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Absolute orientation angle estimation of a quadruped robot using non linear observers

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Abstract: This paper presents the application of two nonlinear observers in order to estimate the absolute orientation angle of a quadruped walking robot. The designed observers are based on high gain and high order sliding mode approaches. A loss of observability during the robot step appears: this problem is dealed by using two different observers structures.

1. INTRODUCTION

Walking biped robots belong to the family of mobile robots which use their environment to move and to make their tasks. One of the main objectives of current research on walking robots is to make dynamical stable gaits with imbalance phases. For this reason, the knowledge of their state variables is crucial in order to plane and to control their motions. This problem is not trivial and is usually solved using sensors such as accelerometers, gyrometers, inertials units ... (for details, see Chaillet [1993], Hirai et al., [1998], Sardin et al., [1999], Löffler et al., [2002], Harada et al., [2004]). In order to limit conception/maintenance costs, and to remove technological features such as noise, bandwidth limits of the sensor w.r.t. dynamics of the walking biped, an exciting challenge in robotics consists in using alternative solutions such as observers to estimate the orientation of a walking biped in imbalance phases. The dynamical behavior of a walking robot, which is a multi-body system, is described by a nonlinear model which implies that observability property depends on state and input (Hermann et al., [1977], Isidori [1995], Plestan et al., [1997]). A consequence is that, over one step, by supposing that only articular positions are measured, and for given trajectories, observability feature can be lost (Lebastard et al., [2006]). In this latter reference, an original strategy has been proposed in order to “cross” this singularity by using two observers based on different structures, each structure having different observability singular points.

Many observers strategies have been proposed for nonlinear systems. In (Xia et al., [1989], Glumineau et al., [1996]), observers based on linearization by input-output injection have been proposed: this approach consists in transforming the nonlinear system into a linear one via a state coordinates transformation and an input-output injection and in designing a classical Luenberger observer for this linear system. Unfortunately, this kind of observers can be applied only with a weak class of systems. In (Gauthier et al., [1992]), high gain observers with an asymptotic convergence are proposed: their design is based on a canonical form (Keller [1987]) and concerns a large class of systems. In (Perruquetti et al., [1998], Hernandez et al., [1996], Hespanha et al., [2002]), observers based on sliding mode are proposed with the objective to get robust estimation and finite time convergence. However, the main drawback of sliding mode being the “chattering” (high frequency oscillation), observers based on high order sliding mode (which ensures better features on robustness and accuracy by decreasing the chattering) have been proposed by (Boukhobza et al., [1998], Davila et al., [2005]). As high gain observers, this latter class of observers can be applied to a large class of physical systems including biped robots. However, to authors' best knowledge, previous works on observers design for walking robots have been mainly done for velocities estimation (in order to noiseless differentiation) by supposing that all angular variables are available (Micheau et al., [2003], Westervelt et al., [2004], Grizzle et al., [2007]). The originality of authors' previous works consists in designing posture and velocities observers which is a hard but more realistic (by a practical point-of-view) task. In Lebastard et al., [2006, 2007], the estimation of posture during imbalance phase for three-links/five links biped robots has been done in simulation by using high gain observers (with asymptotical time convergence convergence) and high order sliding mode observers (with finite time convergence). Furthermore, the proof of the walking gait stability of a walking biped under an observer-based control has been established in Lebastard et al., [2006, 2007].

The main purpose of the current paper consists in evaluating the behavior of nonlinear observers on the walking robot *SemiQuad* Aoustin et al., [2006]: its originality consists in applying nonlinear observers to a structure as *Semiquad*. Furthermore, the objective consists in designing a finite time convergence observer as sliding mode which greatly simplify the proof of closed-loop stability Lebastard et al., [2006]. Previously, the absolute orientation of the experimental platform of the experimental set-up *SemiQuad* has been estimated with success using the Extended Kalman Filter Aoustin et al., [2007]. However, the Extended Kalman Filter has two main drawbacks. The first one is that no convergence proof has been stated for the nonlinear case. The second one is that a loss of observability due to linearization process may happen even if there is no loss of observability for the original nonlinear system. Namely, given linearization point, the observability space based on the linearized model may collapse to null which yields to no means for the estimation of the state from the linearized model. Simulation results of the application of nonlinear observers are displayed, through the comparison of the posture estimation and its simulated value given by each of the four encoder, located in the actuated joints. The obtained results confirm the efficient of our strategy and allow us to prepare experimental tests. They also open us opportunities to extend it to more complex cases such as 3D walking robots. The paper is organized as follows: technological details on *SemiQuad* are recalled in Section 2. The motion equations of *SemiQuad* in sagittal plane and the passive impact model are recalled in Section 3. Section 4 shortly gives some details on the planning motion and the PID controller. Section 5 displays the three designed observers. Numerical results are shown in Section 6. Section 7 contains our conclusion and perspectives.

2. DESCRIPTION OF THE ROBOT *SEMIQUAD*

The prototype (see Figure 1) is composed of a platform and two identical double-link legs with knees. As a matter of fact, the desired gait consists in a quadruped curvet one when both front legs move identically with respect to the body and when both back legs move identically as well. In other words, the angles between the body and the thighs of the two front legs (back legs) are always identical. The knee joints angles are also always identical. It means that the front legs (back legs) of the quadruped are coupled. The legs have uncontrolled feet which extend in the frontal plane. Thus, the robot can only execute $2D$ motions in the sagittal plane. There are four electrical DC motors with gearbox reducers actuating haunche joints between the platform and the thighs and the knee joints. The parameters of the four actuators with their gearbox reducers are specified in Table 1. The lengths, masses and inertia moments of each link of *SemiQuad* are specified in Table 2. A *SemiQuad*'s step is composed of the following

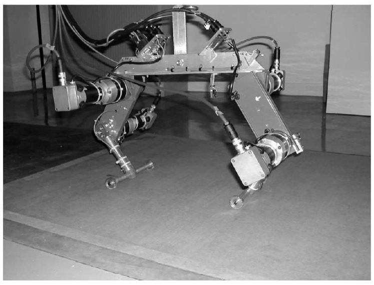

Fig. 1. Photography of *Semiquad*.

Parameter	Units	Haunch	Knee
Length	m	0.23	0.23
Mass	kg	2.82	2.82
Gearbox ratio		50	50
Rotor Inertia	kg.m^2	3.25×10^{-5}	2.26×10^{-5}
Electromagnetical torque constant	N.m/A	0.114	0.086

Table 1. Parameters of actuators

sequence of phases: double support, single support on the back leg, impact, double support, simple support on the

Parameter	Units	Link 1, 5	Link 3	Link 2, 4
Length	m	0.1500	0.3750	0.1500
Mass	kg	0.4000	6.6180	3.2100
Center of mass	m	0.0830	0.1875	0.1390
Moment of inertia	kg.m^2	0.0034	0.3157	0.0043

Table 2. Parameters of SemiQuad

front leg and impact. The first three ones make up an half step and their mathematical models are unlike each other.

3. *SEMIQUAD*'S MODEL

3.1 Dynamic model

Using the second Lagrange method, *SemiQuad*'s motion equations are obtained

$$D_e\ddot{q}_e + C_e\dot{q}_e + G_e = B_e\Gamma + E_{e_1}F_1 + E_{e_2}F_2 \qquad (1)$$

with $q_e = \left[q^T \ x_c \ y_c\right]^T$ (the notation T means transposition) is the vector of generalized coordinates. The vector q is composed by the joint variables and the absolute orientation of the platform $q = [q_1 \ q_2 \ q_3 \ q_4 \ q_5]^T$, and (x_c, y_c) are the Cartesian coordinates of the platform mass center (see Figure 2). $D_e(q_e)(7 \times 7)$ is the symmetric positive inertia

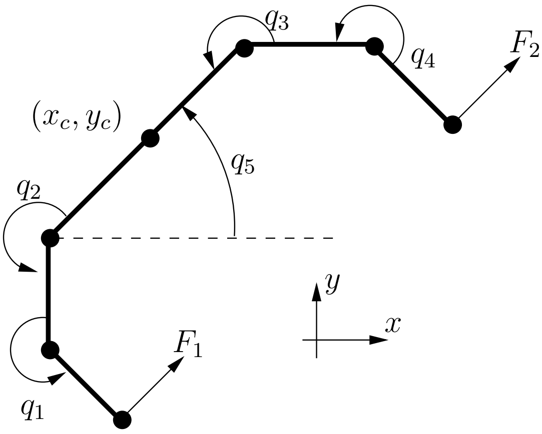

Fig. 2. *SemiQuad*'s diagram: generalized coordinates

matrix, $C_e(q_e, \dot{q}_e)(7 \times 7)$ is the Coriolis and centrifugal effects matrix, $G_e(q_e)(7 \times 1)$ is the gravity effects vector and $B_e(7 \times 4)$ is constant matrix composed of 1 and 0. Each matrix $E_{e_i}(7 \times 2)$ $(i = 1, 2)$ is a jacobian one which represents the relation between the cartesian velocity of the foot i and the angular velocities $\dot{q}$. They allow to take into account the ground reaction F_i on each foot. If the foot j is not in contact with the ground, then $F_i = [0 \ 0]^T$. The term $\Gamma = [\Gamma_1 \ \Gamma_2 \ \Gamma_3 \ \Gamma_4]^T$ is vector of actuator torques. Assume that

H1 During the swing phase of the motion, the stance leg is acting as a pivot then its position is constant.

Then, the model (1) can be simplified and rewritten as:

$$D\ddot{q} + C\dot{q} + G = B\Gamma. \qquad (2)$$

The terms of this reduced model have equal mean as for the model (1), only the dimensions have been changed: $D \in I\!R^{5\times5}$, $H \in I\!R^{5\times5}$, $G \in I\!R^{5\times1}$, $B \in I\!R^{5\times4}$. As the kinetic energy of a robot is invariant under a rotation of the world frame (Spong et al., [1989]) and viewed that q_5 defines the *SemiQuad*'s orientation, the 5×5-symmetric positive inertia matrix is independent of this variable ($D(q_r)$, $q_r = [q_1\ q_2\ q_3\ q_4]^T$). The model (1) can be written under the following state system

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ D^{-1}\left(B\Gamma - Cx_2 - G\right) \end{bmatrix} \\ = f(x) + g(q_r)\Gamma \tag{3}$$

with $x_1 := q$ and $x_2 := \dot{q}$. The state space is defined as $x \in \mathcal{X} := \left\{x := \left[q^T\ \dot{q}^T\right] | q \in \mathcal{M},\quad \dot{q} \in \mathcal{N}\right\}$, with $\mathcal{M} = (-\pi, \pi)^5$ and $\mathcal{N} = \left\{\dot{q} \in I\!R^5 || \dot{q}| < \dot{q}_M < \infty\right\}$. From these definitions, note that all the state variables are bounded.

3.2 Passive impact model

The impact occurs at the end of every single support phase, when the swing leg tip touches the ground. In order to develop the passive impact model the following hypothesis will be used:

H2 The impact is passive and absolutely inelastic.
H3 At the impact the swinging leg does not slip when it touches the ground and the stance leg does not take off.
H4 At the impact, the angular positions are continuous and the angular velocities discontinuous.

Given these hypothesis, the ground reactions at the instant of the impact can be considered as impulsive forces acting only on the swinging leg and consequently they can be modelled by Dirac delta-functions: $F_i = I_{F_i}\delta(t - t_i)$. t_i denotes the impact time and $I_{F_i} := [I_{F_iN}\ I_{F_iT}]^T$ is the vector of magnitudes of impulsive reaction for the leg i. Impact equations can be obtained integrating (2) from t_i^- (just before the impact) to t_i^+ (just after the impact). The torques supplied by the actuators and Coriolis and gravity forces have finite values: thus they do not influence the impact. The impact equations can be written as

$$D_e(q_e)\left(\dot{q}_e^+ - \dot{q}_e^-\right) = E_{e_i}(q_e) I_{F_i} \tag{4}$$

q_e is the *SemiQuad*'s configuration at $t = t_i$ (from **H4**, this configuration does not change at the instant of the impact), $\dot{q}_e^-$ and $\dot{q}_e^+$ are respectively the angular velocities just before and just after the impact. Furthermore, the velocity of the swinging leg tip after the impact is zero ($(E_{e_i}(q_e))^T \dot{q}_e^+ = 0$). The unknowns $\dot{q}_e^+$ and I_{F_i} can be computed using the linear system of equations

$$\begin{bmatrix} D_e(q_e) & -E_{e_i}(q_e) \\ E_{e_i}(q_e)^T & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_e^+ \\ I_{F_i} \end{bmatrix} = \begin{bmatrix} D_e(q_e)\dot{q}_e^- \\ 0 \end{bmatrix} \tag{5}$$

The final result is an expression for $x^- = \left[q^{+T}, \dot{q}^{+T}\right]^T$ (state just after the impact) in terms of $x^- = \left[q^{-T}, \dot{q}^{-T}\right]^T$ (state just before the impact) which is written as $x^+ = \Delta(x^-)$.

4. CONTROL LAW

Reference trajectories for every articulation are specified as time functions using third order polynomials. The tracking of these references is achieved using a proportional-derivative controller. Details of *SemiQuad*'s walking gait strategy can be found in (Aoustin et al., [2006]).

5. OBSERVERS DESIGN

5.1 Observability analysis

Consider system (3) with y the vector composed of the measured variables $y := [y_1\ y_2\ y_3\ y_4]^T = [q_1\ q_2\ q_3\ q_4]^T = q_{rel}$

$$\dot{x} = f(x) + g(y)\Gamma \\ y = \left[I_{4\times4}\ \ 0_{4\times6}\right] x = Cx \tag{6}$$

As $g(y)\Gamma$, the *input-output injection* term of (6), is fully known, an observer for (6) can be designed by the following way. Consider the next nonlinear system, which is the part of (6) without the input-output injection term $g(y)\Gamma$ (with abuse of notation)

$$\dot{x} = f(x) \\ y = Cx \tag{7}$$

The observers designed in the sequel are designed from this latter system.

***Definition** 1.* System (7) is observable if the following conditions (Hermann et al., [1977]) are satisfied

- There exist a set of 4 integers $\{k_1\ \dots\ k_4\}$ called observability index, such that
$$\sum_{i=1}^{4} k_i = 10, \quad k_i \in I\!N.$$
- There exist a subset $\mathcal{T} \subset \mathcal{X}$, for which the transformation
$$\Phi(x) = \left[y_1\ \dots\ y_1^{(k_1-1)}\ \dots\ y_4\ \dots\ y_4^{(k_4-1)}\right]^T \tag{8}$$
is a diffeomorphism for $x \in \mathcal{T}$, which is equivalent to
$$\det\left[\frac{\partial\Phi(x)}{\partial x}\right] \neq 0 \text{ for } x \in \mathcal{T}.$$

■

***Proposition** 1.* There exist $\mathcal{T} \subset \mathcal{X}$ and observability index combination $\{3, 3, 2, 2\}$ such that system (7) is observable for $x \in \mathcal{T}$. ■

Proof. During the single support phase and along the desired trajectoires, the determinant of $\frac{\partial\Phi(x)}{\partial x}$ crosses zero twice (Figure 3). At these singular points, the system is not observable. ■

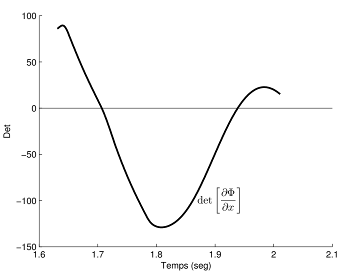

Fig. 3. det $\left[\frac{\partial}{\partial x}\Phi(x)\right]$ versus time along the single support phase.

5.2 Observability canonical form

Suppose that Proposition 1 is fulfilled; then, the following associated state coordinates transformation is invertible for $x \in \mathcal{T}$

$$z = \Phi(x) = [y_1 \; \dot{y}_1 \; \ddot{y}_1 \; y_2 \; \dot{y}_2 \; \ddot{y}_2 \; y_3 \; \dot{y}_3 \; y_3 \; \dot{y}_3]^T \tag{9}$$

Under this state transformation (9), system (7) is equivalent to the canonical form,

$$\begin{aligned} \dot{z} &= Az + \varphi(z) \\ y &= Cz \end{aligned} \tag{10}$$

with $A = \text{diag}\,[A_1 \; \cdots \; A_4]_{10\times 10}$, $C = [C_1 \; \cdots \; C_4]^T_{4\times 10}$, $\varphi(z) = [\varphi_1^T \; \cdots \; \varphi_4^T]^T$, A_i, C_i and φ_i being defined as

$$A_i = \begin{cases} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} & \text{for } k_i = 3 \\ \\ \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} & \text{for } k_i = 2 \end{cases},$$

$C_i = [1 \; 0 \; \ldots \; 0]_{1\times k_i}$ and $\varphi_i = \left[0 \; \ldots \; 0 \; y_i^{(k_i)}\right]^T$.

5.3 Observation strategy

Consider the following system which is supposed to be an observer for system (10)

$$\begin{aligned} \dot{\hat{z}} &= A\hat{z} + \varphi(\hat{z}) + \eta_z(y, \hat{y}) \\ \hat{y} &= C\hat{z} \end{aligned} \tag{11}$$

with $\eta_z(y, \hat{y})$ the observer correction term. Let $\eta_x(y, \hat{y}, \hat{x})$ define as the solution of the following linear system

$$\left[\frac{\partial \Phi(\hat{x})}{\partial \hat{x}}\right] \eta_x = \eta_z \tag{12}$$

At the singular point such that

$$\det\left[\frac{\partial \Phi(\hat{x})}{\partial \hat{x}}\right] = 0,$$

system (12) has not solution or there exist infinity solutions. From $\hat{z} = \Phi(\hat{x})$, one gets

$$\dot{\hat{z}} = \left[\frac{\partial \Phi(\hat{x})}{\partial \hat{x}}\right] \dot{\hat{x}}$$

It yields

$$\begin{aligned} \dot{\hat{x}} &= \left[\frac{\partial \Phi(\hat{x})}{\partial \hat{x}}\right]^{-1} [A\hat{z} + \varphi(\hat{z}) + \eta_z(y, \hat{y})] \\ &= f(\hat{x}) + \left[\frac{\partial \Phi(\hat{x})}{\partial \hat{x}}\right]^{-1} \eta_z(y, \hat{y}) \\ &= f(\hat{x}) + \eta_x(y, \hat{y}) \end{aligned} \tag{13}$$

From (13), an observer for (6) reads as

$$\dot{\hat{x}} = f(\hat{x}) + g(q_r)\Gamma + \eta_x(y, \hat{y}, \hat{x}) \tag{14}$$

In order to overcome the problem of observability singularity, several solutions has been proposed (Lebastard et al., [2006])

***Proposition** 2.* For $x \in \mathcal{X}$, observer (14) is turned into the dynamic system

$$\dot{\hat{x}} = f(\hat{x}) + g(q_r)\Gamma + \Delta_x \eta_x(y, \hat{y}, \hat{x})$$

with

$$\Delta_x = \begin{cases} k_\phi(\hat{x}) & \hat{x} \in (\mathcal{X} \cap \mathcal{T}) \\ 0 & \hat{x} \in (\mathcal{X}/\mathcal{T}) \end{cases}$$

with the function $k_\phi(\hat{x})$ displayed by Figure 4. ■

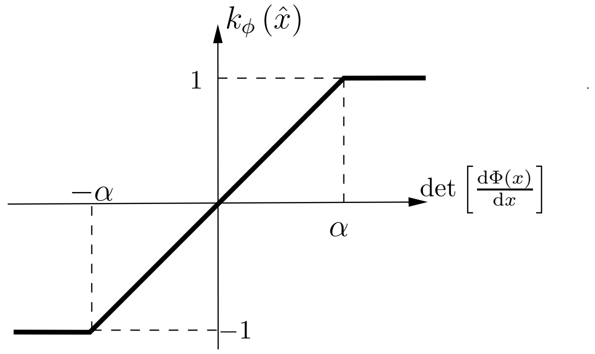

Fig. 4. State correction around the singulier points $\hat{x} \in (\mathcal{X} \cap \mathcal{T})$.

As displayed in Figure 4, α is a positive real constant fixed by the user. The choice of α is made in order to limit the condition number of observability matrix with respect to inversion of $\frac{\partial \Phi(\hat{x})}{\partial \hat{x}}$. Δ_x acts on observer gain values and implies that around the singular points the convergence conditions are not satisfied. It means that the singularity area must be "sufficiently small". An other solution is described in the following proposition which is applicable only when the system admits almost two different observability index combinations.

***Proposition** 3.* For $x \in \mathcal{X}$, observer (14) is turned into the dynamic system

$$\dot{\hat{x}} = f(\hat{x}) + g(q_r)\Gamma + \Delta_x \tag{15}$$

with

$$\Delta_x = \begin{cases} \eta_{x_\alpha}(y, \hat{y}_\alpha) \text{ if } C_\alpha < C_\beta \\ \eta_{x_\beta}(y, \hat{y}_\beta) \text{ if } C_\alpha \geq C_\beta \end{cases},$$

$\eta_{x_\alpha}(y, \hat{y}_\alpha)$ and $\eta_{x_\beta}(y, \hat{y}_\beta)$ being the solutions of linear systems

$$\left[\frac{\partial \Phi_\alpha(\hat{x})}{\partial \hat{x}}\right] \eta_{x_\alpha} = \eta_{z_\alpha} \qquad \left[\frac{\partial \Phi_\beta(\hat{x})}{\partial \hat{x}}\right] \eta_{x_\beta} = \eta_{z_\beta}$$

η_{z_α} and η_{z_β} being the correction terms of observers based on canonical forms with

$$z_\alpha = \Phi_\alpha(x) = [y_1 \; \dot{y}_1 \; \ddot{y}_1 \; y_2 \; \dot{y}_2 \; \ddot{y}_2 \; y_3 \; \dot{y}_3 \; y_4 \; \dot{y}_4]^T$$
$$z_\beta = \Phi_\beta(x) = [y_1 \; \dot{y}_1 \; y_2 \; \dot{y}_2 \; \ddot{y}_2 \; y_3 \; \dot{y}_3 \; \ddot{y}_3 \; y_4 \; \dot{y}_4]^T$$

and C_α (resp. C_β) being the condition number of $\frac{\partial \Phi_\alpha}{\partial \hat{x}}$ (resp. $\frac{\partial \Phi_\beta}{\partial \hat{x}}$). ■

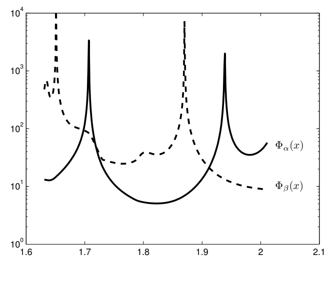

Fig. 5. Observability matrix condition numbers.

5.4 Observers

In this part, three observers are displayed: high gain observer (Gauthier et al., [1992]) and observer based on high order sliding mode differentiation (Davila et al., [2005]). These both observers read as (11), their differences being in the definition of η_z. Whereas high-gain observer ensures an asymptotic convergence of the estimation error, a feature of the sliding mode one is its finite time convergence; this latter point greatly simplifies the proof of stability (Lebastard et al., [2006]).

High gain observer Suppose that function φ of system (10) is globally Lipschitzian with respect to z. Then, system (10) is locally uniformly observable (Gauthier et al., [1981]). Let K denote a matrix of appropriate dimensions, such that $A - KC$ is Hurwitz, and $\Lambda(T) = \mathrm{diag}[\Lambda_1\ \Lambda_2\ \cdots\ \Lambda_p]'$ with $\Lambda_i = \mathrm{diag}[\tau_i\ \tau_i^2\ \cdots\ \tau_i^{k_i-1}]$, with $\tau_i > 0$. Then, the system

$$\dot{\hat{z}} = A\hat{z} + \varphi(\hat{z}) + \Lambda^{-1}K(y - C\hat{z}) \qquad (16)$$

with $\hat{z} \in I\!R^n$, is an asymptotic observer for (10). Furthermore, the dynamics of this observer can be made arbitrarily fast (Tornambe [1989]). Then, with respect to system (15), matrix Δ_x reads as

$$\Delta_x = \begin{cases} \left[\dfrac{\partial \Phi_\alpha(\hat{x})}{\partial \hat{x}}\right]^{-1} \Lambda^{-1}K(y - C\hat{x}) \text{ if } C_\alpha < C_\beta \\ \left[\dfrac{\partial \Phi_\beta(\hat{x})}{\partial \hat{x}}\right]^{-1} \Lambda^{-1}K(y - C\hat{x}) \text{ if } C_\alpha \geq C_\beta \end{cases}$$

Observer based on high order sliding mode differentiator The observer proposed in the sequel is based on high order sliding mode differentiation. As previously, viewed that observability indexes equal 2 or 3, for a sake of clarity and without loss of generality, the observer design for a second (*i.e.* $k_i = 2$) order system and third (*i.e.* $k_i = 3$) is fully displayed in the sequel. Then, in the second order case, subsystem takes the form as (Davila et al., [2005])

$$\dot{z}_{i1} = z_{i2},\ \dot{z}_{i2} = \varphi_{ii}(z),\ y_i = z_{i1} \qquad (17)$$

with $\|\varphi_{ii}(\cdot)\| \leq L_{ii}^2$. Then, an observer for (17) reads as (Davila et al., [2005])

$$\begin{aligned} \dot{\hat{z}}_{i1} &= \hat{z}_{i2} + 1.5\ L_{ii}^{1/2}|z_{i1} - \hat{z}_{i1}|^{1/2}\mathrm{sign}(z_{i1} - \hat{z}_{i1}) \\ &= v_{i1} \\ \dot{\hat{z}}_{i2} &= \varphi_{i2}(\hat{z}_i) + 1.1\ L_{ii}\mathrm{sign}(v_{i1} - \hat{z}_{i1}) \end{aligned} \qquad (18)$$

with $[\hat{z}_{i1}\ \hat{z}_{i2}]^T$ the estimation of $[z_{i1}\ z_{i2}]^T$. In the third order case, subsystem reads as

$$\dot{z}_{i1} = z_{i2},\ \dot{z}_{i2} = z_{i3},\ \dot{z}_{i3} = \varphi_{ii}(z), y_i = z_{i1} \qquad (19)$$

with $\|\varphi_{ii}(\cdot)\| \leq L_{ii}$. Let us propose an observer for the jerk observation based on a third order differentiator (Lebastard et al., [2006])

$$\begin{aligned} \dot{\hat{z}}_{i1} &= \hat{z}_{i2} + 2\ L_{ii}^{1/3}|z_{i1} - \hat{z}_{i1}|^{2/3}\mathrm{sign}(z_{i1} - \hat{z}_{i1}) \\ &= v_{i1} \\ \dot{\hat{z}}_{i2} &= \hat{z}_{i3} + 1.5\ L_{ii}^{1/2}|v_{i1} - \hat{z}_{i2}|^{1/2}\mathrm{sign}(v_{i1} - \hat{z}_{i2}) \\ &= v_{i2} \\ \dot{\hat{z}}_{i3} &= \varphi_{i3}(\hat{z}_i) + 1.1\ L_{ii}\mathrm{sign}(v_{i2} - \hat{z}_{i3}) \end{aligned} \qquad (20)$$

with $[\hat{z}_{i1}\ \hat{z}_{i2}\ \hat{z}_{i3}]^T$ the estimation of $[z_{i1}\ z_{i2}\ z_{i3}]^T$. Then, correction terms of observer (15) are derived from (18)-(20) expressed in x-state space coordinates.

6. SIMULATIONS

This section proposes simulations made only on a single-support swinging phase. As the positions and the orientation angle are known during the double support phase, only initial estimation errors on velocities are considered. Let $x_0 = \left[q(0)^T\ \dot{q}(0)^T\right]^T$ define SemiQuad's initial conditions, the observer's initial condition being taken as $\hat{x}_0 = \left[q(0)^T\ 1.2\dot{q}(0)^T\right]^T$. The observers are simulated as discrete time ones using a sampling period equal to $T_s = 10^{-4}$ seconds. The simulation results are displayed through Figure 6 and show the efficiency of the both observers.

The observers design has been made by supposing the

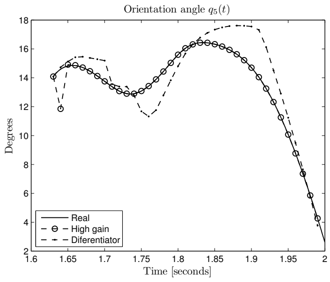

Fig. 6. Orientation angle estimation (*deg.*) versus time (*sec.*).

friction parameters equal to zero. In order to evaluate the observers robustness, friction parameters terms are introduced in the model but not in the observers. In Figure 7, friction terms are stated to $0.1\dot{q}_i$ (for $i = 1, 2, 3, 4$): in this case, only the high order sliding mode observer ensures that the estimated state remains around the real state (with the high gain observer, the estimation error does not converge).

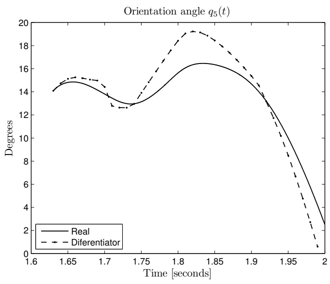

Fig. 7. Orientation angle estimation (*deg.*) versus time (*sec.*).

7. CONCLUSIONS

In this paper, two nonlinear observers have been designed and numerically tested for the quadruped prototype, *SemiQuad*, in order to estimate its posture. The simulation results show that these two nonlinear observers are able to estimate the orientation of *SemiQuad* in single support, and can be a good alternative to sensors. Next step of this work consists, first, in associating a nonlinear dynamic robust control law with posture observers, in order to get experimental results. Furthermore, a main perspective is the extension of this strategy for walking 3D-walking robots.

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