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Adaptive output feedback controller for a class of uncertain nonlinear systems

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Abstract: This paper addresses the problem of asymptotic tracking of an arbitrary smooth bounded reference output sequence in the presence of step like disturbances for a class of uncertain nonlinear multi-outputs systems, namely systems that could be described by the usual lower triangular representation thanks to an appropriate state transformation. These performances are shown to be achieved by combining an appropriate high gain state feedback control with a high gain adaptive observer under persistent of excitation condition. A filtered integral action is incorporated into the underlying state feedback control design to get a robust compensation of step like disturbances while reducing appropriately the noise control system sensitivity. The persistent excitation condition allows to avoid those useful parameter adaptation robustness design features as parameter projection or parameter adaptation freezing. This makes it possible to put the emphasis on the adaptive control design. As an illustrative example, the proposed adaptive control design is used to address the servo problem of a nonlinear double integrator using only the measurements of the position.

Keywords: Nonlinear system, Adaptive output feedback control, Admissible tracking capability, High gain control, Sliding mode control, Adaptive observer, Filtered integral action.

1. INTRODUCTION

The problem of adaptive output feedback control of uncertain nonlinear systems has been very active field of research over the last years. Several adaptive control designs have been proposed with several fundamental results on stabilization, regulation and model following for some classes of uncertain nonlinear systems, namely the lower triangular systems (See for instance Krstic et al. [1995], Marino and Tomei [1995], Khalil [1996], Johansen and Ioannou [1996], Aloliwi and Khalil [1997], Jiang [2000] and references therein). There are three common assumptions of these contributions that are worth to be pointed out. The first one is the global Lipschitz assumption which comes from the involved analysis techniques. The second one concerns the asymptotic stability of the system zeros dynamics which is referred to as the minimum phaseness assumption. The third one stipulates that the uncertain nonlinear functions of the system depend linearly on constant unknown parameters. Though recent approaches of output feedback of a class of non-minimum phase systems with uncertain dynamics have been proposed in Isidori [2000] and Karagiannis et al. [2005], nevertheless they are concerned by the stabilization purpose.
The control designs are commonly carried out using the standard backstepping technique that has been introduced in (Krstic et al. [1995]) and refined using the small gain theorem in (Jiang [2000]) to relax the minimum phaseness assumption. A state feedback Lyapunov based control design approach with an adaptive high gain observer have been used in Khalil [1996] to deal with the tracking problem in the case of single output systems. The underlying fundamental stability and performance results have been obtained under a persistent of excitation condition. Such a requirement has been removed in Aloliwi and Khalil [1997] by incorporating a parameter projection on a priori known compact convex subset of the involved parameter space. The global stability and regulation results derived in Marino and Tomei [1995] for systems with nonlinear parameterizations has been particularly obtained using high gain parameter adaptation laws.
In this paper, one aims at investigating the problem of asymptotic tracking of an arbitrary smooth bounded reference output sequence with bounded derivatives for any initial conditions in the presence of step like disturbances of uncertain nonlinear lower triangular multi-outputs systems. More specifically, we restrict our attention to those nonlinear systems without zeros dynamics where the uncertain nonlinear functions of the system depend linearly on constant unknown parameters. A high gain state feedback control with an appropriate filtered integral action is combined with a high gain adaptive observer to tackle the issue under consideration. To the best of our knowledge, this is the first contribution on the adaptive output feedback control involving a high gain control with global stability and tracking performance results. The adaptive observer performs the twin tasks of state estimation and parameter identification without any a priori knowledge on the system uncertain dynamics provided that a well defined persistence of excitation property is satisfied as it has been shown in Maatoug et al. [2005]. This contribution borrows from the available

results on the design of adaptive observers (See for instance Bastin and Gevers [1988], Besançon [2000], Zhang [2002], Zhu and Pagilla [2006] and references therein). The high gain state feedback control design has been particularly developed from the high gain observer design bearing in mind the control and observation duality in Farza et al. [2005]. This contribution has been particularly inspired from the available high gain observation potentiel (See for instance Gauthier and Kupka [2001], Farza et al. [2004] and references therein). Two design features are worth to be mentioned. Firstly, the controller gain involves a well defined design function which provides a unified framework for the high gain control design, namely several versions of sliding mode like controllers are obtained from particular expressions of the design function. Secondly, a filtered integral action is incorporated into the control design to achieve a robust compensation of step like disturbances while reducing appropriately the noise control system sensitivity.

It is worth noticing that the assumptions on the system zeros and the persistent of excitation have been mainly motivated by exposition simplicity. Indeed, the results can be easily extended to systems with zeros dynamics under the minimum phaseness assumption and the persistent excitation requirement can be removed using a useful parameter adaptation robustness design feature, namely a parameter projection on an a priori known compact convex subset of the involved parameter space or a parameter adaptation freezing throughout an a priori specified dead zone (Ioannou and Sun [1995], Narendra and Annaswamy [1989]).

This paper is organized as follows. The problem formulation is presented in the next section. Section 3 is devoted to the state feedback control design with a full convergence analysis of the tracking error in a free disturbances case. The adaptive output feedback controller is presented in section 4 where the main result of this contribution is given. Section 5 emphasizes the high gain unifying feature of the proposed control design. The possibility to incorporate a filtered integral action into the control design is shown in section 6. Simulation results are given in section 7 to highlight the performance of the proposed controller.

2. PROBLEM FORMULATION

One seeks to an admissible adaptive tracking problem for the folllowing class of MIMO systems

$$\begin{cases} \dot{x} = Ax + Bb(x)u + g(x) + \Psi(s,x)\rho \\ y = Cx = x^1 \end{cases} \quad (1)$$

$$x = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix}, \ \rho = \begin{pmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_m \end{pmatrix}, \ \Psi^T(s,x) = \begin{pmatrix} \Psi_1^T(s,x) \\ \Psi_2^T(s,x) \\ \vdots \\ \Psi_m^T(s,x) \end{pmatrix}$$

$$g(x) = \begin{pmatrix} g^1(x^1) \\ g^2(x^1,x^2) \\ \vdots \\ g^q(x) \end{pmatrix}, \ \Psi_j(s,x) = \begin{pmatrix} \Psi_j^1(s,x^1) \\ \Psi_j^2(s,x^1,x^2) \\ \vdots \\ \Psi_j^{q-1}(s,x^1,\ldots,x^{q-1}) \\ \Psi_j^q(s,x) \end{pmatrix}$$

$$A = \begin{bmatrix} 0 & I_{(q-1)p} \\ 0_p & 0_p \end{bmatrix}; \quad C = [I_p, 0_p, \ldots, 0_p]; \quad B = C^T \quad (2)$$

where the output $y \in \mathbb{R}^p$; the state $x \in \vartheta$ an open subset R^n with $x^k \in \mathbb{R}^p$, $k = 1,\ldots,q$; the input $u(t) \in U$, a compact subset of $\mathbb{R}^s$; s(t) is a bounded known signal; $\rho \in \mathbb{R}^m$ is a vector of unknown constant parameters, $\rho_i \in \mathbb{R}$, $i = 1,\ldots,m$; $g(x) \in \mathbb{R}^n$ with $g^k(x) \in \mathbb{R}^p$, $k = 1,\ldots,q$; $\Psi(s,x)$ is a $n \times m$ matrix and each $\Psi_j(s,x) \in \mathbb{R}^n$, $j = 1,\ldots,m$, denotes its j^{th} column with $\Psi_j^k(s,x) \in \mathbb{R}^p$, $k = 1,\ldots,q$; $b(x)$is rectangular full rank matrix of dimension $p \times m$. The involved system is hence uniformly observable and controllable.

($\mathcal{A}$1) The function $b(x)$ is Lipschitz in x over ϑ and there exists two positive scalars α and β such that for any $x \in \vartheta$, one has $\alpha^2 I_p \le b(x)\,(b(x))^T \le \beta^2 I_p$.

($\mathcal{A}$2) The function $g(x)$ is Lipschitz in its arguments over the domain of interest ϑ.

The control problem to be addressed consists in an asymptotic tracking of an output reference trajectory that will be noted $\{y_r(t)\} \in \mathbb{R}^p$ and assumed to be enough derived, i.e. $\lim_{t\to\infty} (y(t) - y_r(t)) = 0$.

Taking into account the class of systems, it is possible to determine the system state trajectory $\{x_r(t)\} \in \mathbb{R}^n$ and the system input sequence $\{u_r(t)\}$ corresponding to the output trajectory $\{y_r(t)\} \in \mathbb{R}^p$. This allows to define an admissible reference model as follows

$$\begin{cases} \dot{x_r} = Ax_r + Bb(x_r)u_r + g(x_r) + \Psi(s,x_r)\rho \\ y_r = Cx_r \end{cases} \quad (3)$$

The reference model state $x_r \in \mathbb{R}^n$ and its input $u_r \in \mathbb{R}^m$ can be determined as follows

$$\begin{cases} x_r^1 = y_r \\ x_r^k = \dot{x_r}^{k-1} - g^{k-1}(x_r^1,\ldots,x_r^{k-1}) \\ \qquad - \Psi^{k-1}(s,x_r^1,\ldots,x_r^{k-1})\rho \ for \ k \in [2,q] \\ u_r = (b(x_r))^+ \left(\dot{x_r}^q - g^q(x_r) - \Psi^q(s,x_r)\rho\right) \end{cases} \quad (4)$$

By assuming that the reference trajectory is smooth enough, one can recursively determine the reference model state and input from the reference trajectory and its first derivatives, i.e. $y_r^{(i)} = \frac{d^i y_r}{dt^i} \ for \ i \in [1, q-1]$ (see Hajji et al. [2007]). The adaptive output tracking problem can be hence turned to a state trajectory tracking problem defined by

$$\lim_{t\to\infty} (x(t) - x_r(t)) = 0 \quad (5)$$

Such problem can be interpreted as a regulation problem for the tracking error system obtained from the system and model reference state representations (1) and (3), respectively.

$$\begin{cases} \dot{e} = Ae + B\left(b\left(x\right)u\left(x\right) - b\left(x_r\right)u_r\right) \\ \qquad + \ g\left(x\right) - g\left(x_r\right) + \left(\Psi(s,x) - \Psi(s,x_r)\right)\rho \\ e_m = y - y_r \end{cases} \quad (6)$$

3. STATE FEEDBACK CONTROL

As it was early mentioned, the proposed state feedback control design is particularly suggested by the duality from the high gain observer design proposed in Farza et al. [2005]. The underlying state feedback control law is then given by

$$\begin{cases} \nu(e) = -B^T K_c \left(\lambda^q \bar{S} \Delta_\lambda e\right) \\ u(x) = (b(x))^+ \left(\dot{x}_r^q - g^q(x_r) - \Psi^q(s, x_r)\rho + \nu(e)\right) \end{cases} \tag{7}$$

where $(b(x))^+$ denotes the right inverse of the matrix $b(x)$, which exists according to $\mathcal{A}1$, Δ_λ is the block diagonal matrix defined by

$$\Delta_\lambda = diag\left(I_p, \frac{1}{\lambda} I_p, \ldots, \frac{1}{\lambda^{q-1}} I_p\right) \tag{8}$$

where $\lambda > 0$ is a positive scalar, $\bar{S}$ is the unique solution of the following algebraic Lyapunov equation

$$\bar{S} + A^T \bar{S} + \bar{S} A = \bar{S} B B^T \bar{S} \tag{9}$$

and $K_c : \mathbb{R}^n \mapsto \mathbb{R}^n$ is a bounded design function satisfying the following property

$$\forall \xi \in \Omega \ one\ has\ \xi^T B B^T K_c(\xi) \ \geq \ \frac{1}{2} \xi^T B B^T \xi \tag{10}$$

where Ω is any compact subset of $\mathbb{R}^n$.

Remark *3.1.* Taking into account the structure of the matrices B and C and the fact that the following algebraic Lyapunov equation

$$S + A^T S + S A = C^T C \tag{11}$$

has a unique symmetric positive definite solution S (Gauthier et al. [1992]), one can deduce that equation (9) has a unique symmetric positive definite solution $\bar{S}$. Moreover, one can show that (Hajji et al. [2007]): $B^T \bar{S} = [C_q^q I_p \ C_q^{q-1} I_p \ \ldots \ C_q^1 I_p]$.

The above state feedback control law satisfies the tracking objective (5) as pointed out by the following fundamental result.

Theorem *3.1.* The state and output trajectories of system (1)-(2) subject to the assumptions $\mathcal{A}1$ and $\mathcal{A}2$ generated from the input sequence given by (7)-(10) converge globally exponentially to those of the reference model (3) for relatively high values of λ.

Proof. The proof is similar to that given in Hajji et al. [2007] by considering the following Lyapunov function: $V(\bar{e}) = \bar{e}^T \bar{S} \bar{e}$ where $\bar{e} = \lambda^q \Delta_\lambda e$.

Remark *3.2.* Consider the case where the state matrix structure is as follows

$$A = \begin{pmatrix} 0 & A_1 & 0 & \ldots & 0 \\ 0 & 0 & A_2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & A_{q-1} \\ 0 & \ldots & \ldots & 0 & 0 \end{pmatrix}$$

where $A_i \in \mathcal{R}^{p\times p}$ for $i \in [1, q-1]$ are invertible constant matrices. One can easily show that the corresponding control law $\nu(e)$ in the expression of the control law (7) is then given by (see e.g. Hajji et al. [2007]):

$$\nu(e) = -\left(\prod_{i=1}^{q-1} A_i\right)^{-1} B^T K_c \left(\lambda^q \bar{S} \Delta_\lambda \Lambda e\right) \tag{12}$$

$$\text{with} \ \ \Lambda = diag\left(I_p, A_1, A_1 A_2, \ldots, \prod_{i=1}^{q-1} A_i\right) \tag{13}$$

4. ADAPTIVE OUTPUT FEEDBACK CONTROL

The adaptive output feedback control we are concerned by is obtained by invoking the certainty equivalence principle while using the adaptive observer proposed in Maatoug et al. [2005]. This leads to the following adaptive observer based state feedback control law

$$u(\hat{x}, \hat{\rho}) = (b(\hat{x}))^+ \left(\dot{x}_r^q - g^q(x_r) - \Psi^q(s, x_r)\hat{\rho} + \nu(\hat{e})\right) \tag{14}$$

$$\begin{aligned} \text{with} \ \ \nu(\hat{e}) &= -B^T K_c \left(\lambda^q \bar{S} \Delta_\lambda \hat{e}\right) \\ &= -B^T K_c \left(\lambda^q \bar{S} \Delta_\lambda (\hat{x} - x_r)\right) \\ \text{and} \ \ \dot{\hat{x}} &= A\hat{x} + Bu(\hat{x}, \hat{\rho}) + g(\hat{x}) + \Psi(s, \hat{x})\hat{\rho} \\ &\quad - \theta \Delta_\theta^{-1} \left(S^{-1} + \Upsilon \ P \ \Upsilon^T\right) C^T C (\hat{x} - x) \\ \dot{\hat{\rho}} &= -\theta^2 \Omega_\theta^{-1} \ P \ \Upsilon^T \ C^T C (\hat{x} - x) \end{aligned} \tag{15}$$

where
1) $\hat{e} \in \mathbb{R}^n$ denotes the state tracking error estimate, Δ_θ is a diagonal matrix defined in a similar as the matrix Δ_λ (equation 8) for the positive scalar $\theta > 0$ and the matrix S is given by (11).
2) Ω_θ is a $m \times m$ diagonal matrix defined by:

$$\Omega_\theta = diag\left[1, \frac{1}{\theta^{\nu_1}}, \ldots, \frac{1}{\theta^{\nu_{m-1}}}\right] \tag{16}$$

ν_k's, $k = 1, \ldots, m-1$ are positive integers which are chosen such that each term of the matrix $\Delta_\theta \Psi(x) \Omega_\theta^{-1}$ is a polynomial in $\frac{1}{\theta}$ (see e.g. Maatoug et al. [2005]).
3) $\Upsilon_\xi(t)$ is a $n \times m$ matrix satisfying the following Ordinary Differential Equation (ODE):

$$\dot{\Upsilon}_\xi(t) = \theta\left(A - S^{-1} C^T C\right) \Upsilon_\xi(t) + \theta \Delta_\theta \Psi(s, \xi) \Omega_\theta^{-1} \tag{17}$$

for any $\xi \in \mathbb{R}^n$.
4) $P(t)$ is the $m \times m$ symmetric matrix governed by the following ODE:

$$\dot{P}(t) = -\theta P(t) \Upsilon_\xi^T(t) C^T C \Upsilon_\xi(t) P(t) + \theta P(t) \tag{18}$$

$P(t_0) \in \mathbb{R}^m \times \mathbb{R}^m$ is chosen symmetric positive definite and the matrix $\Upsilon_\xi(t)$ is governed by (17).
Bearing in mind the tracking error state equation (6) and the adaptive output feedback control law (14)-(15), the adaptive output feedback control system can be described by the state equation of the tracking error estimate, $\hat{e}$ together with that of the observation error, $\varepsilon = \hat{x} - x$ and $\tilde{\rho} = \hat{\rho} - \rho$, respectively given by

$$\begin{aligned} \dot{\hat{e}} &= A\hat{e} + B\nu(\hat{e}) + g(\hat{e} + x_r) - g(x_r) \\ &\quad + \left(\Psi(s, \hat{e} + x_r) - \Psi(s, x_r)\right)\hat{\rho} \\ &\quad - \theta \Delta_\theta^{-1} \left(S^{-1} + \Upsilon \ P \ \Upsilon^T\right) C^T C \varepsilon \\ \dot{\varepsilon} &= A\varepsilon + B\left(b(\hat{x}) - b(x)\right) u(\hat{x}) + g(\hat{x}) - g(x) \\ &\quad + \Psi(s, \hat{x})\hat{\rho} - \Psi(s, x)\rho - \theta \Delta_\theta^{-1} \left(S^{-1} + \Upsilon \ P \ \Upsilon^T\right) C^T C \varepsilon \\ \dot{\tilde{\rho}} &= -\theta^2 \Omega_\theta^{-1} \ P \ \Upsilon^T \ C^T C \varepsilon \end{aligned} \tag{19}$$

Before stating our main result, one assumes the additional assumptions :
($\mathcal{A}3$) The matrix $\Psi(s, x(t))$ is uniformly bounded for all $t \geq 0$ and $x \in \mathbb{R}^n$.
($\mathcal{A}4$) The function g(x) and $\Psi(s, x))$ are globally Lipschitz with respect to x uniformly in s.
($\mathcal{A}5$) The output designed reference signals are such that for any $\hat{x}(0) \in \mathbb{R}^n$, $\hat{\rho}(0) \in \mathbb{R}^m$, the matrix $C\Upsilon_{\hat{x}}$, where $\Upsilon(\hat{x})$ is governed by (17) with $\xi \equiv \hat{x}$, is persistently exciting.

Remark *4.1.* Notice that, according to assumptions ($\mathcal{A}3$) and ($\mathcal{A}5$), it is well known that equation (18) admits a unique symmetric positive definite uniformly bounded matrix $P(t)$.

The resulting control system achieves the required tracking performances as pointed out by the following fundamental result.

Theorem *4.1.* The control system corresponding to the adaptive output feedback controller (19) leads to an asymptotically exponentially vanishing tracking , i.e. $\lim_{t\to\infty} e(t) = 0$, provided that the assumptions $\mathcal{A}1$ to $\mathcal{A}5$ hold.

Proof. One shall firstly show that the observation error converges exponentially to zero, i.e. $\lim_{t\to\infty} \varepsilon(t) = 0$ and $\lim_{t\to\infty} \tilde{\rho}(t) = 0$, and then conclude to the exponential convergence to zero of the tracking error estimate, i.e. $\lim_{t\to\infty} \hat{e}(t) = 0$. The first part is established from a Lyapunov function using the estimation errors $\bar{\varepsilon} = \Delta_\theta \varepsilon$ and $\bar{\rho} = \frac{1}{\theta}\Omega_\theta \tilde{\rho}$ which are governed by the equations

$$\begin{aligned}\dot{\bar{\varepsilon}} &= \theta\left(A - S^{-1}C^TC\right)\bar{\varepsilon} + \Upsilon\dot{\bar{\rho}} \\ &+ \Delta_\theta B\left(b(\hat{x}) - b(x)\right)u(\hat{x}) + \Delta_\theta\left(g(\hat{x}) - g(x)\right) \\ &+ \Delta_\theta\left(\Psi(s,\hat{x}) - \Psi(s,x)\right)\rho + \theta\Delta_\theta\Psi(s,\hat{x})\Omega_\theta^{-1}\bar{\rho}\end{aligned}$$

$$\dot{\bar{\rho}} = -\theta P\Upsilon^TC^TC\bar{\varepsilon}$$

Now, define : $\eta = \bar{\varepsilon} - \Upsilon\bar{\rho}$ where the matrix $\Upsilon \in \mathbb{R}^{n_pq\times m}$ is governed by equation (17) with $\xi \triangleq \hat{x}$. One can show that:

$$\begin{aligned}\dot{\eta} &= \theta\left(A - S^{-1}C^TC\right)\eta + \Delta_\theta\left(g(\hat{x}) - g(x)\right) \\ &+ \Delta_\theta B\left(b(\hat{x}) - b(x)\right)u(\hat{x}) + \Delta_\theta\left(\Psi(s,\hat{x}) - \Psi(s,x)\right)\rho\end{aligned}$$

Set $V_1(\eta) = \eta^TS\eta$, $V_2(\bar{\rho}) = \bar{\rho}^TP^{-1}\bar{\rho}$ and let $V_o(\eta,\bar{\rho}) = \theta^{2q}(V_1 + V_2)$ be the Lyapunov candidate function. Using the algebraic Lyapunov equation (11), one can show that

$$\begin{aligned}\dot{V}_1 \;=\; &-\theta V_1 - \theta\eta^TC^TC\eta + 2\eta^TS\Delta_\theta\left(g(\hat{x}) - g(e)\right) \\ &+2\eta^TS\Delta_\theta B\left(b(\hat{x}) - b(x)\right)u(\hat{x}) \\ &+2\eta^TS\Delta_\theta\left(\Psi(s,\hat{x}) - \Psi(s,x)\right)\rho\end{aligned} \quad (20)$$

It is obvious that $\|\bar{\varepsilon}\| \leq \|\eta\| + \|\Upsilon(t)\|\|\bar{\rho}\|$. In others respects, using triangular structures and the Lipschitz conditions on $\Psi(x)$, $g(x)$ as well as the boundedness of the input u, one can show that :

$$\|\Delta_\theta B\left(b(\hat{x}) - b(x)\right)u(\hat{x})\| \leq \alpha\bar{\varepsilon} \leq C_1\|\eta\| + C_2\|\bar{\rho}\|$$
$$\|\;\Delta_\theta\left(g(\hat{x}) - g(x)\right)\| \leq \beta\bar{\varepsilon} \leq C_3\|\eta\| + C_4\|\bar{\rho}\|$$
$$\|\Delta_\theta\left(\Psi(s,\hat{x}) - \Psi(s,(x))\,\rho\right\| \leq \gamma\bar{\varepsilon} \leq C_5\|\eta\| + C_6\|\bar{\rho}\|$$

where α, C_1, C_2, C_3, C_4, C_5 and C_6 are positive constants which do not depend on θ.

Otherwise, one has:

$$\dot{V}_2 = -\theta V_2 - 2\theta(\Upsilon\bar{\rho})^TC^TC(\eta + \Upsilon\bar{\rho}) + \theta(\Upsilon\bar{\rho})^TC^TC\Upsilon\bar{\rho}$$

Then, one can show that

$$\dot{V}_1 + \dot{V}_2 \leq -(\theta - k_1)V_1 - \theta V_2 + k_2\sqrt{V_1}\sqrt{V_2}$$

where $k_1, k_2 > 0$ are positive constants which do not depend on θ. Finally, set $V_1^\star = (\theta - k_1)V_1$ and $V_2^\star = \theta V_2$.

One has $\dot{V}_1 + \dot{V}_2 \leq -(1 - \dfrac{k_2}{\sqrt{2\theta(\theta - k_1)}})(V_1^\star + V_2^\star)$ (21)

Now, choose θ such that $(1 - \dfrac{k_2}{\sqrt{2\theta(\theta - k_1)}}) > 0$, one obtains:

$$\dot{V}_o(\eta,\bar{\rho}) \leq -(\theta - k_1 - \frac{k_2}{\sqrt{2\theta}})V_o(t) \quad (22)$$

Remark *4.2.* Since ρ is bounded and $\lim_{t\to\infty} \tilde{\rho}(t) = 0$ it follows that $\hat{\rho}$ is bounded.

The second part of the proof is carried out from a Lyapunov function involving the state estimate $\bar{e} = \lambda^q\Delta_\lambda\hat{e}$. From the fact that $\lambda^q\Delta_\lambda B = \lambda B$ and $C\Delta_\theta = C$, one can show:

$$\begin{aligned}\dot{\bar{e}} &= \lambda A\bar{e} - \lambda BB^TK_c\left(\bar{S}\bar{e}\right) + \lambda^q\Delta_\lambda\left(g(\hat{e} + x_r) - g(x_r)\right) \\ &+ \lambda^q\Delta_\lambda\left(\Psi(s,\hat{e} + x_r) - \Psi(s,x_r)\right)\hat{\rho} \\ &- \theta\lambda^q\Delta_\lambda\Delta_\theta^{-1}\left(S^{-1} + \Upsilon P\Upsilon^T\right)C^TC\bar{\varepsilon}\end{aligned}$$

Let us now show that $V_c : \bar{e} \mapsto V_c(\bar{e}) = \lambda^{-2q}\bar{e}^T\bar{S}\bar{e}$ is a Lyapunov function for the control system. Proceeding as above, one can easily derive the following property on the derivative of the function V_c:

$$\begin{aligned}\dot{V}_c \;\leq\; &-\left(\lambda - \gamma_c\right)V_c \\ &+\; 2\|\theta\lambda^{-q}\bar{e}^T\bar{S}\Delta_\lambda\Delta_\theta^{-1}\left(S^{-1} + \Upsilon P\Upsilon^T\right)C^TC\bar{\varepsilon}\|\end{aligned} \quad (23)$$

where γ_c is a positive scalar which does not depend on λ. Now, from the fact that Υ and P are uniformly bounded and using the expression (9) of the matrix $\bar{S}$, one can show that for $\theta \geq 1$ and $\lambda \geq 1$, one gets

$$\begin{aligned}&2\|\theta\lambda^{-q}\bar{e}^T\bar{S}\Delta_\lambda\Delta_\theta^{-1}\left(S^{-1} + \Upsilon P\Upsilon^T\right)C^TC\bar{\varepsilon}\| \leq \\ &2\lambda_{min}^2(S)\theta^q\lambda^{-q}\|\bar{\varepsilon}\|\|\bar{e}\| \leq k_5\sqrt{V}_o\sqrt{V_c}\end{aligned} \quad (24)$$

where $k_5 > 0$ is a constant which does not depend on λ.

Combining inequalities (23) and (24), one obtains

$$\dot{V}_c \;\leq\; -\left(\lambda - \gamma_c\right)V_c + k_5\sqrt{V_o}\sqrt{V_c} \quad (25)$$

This makes it possible to conclude to the exponential convergence of the tracking error estimate provided that $\lambda > \gamma_c$ and $\theta > \gamma_o(\lambda)$ as pointed out by the following property

$$\begin{aligned}\sqrt{V_c(z(t))} \;\leq\; &e^{-\left(\frac{\lambda - \gamma_c}{2}\right)t}\sqrt{V_c(z(0))} + \\ &\frac{k_5}{\theta - \lambda - \gamma_o(\lambda) + \gamma_c}\left(e^{-\left(\frac{\theta - \gamma_o(\lambda)}{2}\right)t} - e^{-\left(\frac{\lambda - \gamma_c}{2}\right)t}\right)\end{aligned}$$

Remark *4.3.* The adopted high gain concept allows to recover the famous separation theorem for the class of nonlinear system under consideration. Furthermore, one can easily check that the result 4.1 is still valid if one uses the measure of the tracking error, rather than its estimate, in the adaptive output feedback control law (14)-(15). This legitimates the use of a reduced order observer.

5. PARTICULAR DESIGN FUNCTIONS

The control law involves a gain depending on the bounded design function K_c which is completely characterized by the fundamental property (10). Some useful design functions are given below to emphasize the unifying feature of the proposed high gain concept.

• The usual high gain design function given by $K_c(\xi) = k_c\xi$ where k_c is a positive scalar satisfying $k_c \geq \frac{1}{2}$.
• The design function involved in the actual sliding mode framework $K_c(\xi) = k_c \; sign(\xi)$ where k_c is a positive scalar and 'sign' is the usual signum function (for $x \in \mathbb{R}^n$ with components $x_i \in \mathbb{R}$, $sign(x)$ is a vector and its ith component is $sign(x_i)$).
• The design functions that are commonly used in the sliding mode practice, namely

$$K_c(\xi) \;=\; k_c \; tanh(k_o\xi) \tag{26}$$

where $tanh$ denotes the hyperbolic tangent function and k_c and k_o are positive scalars.

6. FILTERED INTEGRAL ACTION

One can easily incorporate a filtered integral action into the proposed state feedback control design, for performance enhancement considerations, by simply introducing suitable state variables as follows

$$\begin{cases} \dot{\sigma}^f = e^f \\ \dot{e}^f = -\Gamma e^f + \Gamma e^1 \end{cases} \tag{27}$$

where $\Gamma = Diag\{\gamma_i\}$ is a design matrix that has to be specified according to the desired filtering action. The state feedback gain is then determined from the control design model

$$\begin{cases} \dot{e}_a = A_a e_a + g_a(x_r, e_a) - g_a(x_r, 0) \\ \qquad + B_a\left(b(e + x_r)u_a(e_a) - b(x_r)u_r\right) \\ \qquad + \left(\Psi_a(s, x_r, e_a) - \Psi_a(s, x_r, 0)\right)\rho \\ y_a = \sigma^f \end{cases} \tag{28}$$

with $e_a = \begin{pmatrix} \sigma^f \\ e^f \\ e \end{pmatrix}$, $A_a = \begin{pmatrix} 0 & I_p & 0 \\ 0 & 0 & \Gamma \\ 0 & 0 & A \end{pmatrix}$, $B_a = \begin{pmatrix} 0_p \\ 0_p \\ B \end{pmatrix}$

$g_a(x_r, e_a) = \begin{pmatrix} 0_p \\ -\Gamma e^f \\ g(e + x_r) \end{pmatrix}$, $\Psi_a(s, x_r, e_a) = \begin{pmatrix} 0_p \\ 0_p \\ \Psi(s, e + x_r) \end{pmatrix}$

Indeed, the control design model structure (28) is similar to that of the error system (6) and hence the underlying state feedback control design is the same. The adaptive output feedback control law incorporating a filtered integral action is then given by

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu\left(\hat{x}\right) + g(\hat{x}) + \Psi(s, \hat{x})\hat{\rho} \\ \qquad - \theta\Delta_\theta^{-1}\left(S^{-1} + \Upsilon P \Upsilon^T\right) C^T C\left(\hat{x} - x\right) \\ \dot{\hat{\rho}} = -\theta^2\Omega_\theta^{-1} \; P \; \Upsilon^T \; C^T C\left(\hat{x} - x\right) \\ u(\hat{e}_a, \hat{\rho}) = \left(b(\hat{x})\right)^+ \left(\dot{x}_r^q - g^q(x_r) - \Psi^q(s, x_r)\hat{\rho} + \nu(\hat{e}_a)\right) \\ \nu(\hat{e}_a) = -\Gamma^{-1}B_a^T K_{ac}(\lambda^{q+2}\bar{S}_a\Delta_{a\lambda}\Lambda\hat{e}_a) \end{cases} \tag{29}$$

$$\text{with } \hat{e}_a = \begin{pmatrix} \sigma^f \\ e^f \\ \hat{e} \end{pmatrix}; \; \Lambda = (I_p, I_p, \Gamma, \ldots, \Gamma)$$

$$\Delta_{a\lambda} = diag\left(I_p, \frac{1}{\lambda}\; I_p, \ldots, \frac{1}{\lambda^q}\; I_p, \frac{1}{\lambda^{q+1}}\; I_p\right)$$

where $\bar{S}_a$ is the unique symmetric positive definite matrix solution of the following Lyapunov algebraic equation

$$\bar{S}_a + \bar{S}_a\bar{A}_a + \bar{A}_a^T\bar{S}_a = \bar{S}_a\bar{B}_a\bar{B}_a^T\bar{S}_a \tag{30}$$

and $K_{ac} : \mathbb{R}^{n+2p} \to \mathbb{R}^{n+2p}$ is a bounded design function satisfying a similar inequality as (10), namely

$$\xi_a^T B_a B_a^T K_{ac}(\xi_a) \;\geq\; \frac{1}{2}\; \xi_a^T B_a B_a^T \xi_a \;\forall\; \xi_a \in \Omega \tag{31}$$

where Ω is any compact subset of $\mathbb{R}^{n+2p}$.
It can be easily shown that the resulting adaptive output feedback control system is globally stable and performs an asymptotic rejection of state and/or output step like disturbances.

7. ILLUSTRATIVE EXAMPLE

Let consider an academic tracking problem for the nonlinear double integrator which belongs to the class of systems (1) described by

$$\begin{cases} \dot{x}_1 = x_2 + \left(-x_1^3 + sin\omega_1 t\right)\; \rho_1 \\ \dot{x}_2 = (2 + tanh(x_2))u - atan(x_2)\; \rho_1 + \dfrac{cos\omega_1 t}{1 + x_2^2}\; \rho_2 \\ y = x_1 \end{cases}$$

where the state vector is $x = [x_1\; x_2]^T \in \mathbb{R}^2$, $\rho = [\rho_1\; \rho_2]^T \in \mathbb{R}^2$. The value of w_1 used in simulation is 20. The desired output reference trajectory is generated from a second order generator with unitary static gain and two equal poles $p_1 = p_2 = -5$ which input sequence is a rectangular wave. An adaptive output feedback control with a filtered integral action of the form (29) has been synthesized and intensive simulation study has been made using all the design functions that has been described above. As the performances were almost comparable, one will present only those obtained with the design function given by the expression (26). The design parameters have been specified as follows

$$k_c = 1,\; k_o = 7,\; \lambda = 4.51,\; \tau = 50 \; and \; \theta = 15;$$

Figure 1 shows the dynamical behavior of the control system together with its the corresponding observation error, i.e. there is no modelling errors. Two remarks are worth to be mentioned. Firstly, the proposed controller achieve the required tracking performances . Secondly, the obtained results clearly show the good performances of the adaptive observer in providing satisfactory estimates of the states as well as of the unknown parameters.

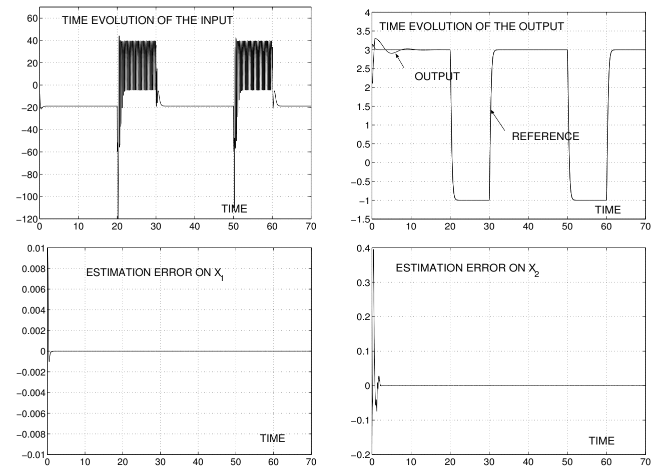

Fig. 1. Performances of the adaptive output controller

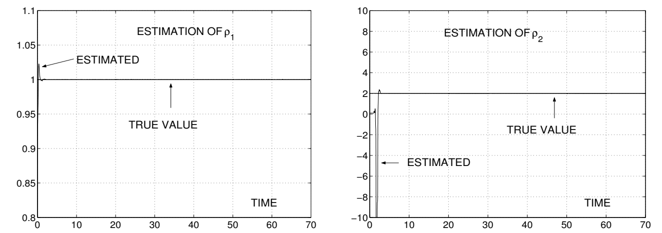

Fig. 2. Comparison of parameter estimates with their respective true values

8. CONCLUSION

The motivation of this paper was twofold. Firstly, a unified high gain state feedback control design framework has been developed to address an admissible tracking problem for a class of controllable and observable nonlinear systems. Such a framework has been particularly suggested thanks to the duality from the high gain system observation. The unifying feature is provided through a suitable design function that allows to rediscover all those well known high gain control methods, namely the sliding modes control. A Lyapunov approach has been adopted to show that the required tracking performances are actually handled. Secondly, the proposed state feedback control is combined with adaptive observer to provide an adaptive output feedback controller according to the well known separation theorem. Of practical purpose, a filtered integral action has been incorporated into the proposed control design to deal with step like disturbances while ensuring an adequate insensitivity to measurement noise. The effectiveness of the proposed adaptive output feedback control method has been emphasized throughout simulation results involving a nonlinear double integrator.

There are two remarks that are worth to be mentioned. Firstly, the proposed adaptive control design framework can be extended to systems with asymptotically stable zeros dynamics thanks to an appropriate technical effort. Secondly, the persistent excitation requirement can be removed thanks to a useful parameter adaptation robustness design feature, namely a parameter projection on an a priori known compact convex subset of the involved parameter space or a parameter adaptation freezing throughout an a priori specified dead zone.

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