent a model reduction procedure to remove the uncontrollable rotor flapping modes, and the heading integration mode was truncated, to leave the 8^{th} order system used here. In addition, the state order was changed for the sake of compatibility with the desired eigenvector set presented in Box 1, and scaling factors have been applied to the states in order to non-dimensionalise them as far as possible - complementary scalings have therefore been applied to the input and output matrices at every stage.

Since the implications of these scalings and re-orderings are rather complex, the reader is referred to Griffin [1997] for full details, and a full model will not be presented here. However, it is informative to note the original open-loop pole locations as

$$\begin{aligned}&\{-9.30, -1.95, 0.27 - 0.35i, 0.27 + 0.35i, \ldots\\&-0.26 - 0.50i, -0.26 + 0.50i, -0.30, -0.34\}\end{aligned}$$

and hence that the helicopter is open-loop unstable. The associated eigenvectors, in the same mode order as above, and the same state order as the ideal eigenvector set in Box 1, are

$$V = \begin{bmatrix} -0.07 & -0.25 & 0.90 & 0.90 & \\ -0.97 & -0.21 & -0.02+0.06i & -0.02-0.06i & \\ 0.11 & 0.11 & 0.08+0.10i & 0.08-0.10i & \cdots \\ 0 & 0.63 & 0.13+0.35i & 0.13-0.35i & \\ -0.10 & -0.62 & 0.02+0.00i & 0.02-0.00i & \\ 0.01 & 0.32 & 0.03-0.05i & 0.03+0.05i & \cdots \\ 0 & 0.02 & 0.04+0.04i & 0.04-0.04i & \\ -0.17 & -0.01 & 0.10-0.07i & 0.10+0.07i & \end{bmatrix}$$

$$\begin{matrix} \\ \\ \cdots \\ \\ \\ \cdots \\ \\ \\ \end{matrix}\begin{bmatrix} 0.91 & 0.91 & -0.64 & -0.61 \\ -0.05-0.06i & -0.05+0.06i & -0.05 & -0.05 \\ -0.07+0.14i & -0.07-0.14i & 0.05 & 0.05 \\ -0.05-0.33i & -0.05+0.33i & -0.44 & -0.40 \\ 0.02-0.01i & 0.02+0.01i & -0.02 & -0.02 \\ -0.05-0.02i & -0.05+0.02i & -0.04 & -0.04 \\ 0.04+0.01i & 0.04-0.01i & 0.14 & -0.44 \\ -0.02-0.17i & -0.02+0.17i & 0.61 & 0.52 \end{bmatrix}$$

and these demonstrate that a high degree of cross-coupling is present open-loop.