Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Observer-based output feedback controller for a class of nonlinear systems

S. Hajji [*,**] **M. Farza** [**] **M. M'Saad** [**] **M. Kamoun** [*]

[*] *ENIS, Département de Génie électrique, BP W, 3038 Sfax, Tunisia (e-mail: Med.KAMOUN@enis.rnu.tn)*
[**] *GREYC, UMR 6072 CNRS, Université de Caen, ENSICAEN, 6 Boulevard Maréchal Juin, 14050 Caen Cedex, France (e-mail: mfarza,msaad@greyc.ensicaen.fr)*

Abstract: The design of an observer based output feedback controller for a class of uniformly observable nonlinear systems with an admissible tracking capability, is proposed. Two fundamental features of the proposed control scheme are worth to be mentioned. The first one consists in the high gain nature of the underlying state feedback control and observer designs. More specifically, a unified high gain control design framework is proposed thanks to the duality between control and observation. The second feature consists in incorporating a filtered integral action into the control design. The filtering is mainly motivated by measurement noise sensitivity reduction while the integral action allows to achieve a robust offset free performance in the presence of step like disturbances. The features of the proposed approach are illustrated using the induction motor model and simulation are performed in order to highlight the performance of the underlying observer based output feedback controller.

Keywords: Nonlinear system, Output feedback control, High gain control, Sliding mode control, High gain observer, Filtered integral action, Induction motor.

1. INTRODUCTION

The problems of observation and control of nonlinear systems have received a particular attention throughout the last four decades (Agrawal and Sira-Ramirez [2004], Gauthier and Kupka [2001], Isidori [1995], Nijmeijer and van der Schaft [1991], Krstič et al. [1995], Sepulchre et al. [1997]). Considerable efforts were dedicated to the analysis of the structural properties to understand better the concepts of controllability and of observability of nonlinear systems (Hammouri and Farza [2003], Gauthier and Kupka [2001], Rajamani [1998], Isidori [1995], Fliess and Kupka [1983], Nijmeijer [1981], Gauthier and Bornard [1981], Fliess and Kupka [1983]). Several control and observer design methods were developed thanks to the available techniques, namely feedback linearisation, flatness, high gain, variable structure, sliding modes and backstepping (Farza et al. [2005], Agrawal and Sira-Ramirez [2004], Boukhobza et al. [2003], Gauthier and Kupka [2001], Fliess et al. [1999], Sepulchre et al. [1997], Isidori [1995], Krstič et al. [1995]). The main difference between these contributions lies in the design model, and henceforth the considered class of systems, and the nature of stability and performance results. A particular attention has been devoted to the design of state feedback control laws incorporating an observer satisfying the separation principle requirements as in the case of linear systems (Mahmoud and Khalil [1996]). Furthermore, various control design features have been used to enhance the performance, namely the robust compensation of step like disturbances by incorporating an integral action in the control design (Seshagiri and Khalil [1996]) and the filtering to reduce the control system sensitivity in the presence of noise measurements.

In this paper, one proposes an output feedback controller for a class of uniformly observable nonlinear systems. More specifically, one will address an admissible tracking problem for minimum phase nonlinear systems. The output feedback controller is obtained by simply combining an appropriate high gain state feedback control with a standard high gain observer (Gauthier and Kupka [2001], Farza et al. [2005]). The state feedback control design was particularly suggested from the high gain observer design bearing in mind the control and observation duality. Of particular interest, the controller gain involves a well defined design function which provides a unified framework for the high gain control design, namely several versions of sliding mode controllers are obtained by considering particular expressions of the design function. Furthermore, it is shown that a filtered integral action can be simply incorporated into the control design to carry out a robust compensation of step like disturbances while reducing appropriately the noise control system sensitivity .

This paper is organized as follows. The problem formulation is presented in the next section. Section 3 is devoted to the state feedback control design with a full convergence analysis of the tracking error in a free disturbances case. The output feedback controller is presented in section 4 where the main result of this contribution is given. Section 5 emphasizes the high gain unifying feature of the proposed control design. The possibility to incorporate a filtered integral action into the control design is shown in section 6. The main features of the proposed approach are illustrated in section 7 through an example dealing with the induction motor. Simulation results are given throughout this section in order to highlight the performance of the proposed approach.

2. PROBLEM FORMULATION

One seeks to an admissible tracking problem for the following class of MIMO uniformly observable systems

$$\begin{cases} \dot{x} = Ax + \varphi(u, x) \\ y = Cx = x^1 \end{cases} \tag{1}$$

$$\text{with } x = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix}, \ \varphi(u, x) = \begin{pmatrix} \varphi^1(u, x^1) \\ \varphi^2(u, x^1, x^2) \\ \vdots \\ \varphi^{q-1}(u, x^1, \ldots, x^{q-1}) \\ \varphi^q(u, x) \end{pmatrix} \tag{2}$$

$$A = \begin{pmatrix} 0 & I_{(q-1)n_1} \\ 0 & 0 \end{pmatrix}, \ C = \begin{pmatrix} I_{n_1} & 0_{n_1} & \ldots & 0_{n_1} \end{pmatrix} \tag{3}$$

where the state $x \in \vartheta$ an open subset $\mathbb{R}^n$ with $x^k \in \mathbb{R}^{n_1}$ ($n_1 = p$), the input $u \in U$ a compact set of $\mathbb{R}^m$.
Set $z^1(t) = h(x) \in \mathbb{R}^p$ where $h(x)$ a smooth function. The control problem to be addressed consists in an asymptotic tracking of a reference trajectory of $z^1(t)$ that will be denoted by $\{z_d^1(t)\} \in \mathbb{R}^p$, i.e.

$$\lim_{t\to\infty} \left(z^1(t) - z_d^1(t)\right) = 0 \tag{4}$$

To deal with the tracking problem, one shall need some hypotheses which will be stated at due courses. At this step, one assumes the following:
(H1) There exists a lipschitzian diffeomorphism $\Phi = \begin{pmatrix} \Phi_z \\ \Phi_\xi \end{pmatrix}$: $\mathbb{R}^n \to \mathbb{R}^n$, $x \mapsto \Phi(x) = \begin{pmatrix} z = \Phi_z(x) \\ \xi = \Phi_\xi(x) \end{pmatrix}$ that puts system (1) under the following form:

$$\begin{cases} \dot{z} = A_r z + B_r \left(b(\xi, z)u + g(\xi, z)\right) + \psi(z) \\ \dot{\xi} = \eta(\xi, z, u) \end{cases} \tag{5}$$

$$\text{with } z = \begin{pmatrix} z^1 \\ z^2 \\ \vdots \\ z^r \end{pmatrix}, \ \psi(z) = \begin{pmatrix} \psi^1(z^1) \\ \psi^2(z^1, z^2) \\ \vdots \\ \psi^{r-1}(z^1, \ldots, z^{r-1}) \\ 0 \end{pmatrix} \tag{6}$$

$$A_r = \begin{pmatrix} 0 & I_{(r-1)p} \\ 0 & 0 \end{pmatrix} \text{ and } B_r = \begin{pmatrix} 0_p \\ \vdots \\ 0_p \\ I_p \end{pmatrix} \tag{7}$$

are respectively $(rp)\times(rp)$ and $(rp)\times p$ matrices; $z^k \in \mathbb{R}^p$, $k = 1, \ldots, r$; $\xi \in \mathbb{R}^{n-rp}$, $g(\xi, z) \in \mathbb{R}^p$ and $b(\xi, z)$ is a rectangular matrix of dimension $p \times m$ with $p \leq m$.
(H2) The function ψ is globally lipschitz in z and the functions b and g are globally lipschitz in z uniformly in ξ. Moreover, the matrix $b(\xi, z)$ is of full row rank and satisfies the following condition

$$\begin{aligned} &\exists \underline{b}^2, \bar{b}^2 > 0, \ \forall \xi \in \mathbb{R}^{n-rp}, \ \forall z \in \mathbb{R}^{rp} : \\ &\qquad \underline{b}^2 I_p \leq b(\xi, z) \left(b(\xi, z)\right)^T \leq \bar{b}^2 I_p \end{aligned} \tag{8}$$

Unfortunately, as all results for the class of systems (5), we do require a minimum phase assumption for the inverse dynamics which we state as follows (Praly [2003], Krishnamurthy et al. [2003]):
(H3) The system : $\dot{\xi} = \eta\left(\xi, v_1, (b(\xi, v_1))^+ (v_2 - g(\xi, v_3))\right)$ with input (v_1, v_2, v_3) and state ξ is Input-to-State Stable where $(\cdot)^+$ denotes the right inverse of $(\cdot)$.
Taking into account the structure of system (5), it is possible to derive the subsystem state trajectory $\{z_d(t)\} \in \mathbb{R}^{rp}$ and the associated input sequence $\{u_d(t)\}$ corresponding to the desired trajectory $\{z_d^1(t)\} \in \mathbb{R}^p$. This allows to define an admissible reference model as follows

$$\dot{z}_d = A_r z_d + B_r (b(\xi, z_d)u_d + g(\xi, z_d)) + \psi(z_d) \tag{9}$$

where $z_d = \begin{pmatrix} z_d^1 \\ z_d^2 \\ \vdots \\ z_d^r \end{pmatrix} \in \mathbb{R}^{rp}$ is the reference model state.

Notice that the components $z_d^k \in \mathbb{R}^p$, $k = 2, \ldots, r$ as well as the associated input $u_d \in \mathbb{R}^m$ can be computed from system (9) as follows

$$\begin{cases} z_d^k = \dot{z_d}^{k-1} - \psi^{k-1}(z_d^1, \ldots, z_d^{k-1}) \ for \ k \in [2, r] \\ u_d = (b(\xi, z_d))^+ \left(\dot{z_d}^r - g(\xi, z_d)\right) \end{cases} \tag{10}$$

By assuming that the reference trajectory is smooth enough, one can recursively determine the state and the input of the reference model from the reference trajectory and its first time derivatives, i.e. $z_d^{1(i)} = \dfrac{d^i z_d^1}{dt^i}$ $for \ i \in [1, r-1]$ (see Hajji et al. [2007]).
The tracking problem (4) can be hence turned to a state trajectory regulation problem defined by

$$\lim_{t\to\infty} e(t) = 0 \ \text{ where } \ e(t) = (z(t) - z_d(t)) = 0 \tag{11}$$

Such problem can be interpreted as a regulation problem for the tracking error system obtained from the system and model reference state representations (5) and (9), respectively:

$$\begin{aligned} \dot{e} &= A_r e + B_r \left(b(\xi, e + z_d)u - b(\xi, z_d)u_d\right) \\ &\quad + B_r \left(g(\xi, e + z_d) - g(\xi, z_d)\right) + \psi(e + z_d) - \psi(z_d) \\ \dot{\xi} &= \eta(\xi, e + z_d, u) \end{aligned} \tag{12}$$

3. STATE FEEDBACK CONTROL

As it was previously mentioned, the proposed state feedback control design is particularly suggested by the duality from the high gain observer design proposed in Farza et al. [2005]. The underlying state feedback control law is then given by

$$\begin{cases} \nu(e) = -B_r^T K_c \left(\bar{S}\Gamma_\lambda e\right) \\ u = (b(\xi, z))^+ \left(b(\xi, z_d)u_d + \nu(e)\right) \\ \quad = (b(\xi, e + z_d))^+ \left(\dot{z}_d^r - g(\xi, z_d) + \nu(e)\right) \end{cases} \tag{13}$$

where Γ_λ is the block diagonal matrix defined by

$$\Gamma_\lambda = \lambda^r \Delta_\lambda = diag\left(\lambda^r I_p, \lambda^{r-1}\ I_p, \ldots, \lambda\ I_p\right) \quad (14)$$

$$\text{i.e. } \Delta_\lambda = diag\left(I_p, \frac{1}{\lambda}\ I_p, \ldots, \frac{1}{\lambda^{r-1}}\ I_p\right) \quad (15)$$

where $\lambda > 0$ is a positive scalar, $\bar{S}$ is the unique symmetric positive definite solution of the following algebraic Lyapunov equation

$$\bar{S} + A_r^T \bar{S} + \bar{S} A_r = \bar{S} B_r B_r^T \bar{S} \quad (16)$$

and $K_c : \mathbb{R}^{rp} \mapsto \mathbb{R}^{rp}$ is a bounded design function satisfying the following property

$$\forall \omega \in \Omega \text{ one has } \omega^T B_r B_r^T K_c(\omega) \ \geq\ \frac{1}{2}\ \omega^T B_r B_r^T \omega \quad (17)$$

where Ω is any compact subset of $\mathbb{R}^{rp}$.

Remark *3.1.* From the fact that the following algebraic Lyapunov equation

$$S + A_r^T S + S A_r = C_r^T C_r \quad (18)$$

where $C_r = B_r^T$, has a unique symmetric positive definite solution S Gauthier et al. [1992], one can deduce that equation (16) has a unique symmetric positive definite solution $\bar{S}$. Moreover, one can show that (Hajji et al. [2007])

$$B_r^T \bar{S} = [C_q^q I_p\ C_q^{q-1} I_p\ \ldots\ C_r^1 I_p] \quad (19)$$

The above state feedback control law satisfies the tracking objective (11) as pointed out by the following fundamental result

Theorem *3.1.* Under assumptions $H1$ to $H3$, the tracking error $e(t)$ of system (12) generated from the input sequence given by (13)-(17) converges globally exponentially to zero for relatively high values of λ.

Proof. The proof is similar to that given in Hajji et al. [2007] by considering the following Lyapunov function: $V(\bar{e}) = \bar{e}^T \bar{S} \bar{e}$ where $\bar{e} = \Gamma_\lambda e$.

Remark *3.2.* Consider the case where the matrix A_r has the following structure:

$$A_r = \begin{pmatrix} 0 & A_1 & 0 & \ldots & 0 \\ 0 & 0 & A_2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & A_{r-1} \\ 0 & \ldots & \ldots & 0 & 0 \end{pmatrix}$$

where $A_i \in \mathcal{R}^{p\times p}$ for $i \in [1, r-1]$ are invertible constant matrices. One can easily show that the corresponding control law $\nu(e)$ in the expression of the control law (13) is then given by (see e.g. Hajji et al. [2007])

$$\nu(e) = -\left(\prod_{i=1}^{q-1} A_i\right)^{-1} B_r^T K_c\left(\bar{S}\Gamma_\lambda \Lambda e\right) \quad (20)$$

$$\text{with} \quad \Lambda = diag(I_p, A_1, A_1A_2, \ldots, \prod_{i=1}^{r-1} A_i) \quad (21)$$

4. OUTPUT FEEDBACK CONTROL

The output feedback control we are concerned by is obtained by invoking the certainty equivalence principle while using the high gain observer proposed in Farza et al. [2005] which takes here the following form:

$$\dot{\hat{x}} = A\hat{x} + \varphi(u(\hat{x}), \hat{x}) - \theta \Delta_\theta^{-1} S^{-1} C^T C\left(\hat{x} - x\right) \quad (22)$$

where Δ_θ is a diagonal matrix defined in a similar way as the matrix Δ_λ (equation (15) with $r = q$) for the positive scalar $\theta > 0$ and the matrix S is given by (18).
Let $\varepsilon(t) = \hat{x}(t) - x(t)$ be the observation error. One has:

$$\dot{\varepsilon} = A\varepsilon + \varphi(u(\hat{x}), \hat{x}) - \varphi(u(\hat{x}), x) - \theta\Delta_\theta^{-1} S^{-1} C^T C\varepsilon \quad (23)$$

The equations of the observer providing the estimate, $\hat{e}$, of the tracking error can be written in (z, ξ) coordinates as follows:

$$\begin{cases} \dot{\hat{e}} = A_r \hat{e} + B_r\left(b(\hat{\xi}, \hat{e} + z_d)\hat{u} - b(\hat{\xi}, z_d)u_d\right) \\ \quad + B_r\left(g(\hat{\xi}, \hat{e} + z_d) - g(\hat{\xi}, z_d)\right) + \psi(\hat{e} + z_d) - \psi(z_d) \\ \quad - \theta\dfrac{\partial \Phi_z}{\partial x}(\Phi^c(\hat{z}, \hat{\xi}))\Delta_\theta^{-1} S^{-1} C^T C\varepsilon \\ \dot{\hat{\xi}} = \eta(\hat{\xi}, \hat{e} + z_d, \hat{u}) - \theta\dfrac{\partial \Phi_\xi}{\partial x}(\Phi^c(\hat{z}, \hat{\xi}))\Delta_\theta^{-1} S^{-1} C^T C\varepsilon \end{cases} \quad (24)$$

where
• Φ^c is the converse function of Φ, i.e. $x = \Phi^c(z, \xi)$.
• $\dfrac{\partial \Phi}{\partial x}(\hat{x}) = \begin{pmatrix} \dfrac{\partial \Phi_z}{\partial x}(\Phi^c(\hat{z}, \hat{\xi})) \\ \dfrac{\partial \Phi_\xi}{\partial x}(\Phi^c(\hat{z}, \hat{\xi})) \end{pmatrix}$ is the jacobian of the transformation Φ evaluated at $\hat{x}$.
• $\hat{u}$ is the output feedback control which is obtained using the certainty equivalence principle, i.e.

$$\begin{cases} \hat{u} \triangleq u\left(\hat{\xi}, \hat{z}\right) = \left(b(\hat{\xi}, \hat{z})\right)^+ \left(\dot{z}_d^r - g(\hat{\xi}, z_d) + \nu\left(\hat{e}\right)\right) \\ \nu\left(\hat{e}\right) = -B_r^T K_c\left(\bar{S}\Gamma_\lambda \hat{e}\right) \end{cases} \quad (25)$$

Before giving our main result and in order to prove the convergence to zero of the state estimation error ε, one needs the following classical technical hypothesis used in all works related to high observer synthesis:
(H4) The function φ is globally Lipschitz in x uniformly in u.
Now, one states the following.

Theorem *4.1.* The control system corresponding to the output feedback controller (23)-(25) leads to an asymptotically exponentially vanishing tracking , i.e. $\lim\limits_{t\to\infty} e(t) = 0$, provided that the assumptions $H1$ to $H4$ hold.

Proof. One shall firstly show that the observation error converges exponentially to zero, i.e. $\lim\limits_{t\to\infty} \varepsilon(t) = 0$, and then conclude to the exponential convergence to zero of the tracking error estimate, i.e. $\lim\limits_{t\to\infty} \hat{e}(t) = 0$. The first part is established from a Lyapunov function using the error $\bar{\varepsilon} = \theta^q \Delta_\theta \varepsilon$ which is governed by the equation

$$\dot{\bar{\varepsilon}} = \theta A\bar{\varepsilon} - \theta S^{-1} C^T C\bar{\varepsilon} + \theta^q \Delta_\theta\left(\varphi(u(\hat{x}), \hat{x}) - \varphi(u(\hat{x}), x)\right)$$

Indeed, let $V_o(\bar{\varepsilon}) = \bar{\varepsilon}^T S\bar{\varepsilon}$ be the Lyapunov candidate function for the observer. From the fact that φ is Lipschitz and from the boundedness of the design function $K_c(\hat{z})$, one can show that (Hajji et al. [2007]):

$V_o(\bar{\varepsilon}) \leq e^{-(\theta-\gamma_o)t} V_o(\bar{\varepsilon}(0))$ where $\gamma_o > 0$ is a positive constant which does not depend on θ.
The second part of the proof is carried out from a Lyapunov function involving the estimate $\bar{e} = \Gamma_\lambda \hat{e}$. The underlying description can be deduced from equation (24) as follows

$$\begin{aligned}\dot{\bar{e}} &= \lambda A_r \bar{e} - \lambda B_r B_r^T K_c \left(\bar{S}\bar{e}\right) - \theta \Gamma_\lambda \frac{\partial \Phi_z}{\partial x}(\hat{x}) \Delta_\theta^{-1} S^{-1} C^T C \varepsilon \\ &+ \lambda B_r \left(g(\hat{\xi}, \hat{e} + z_d) - g(\hat{\xi}, z_d) \right) + \Gamma_\lambda \left(\psi(\hat{e} + z_d) - \psi(z_d) \right)\end{aligned}$$

Let us now show that $V_c : \bar{e} \mapsto V_c(\bar{e}) = \lambda^{-2r} \bar{e}^T \bar{S} \bar{e}$ is a Lyapunov function for the control system. Proceeding as in Hajji et al. [2007], one can show that

$$\begin{aligned}\dot{V}_c &\leq -(\lambda - \gamma_c) V_c \\ &+ 2\|\theta^{1-q} \lambda^{-r} \bar{e}^T \bar{S} \Delta_\lambda \frac{\partial \Phi_z}{\partial x}(\hat{x}) \Delta_\theta^{-1} S^{-1} C^T C \bar{\varepsilon}\| \quad (26)\end{aligned}$$

where γ_c is a positive scalar. Since $\Phi(x)$ is globally Lipschitz, one has for $\theta, \lambda \geq 1$:

$$\|\theta^{1-q} \lambda^{-r} \bar{e}^T \bar{S} \Delta_\lambda \frac{\partial \Phi_z}{\partial x}(\hat{x}) \Delta_\theta^{-1} S^{-1} C^T C \bar{\varepsilon}\| \leq k \lambda^{-r} \|\bar{\varepsilon}\| \|\bar{e}\| \quad (27)$$

where $k > 0$ is a positive constant that does not depend on θ, nor λ. Combining the inequalities (26) and (27), one obtains

$$\begin{aligned}\dot{V}_c &\leq -(\lambda - \gamma_c) V_c + 2k\rho \|\bar{\varepsilon}\| \|\lambda^{-r} \bar{e}\| \\ &\leq -(\lambda - \gamma_c) V_c + c\sqrt{V_o}\sqrt{V_c} \quad (28)\end{aligned}$$

where $c > 0$ is a positive constant that does not depend on θ, nor λ. This leads to

$$\begin{aligned}\sqrt{V_c(\bar{e}(t))} &\leq e^{-\left(\frac{\lambda-\gamma_c}{2}\right)t} \sqrt{V_c(\bar{e}(0))} + \\ &\frac{c}{\theta - \lambda - \gamma_o + \gamma_c} \left(e^{-\left(\frac{\theta-\gamma_o}{2}\right)t} - e^{-\left(\frac{\lambda-\gamma_c}{2}\right)t} \right)\end{aligned}$$

Now, it suffices to choose $\lambda > \gamma_c$ and $\theta > \gamma_o$.

5. PARTICULAR DESIGN FUNCTIONS

The control law involves a gain depending on the bounded design function K_c which is completely characterized by the fundamental property (17). Some useful design functions are given below to emphasize the unifying feature of the proposed high gain concept.

- The usual high gain design function given by $K_c(\xi) = k_c \xi$ where k_c is a positive scalar satisfying $k_c \geq \frac{1}{2}$.
- The design function involved in the actual sliding mode framework $K_c(\xi) = k_c\ sign(\xi)$ where k_c is a positive scalar and 'sign' is the usual signum function (for $x \in \mathbb{R}^n$ with components $x_i \in \mathbb{R}$, $sign(x)$ is a vector and its ith component is $sign(x_i)$).
- The design functions that are commonly used in the sliding mode practice, namely

$$K_c(\xi) = k_c\ tanh(k_o \xi) \quad (29)$$

where $tanh$ denotes the hyperbolic tangent function and k_c and k_o are positive scalars.

6. FILTERED INTEGRAL ACTION

One can easily incorporate a filtered integral action into the proposed state feedback control design, for performance enhancement considerations, by simply introducing suitable state variables as follows

$$\begin{cases} \dot{\sigma}^f = e^f \\ \dot{e}^f = -\Theta e^f + \Theta e^1 \end{cases} \quad (30)$$

where $e^1 = z^1 - z_d^1$ and $\Theta = Diag\left(\frac{1}{\tau_1}, \ldots, \frac{1}{\tau_p}\right)$ is a design matrix that has to be specified according to the desired filtering action ($\tau_i > 0$, $i = 1, \ldots, p$, are real numbers). The state feedback gain is then derived from the control design model

$$\begin{aligned}\dot{e}_a &= A_a e_a + B_a \left(b(\xi, e + z_d) u_a - b(\xi, z_d) u_d \right) \\ &+ B_a \left(g(\xi, e + z_d) - g(\xi, z_d) \right) + \psi_a(z_d, e_a) - \psi(z_d, 0) \\ \dot{\xi} &= \eta(\xi, e + z_d, u_a) \quad (31)\end{aligned}$$

with $e_a = \begin{pmatrix} \sigma^f \\ e^f \\ e \end{pmatrix}$, $A_a = \begin{pmatrix} 0 & I_p & 0 \\ 0 & 0 & \Theta \\ 0 & 0 & A \end{pmatrix}$, $B_a = \begin{pmatrix} 0_p \\ 0_p \\ B_r \end{pmatrix}$ and $\psi_a(z_d, e_a) = \begin{pmatrix} 0_p \\ -\Theta e^f \\ \psi(e + z_d) \end{pmatrix}$. Indeed, the control design model structure (31) is similar to that of the error system (12) and hence the underlying state feedback control design is the same. The output feedback control law incorporating a filtered integral action is then given by

$$\begin{cases} \dot{\hat{x}} &= A\hat{x} + \varphi(u_a(\hat{e}_a), \hat{x}) - \theta \Delta_\theta^{-1} S^{-1} C^T C (\hat{x} - x) \\ u_a(\hat{e}_a) &= \left(b(\hat{\xi}, \hat{e} + z_d) \right)^+ \left(b(\hat{\xi}, z_d) u_d + \nu_a(\hat{e}_a) \right) \\ &= \left(b(\hat{\xi}, \hat{e} + z_d) \right)^+ \left(\dot{z}_d^q - g(\hat{\xi}, z_d) + \nu_a(\hat{e}_a) \right) \\ \nu_a(\hat{e}_a) &= -\Theta^{-1} B_a^T K_{ac}(\bar{S}_a \Theta_{a\lambda} \Lambda \hat{e}_a) \end{cases} \quad (32)$$

where $\hat{e}_a = \begin{pmatrix} \sigma^f \\ e^f \\ \hat{e} \end{pmatrix}$, $\Theta_{a\lambda} = diag\left(\lambda^{r+2} I_p, \lambda^{r+1}\ I_p, \ldots, \lambda\ I_p\right)$, $\Lambda = diag(I_p, I_p, \Theta, \Theta, \ldots, \Theta)$, $\bar{S}_a$ is the unique symmetric positive definite matrix solution of the following Lyapunov algebraic equation $\bar{S}_a + \bar{S}_a \bar{A}_a + \bar{A}_a^T \bar{S}_a = \bar{S}_a \bar{B}_a \bar{B}_a^T \bar{S}_a$ and $K_{ac} : \mathbb{R}^{(r+2)p} \to \mathbb{R}^{(r+2)p}$ is a bounded design function satisfying a similar inequality as (17), namely $e_a^T B_a B_a^T K_{ac}(e_a) \geq \frac{1}{2}\ e_a^T B_a B_a^T e_a\ \forall\ e_a \in \Omega$ where Ω is any compact subset of $\mathbb{R}^{(r+2)p}$. It can be easily shown that the resulting output feedback control system is globally stable and performs an asymptotic rejection of state and/or output step like disturbances.

7. APPLICATION TO INDUCTION MOTOR

In this section, rather than to consider an academic example, we suggest illustrating the performance of the approach proposed through the well known problem related to the control of the induction motor.

7.1 The induction motor model

The electrical behavior of an induction motor can be described in the (α, β) coordinate system in stationary reference frame fixed with the stator by the following dynamical system (Leonard [2001]):

$$\begin{cases} \dot{i} = KF(\omega)\psi - \gamma i + \dfrac{1}{\sigma L_s}u \\ \dot{\psi} = -F(\omega)\psi + \dfrac{M}{T_r}i \\ \dot{\omega} = \dfrac{pM}{JL_r}i^T J_2\psi - \dfrac{1}{J}\tau_L \end{cases} \quad (33)$$

where $i = (i_1, i_2)^T$, $\psi = (\psi_1, \psi_2)^T$, $u = (u_1, u_2)^T$ are respectively the stator current, the rotor fluxes and the voltage; ω and τ_L respectively denote the motor speed and the load torque; $F(\omega) = \frac{1}{T_r}I_2 - p\omega J_2$, I_2 is the 2×2 matrix identity and $J_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$; J is the motor moment of inertia; p is the number of pairs of poles. The parameters T_r, σ, K and γ are defined as: $Tr = \dfrac{L_r}{R_r}$, $\sigma = 1 - \dfrac{M^2}{L_sL_r}$, $K = \dfrac{M}{\sigma L_s L_r}$, $\gamma = \dfrac{R_s}{\sigma L_s} + \dfrac{R_r M^2}{\sigma L_s L_r^2}$ where R_s, R_r are stator (resp. rotor) per-phase resistances, L_s, L_r are stator (resp. rotor) per-phase inductances and M is the mutual mutual inductance.
The control objective consists in regulating the square of the fluxes vector norm, i.e. $\|\psi\|^2 = \psi_1^2 + \psi_2^2$ at a desired constant value while tracking a predescribed profile for the motor speed ω. Let us denote by $z^1 = \begin{pmatrix} z_1^1 \\ z_2^1 \end{pmatrix}$ where $z_1^1 = \omega$ and $z_2^1 = \|\psi\|^2$ the variables to be controlled and let $z_d^1 = \begin{pmatrix} z_{d1}^1 \\ z_{d2}^1 \end{pmatrix}$ be the corresponding desired trajectory. With these notations, the control objective consists in defining an admissible control input such that

$$\lim_{t\to\infty}(z_1^1(t) - z_{d1}^1(t)) = \lim_{t\to\infty}(z_2^1(t) - z_{d2}^1) = 0$$

7.2 Control design

We shall perform a change of variables to bring the equations of the motor model (33) into coordinates that will be easier to work with. Indeed, let $\Phi : \mathbb{R}^5 \to \mathbb{R}^5$, $x = \begin{pmatrix} i \\ \psi \\ \omega \end{pmatrix} \mapsto z = \begin{pmatrix} z^1 \\ z^2 \\ \xi \end{pmatrix}$ where $z^1 = \begin{pmatrix} z_1^1 \\ z_2^1 \end{pmatrix}$, $z^2 = \begin{pmatrix} z_1^2 \\ z_2^2 \end{pmatrix}$

and
$$\begin{cases} z_1^1 = w;\ z_2^1 = \|\psi\|^2 = \psi_1^2 + \psi_2^2; \\ z_1^2 = \dfrac{pM}{L_r}i^T J_2\psi;\ z_2^2 = i^T\psi;\ \xi = \arctan(\dfrac{\psi_2}{\psi_1}) \end{cases} \quad (34)$$

System (33) can be written in the new coordinates as follows:

$$\begin{cases} \dot{z}^1 &= A_1 z^2 + \Psi^1(\tau_L(t), z^1) \\ \dot{z}^2 &= b(\xi, z)u + g(z) \\ \dot{\xi} &= pz_1^1 + \dfrac{L_r}{pT_r}\dfrac{z_1^2}{z_2^1} \end{cases} \quad (35)$$

where $A_1 = \begin{pmatrix} \dfrac{1}{J} & 0 \\ 0 & 2\dfrac{M}{T_r} \end{pmatrix}$, $\Psi^1(\tau_L(t), z^1) = \begin{pmatrix} -\dfrac{1}{J}\tau_L \\ -\dfrac{2}{T_r}z_2^1 \end{pmatrix}$, $b(\xi, z) = \frac{1}{\sigma L_s}\sqrt{z_2^1}\begin{pmatrix} -\dfrac{pM}{L_r}sin(\xi) & \dfrac{pM}{L_r}cos(\xi) \\ cos(\xi) & sin(\xi) \end{pmatrix}$ and g has a triangular structure with respect to z. It is easy to see that system (35) is under form (5) and an output control law of the form (32) can be synthesized in order to achieve the control objective. Indeed, such a law, with a design function given by the expression (29), has been synthesized and corresponding simulation results are given in the next section. However, before giving these results, one notices that the underlying control scheme necessitates the knowledge of the load torque and its first time derivative. Since these variables are not measured, their are estimated through a high gain nonlinear observer (see e.g. Rossignol et al. [2003]).

7.3 Simulation results

In order to illustrate the performance of the proposed control scheme, simulation results on a 3 kW controlled induction motor are shown. The controllers are tested on a wide operating domain through a classical benchmark. The reference trajectory speed of the motor speed is shown in figure (1) and the set-point corresponding to the norm of the fluxes vector is constant and equal to $0.5Wb$. In order to simulate practical conditions, the measurements i_1, i_2 and ω have been corrupted with additive noises with zero mean value and standard variations which are respectively equal to 0.031, 0.032 and 0.32. The following parameter values are used for the numerical integration of the proposed scheme :

$$p = 2;\quad Ls = 0.13\ H;\quad M = 0.083\ H;\quad Lr = 0.069\ H;$$
$$Rs = 3.9\ Ohm;\quad Rr = 3\ Ohm;\quad J = 0.22\ Kg.m^2$$

A satisfactory shaping of the control system input-

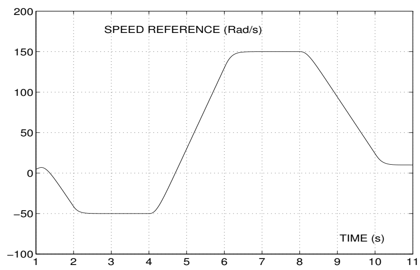

Fig. 1. The desired speed reference

output performance has been achieved with the following specifications: $k_c = k_o = \lambda = 5$, $\tau_1 = \tau_2 = 100$
Figure 2 presents the tracking error corresponding to ω and the time evolution of $\|\psi\|^2$. The resulting input time evolution is given in figure 3. Notice that tracking error related to the motor speed is less than 1.5% of the speed maximal value while the fluxes tracking is less than 2% of the nominal value.

8. CONCLUSION

The motivation of this paper was twofold. Firstly, a unified high gain state feedback control design framework has been developed to address an admissible tracking problem for a class of minimum phase uniformly observable nonlinear systems. Such a framework has been particularly suggested thanks to the duality from the high gain system observation. The unifying feature is provided through a suitable design function that allows to rediscover all those

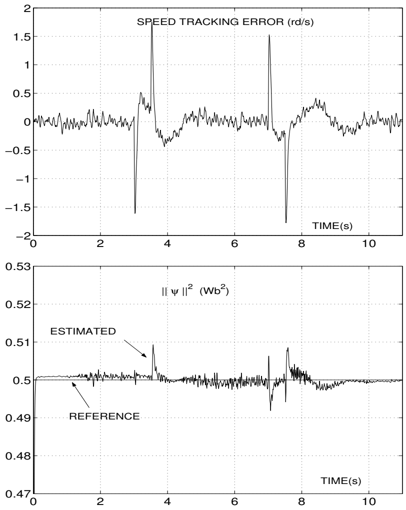

Fig. 2. Tracking error of ω and time evolution of the fluxes norm

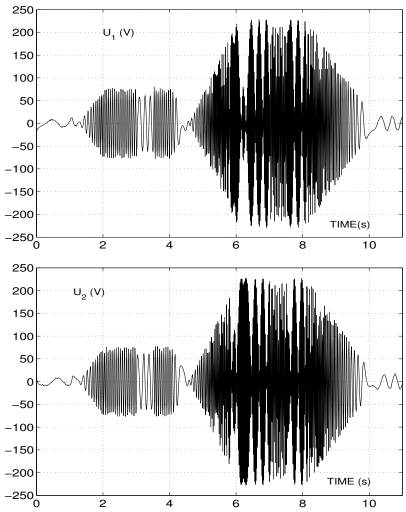

Fig. 3. Time evolution of the input voltage

well known high gain control methods, namely the sliding modes control. A Lyapunov approach has been adopted to show that the required tracking performance are actually handled. Secondly, the proposed state feedback control is combined with a high gain observer to provide an output feedback controller according to the well known separation theorem. Of practical purpose, a filtered integral action has been incorporated into the proposed control design to deal with step like disturbances while ensuring an adequate insensitivity to measurement noise. The proposed approach has been illustrated through an example dealing with the induction motor and the performance of the underlying output feedback controller were highlighted through realistic simulations.

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