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Novel delay-dependent stability criterion for delayed neural networks

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Abstract: A new augmented Lyapunov functional is constructed for delayed neural networks, free-weighting matrix technique is employed to derive the delay- dependent stability criterion. The derived criterion is formulated in terms of linear matrix inequality (LMI). A numerical example is given to demonstrate the effectiveness and applicability of the criterion.

Index Terms: Delay-dependent, neural networks (NNs), delay, linear matrix inequality (LMI), stability.

I. INTRODUCTION

Neural networks (NNs) have received considerable attention due to their extensive applications in a variety of areas, such as signal processing, pattern recognition, static image processing, associative memory, and combinatorial optimization in the past decades See e.g. Borkar, and Soumyanatha (1997), Nmichel and Liu (2002). In the real world, time delay is frequently encountered in NNs, and it can easily cause instability and oscillations in a system. Because the applications of delayed neural networks rely heavily on the dynamical behavior of the networks, the stability of delayed neural network has been investigated by many researchers and presented a number of useful and interesting results. See e.g. Arik (2000,2003), Liao and Wang (1999,2000), Cao and zhou (1998), Zhang, Ma, Xu (2001), Liao and Chen (2002), Xu, Lam, Ho and Zou (2005), Hua, Long and Guan (2006), He, Liu and Rees (2007a), Park,and Cho (2007). According to the results, the stability criteria for delayed NNs can be classified into two categories, namely, delay-independent and delay-dependent. The delay-dependent stability criteria have attracted much attention because delay-dependent criteria make use of information on the length of delays, and less conservative than delay-independent ones. Among them, Cao and zhou (1998), Zhang, Ma, Xu (2001) presented delay-independent stability criteria for a class of NNs with delay, Liao and Chen (2002) derived sufficient conditions of delayed NNs by using the LMI approach, the results in Liao and Chen (2002) were improved in Arik (2003) by constructing different Lyapunov-Krasovskii fuctionals. Xu, Lam, Ho and Zou (2005) presented a less conservative delay-dependent stability criteria than Arik (2003) by constructing a more general Lyapunov-Krasovskii functional and using LMI approach. Despite these improvements, it is still hard to further reduce the conservatism by using the same types of Lyapunov functional as in the above works, so a new augmented Lyapunov fuctional is constructed to study the problem of stability of delayed NNs. He, Wang ,Lin and Wu (2005a) Moreover, the free-weighting matrix technique is proved very effective for deriving the delay-dependent stability criteria of time-delay systems He, Wu, She and Liu (2004a,2004b), Wu, He, She and Liu (2004), He, Wang and Wu (2005b), He, Wang and Xie (2007b). Therefore, it is a good idea to use augmented Lyapunov functional and free-weighting matrix approach to derive a less conservative stability criteria of the delayed NNs.

In this paper, the problem of asymptotic stability for delayed neural networks is considered. Attention is focused on derivation of the stability criterion for the delayed neural networks. By constructing a new augmented Lyapunov functional, employing free-weighting matrix approach, a delay-dependent criterion in terms of LMI for delayed neural networks is obtained.

II. PROBLEM FORMULATION

We consider the following delayed neural network:

$$\dot{x}(t) = -Cx(t) + Ag(x(t)) + Bg(x(t-\tau)) + u \tag{1}$$

where $x(\cdot) = [x_1(\cdot), x_2(\cdot), \ldots, x_n(\cdot)]^{\mathrm{T}} \in R^n$ is the neuron state vector, $g(x(\cdot)) = [g_1(x_1(\cdot)), g_2(x_2(\cdot)), \ldots, g_n(x_n(\cdot))]^{\mathrm{T}} \in R^n$ denotes the neuron activation function, and $u = [u_1, u_2, \ldots, u_n]^{\mathrm{T}} \in R^n$ is a constant input vector. $C = diag\{c_1, c_2, \ldots, c_n\}$ is a diagonal matrix with positive entries, $c_i > 0$ $(i = 1,2,\ldots,n)$, A and B are the connection weight matrix and delayed connection weight matrix, respectively. The time delay τ is a constant.

Assumption 1:

The activation function $g_i(\cdot)$, $i = 1,2,\ldots,n$, satisfies the following condition:

$$0 \le \frac{g_i(\xi_1) - g_i(\xi_2)}{\xi_1 - \xi_2} \le k_i \, , \text{ for any } \xi_1 \, , \xi_2 \in R \, , \; \xi_1 \ne \xi_2 \, , \; i = 1,2,\ldots,n \tag{2}$$

where k_i, $i = 1,2,\ldots,n$ are positive constants.

In the following, the equilibrium point $x^* = [x_1^*, x_2^*, \ldots, x_n^*]^{\mathrm{T}}$, of the system (1) will be shifted to the origin by the transformation $z(\cdot)=x(\cdot)-x^*$ which puts the system (1) into the following form

$$\dot{z}(t)=-Cz(t)+Af(z(t))+Bf(z(t-\tau)) \tag{3}$$

where $z(t)=[z_1(\cdot),z_2(\cdot),\ldots,z_n(\cdot)]^{\mathrm{T}}$ is the state vector of the transformed system, and $f(z(\cdot))=[f_1(z_1(\cdot)),f_2(z_2(\cdot)),\ldots,f_n(z_n(\cdot))]^{\mathrm{T}}$ with $f_i(z_i(\cdot))=g_i(z_i(\cdot)+z_i^*)-g_i(z_i^*)$, $i=1,2,\ldots,n$ Note that the functions $f_i(.)$, $i=1,2,\ldots,n$ satisfy the following conditions:

$$0\le\frac{f_i(z_i)}{z_i}\le k_i, f_i(0)=0, \forall z_i\ne 0,\quad i=1,2,\cdots,n \tag{4}$$

which is equivalent to

$$f_i(z_i)[f_i(z_i)-k_iz_i]\le 0,\quad f_i(0)=0,\quad i=1,2,\cdots,n \tag{5}$$

In the following section, we will develop delay-dependent condition such that origin of the delayed network (3) is asymptotically stable.

III. STABILITY CRITERION

Now, we present the following new asymptotic condition for system (3), which is dependent of the size of delay.

Theorem 1: The origin of the delayed neural network in (3)is asymptotically stable, if there exist matrices

$L=\begin{bmatrix}L_{11} & L_{12}\\ L_{12}^{\mathrm{T}} & L_{22}\end{bmatrix}\ge 0$ with $L_{11}>0$; $Q=\begin{bmatrix}Q_{11} & Q_{12}\\ Q_{12}^{\mathrm{T}} & Q_{22}\end{bmatrix}\ge 0$; $Z=\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{12}^{\mathrm{T}} & Z_{22}\end{bmatrix}>0$ and $M=[M_1^{\mathrm{T}}\ M_2^{\mathrm{T}}\ M_3^{\mathrm{T}}\ M_4^{\mathrm{T}}]^{\mathrm{T}}$, diagonal matrices $R\ge 0, T\ge 0, S\ge 0,$ such that the following LMI is feasible

$$\Phi=\begin{bmatrix}\Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} & -\tau C^{\mathrm{T}}Z_{22}\\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} & 0\\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} & \tau A^{\mathrm{T}}Z_{22}\\ * & * & * & \Phi_{44} & \Phi_{45} & \Phi_{46} & \tau B^{\mathrm{T}}Z_{22}\\ * & * & * & * & \Phi_{55} & \Phi_{56} & 0\\ * & * & * & * & * & \Phi_{66} & 0\\ * & * & * & * & * & * & -\tau Z_{22}\end{bmatrix}<0 \tag{6}$$

where

$\Phi_{11}=-L_{11}C-C^{\mathrm{T}}L_{11}^{\mathrm{T}}+\tau Z_{11}+L_{12}+L_{12}^{\mathrm{T}}+Q_{11}+M_1+M_1^{\mathrm{T}}-\tau(Z_{12}C+C^{\mathrm{T}}Z_{12}^{\mathrm{T}})$; $\Phi_{12}=-L_{12}-M_1+M_2^{\mathrm{T}}$;

$\Phi_{13}=L_{11}A+Q_{12}+\tau Z_{12}A+KT-C^{\mathrm{T}}R+M_3^{\mathrm{T}}$; $\Phi_{14}=\tau Z_{12}B+M_4^{\mathrm{T}}+L_{11}B$; $\Phi_{15}=-\tau C^{\mathrm{T}}L_{12}+\tau L_{22}^{\mathrm{T}}$; $\Phi_{16}=-\tau M_1$; $\Phi_{22}=-M_2-M_2^{\mathrm{T}}-Q_{11}$; $\Phi_{23}=-M_3^{\mathrm{T}}$; $\Phi_{24}=-Q_{12}+KS-M_4^{\mathrm{T}}$; $\Phi_{25}=-\tau L_{22}^{\mathrm{T}}$; $\Phi_{26}=-\tau M_2$; $\Phi_{33}=R^{\mathrm{T}}A+A^{\mathrm{T}}R+Q_{22}-2T$; $\Phi_{34}=R^{\mathrm{T}}B$; $\Phi_{35}=\tau A^{\mathrm{T}}L_{12}$; $\Phi_{36}=-\tau M_3$; $\Phi_{44}=-Q_{22}-2S$; $\Phi_{45}=\tau B^{\mathrm{T}}L_{12}$; $\Phi_{46}=-\tau M_4$; $\Phi_{55}=-\tau Z_{11}$; $\Phi_{56}=-\tau Z_{12}$; $\Phi_{66}=-\tau Z_{22}$; $R=diag(r_1,r_2,\ldots,r_n)$; $K=diag(k_1,k_2,\ldots,k_n)$; $T=diag(t_1,t_2,\ldots,t_n)$; $S=diag(s_1,s_2,\ldots,s_n)$;

and $*$ denotes the symmetric terms in a symmetric matrix.

Proof: Choose an augmented Lyapunov functional as

$$V(z(t))=V_1(z(t))+V_2(z(t))+V_3(z(t))+V_4(z(t)) \tag{7}$$

$$V_1(z(t))=\zeta_1^{\mathrm{T}}(t)L\zeta_1(t)$$

$$V_2(z(t))=\int_{t-\tau}^{t}\zeta_2^{\mathrm{T}}(s)Q\zeta_2(s)\mathrm{d}s$$

$$V_3(z(t))=\int_{-\tau}^{0}\int_{t+\theta}^{t}\zeta_3^{\mathrm{T}}(s)Z\zeta_3(s)\mathrm{d}s\mathrm{d}\theta$$

$$V_4(z(t))=2\sum_{i=1}^{n}r_i\int_0^{z_i(t)}f(s)\mathrm{d}s$$

where

$L=\begin{bmatrix}L_{11} & L_{12}\\ L_{12}^{\mathrm{T}} & L_{22}\end{bmatrix}\ge 0$ with $L_{11}>0$; $Q=\begin{bmatrix}Q_{11} & Q_{12}\\ Q_{12}^{\mathrm{T}} & Q_{22}\end{bmatrix}\ge 0$ $Z=\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{12}^{\mathrm{T}} & Z_{22}\end{bmatrix}>0$ are to be determined

and $\zeta_1(t)=\begin{bmatrix}z(t)\\ \int_{t-\tau}^{t}z(s)\mathrm{d}s\end{bmatrix}$, $\zeta_2(s)=\begin{bmatrix}z(s)\\ f(z(s))\end{bmatrix}$, $\zeta_3(s)=\begin{bmatrix}z(s)\\ \dot{z}(s)\end{bmatrix}$.

Calculating the derivatives of $V_i(z(t))$, $i=1,2,...,4$ defined in (7) along the trajectories of (3) yields

$$\dot{V}_1(z(t))=2\zeta_1^{\mathrm{T}}(t)L\dot{\zeta}_1(t)$$

$$=2\begin{bmatrix}z(t)\\ \int_{t-\tau}^{t}z(s)ds\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}L_{11} & L_{12}\\ L_{12}^{\mathrm{T}} & L_{22}\end{bmatrix} * \begin{bmatrix}-Cz(t)+Af(z(t))+Bf(z(t-\tau))\\ z(t)-z(t-\tau)\end{bmatrix}$$

$$\dot{V}_2(z(t))=\zeta_2^{\mathrm{T}}(t)Q\zeta_2(t)-\zeta_2^{\mathrm{T}}(t-\tau)Q\zeta_2(t-\tau)$$

$$= \begin{bmatrix} z(t) \\ f(z(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^{\mathrm{T}} & Q_{22} \end{bmatrix} \begin{bmatrix} z(t) \\ f(z(t)) \end{bmatrix} - \begin{bmatrix} z(t-\tau) \\ f(z(t-\tau)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^{\mathrm{T}} & Q_{22} \end{bmatrix} \begin{bmatrix} z(t-\tau) \\ f(z(t-\tau)) \end{bmatrix}$$

$$\dot{V}_3(z(t)) = \tau\zeta_3^{\mathrm{T}}(t)Z\zeta_3(t) - \int_{t-\tau}^{t} \zeta_3^{\mathrm{T}}(s)Z\zeta_3(s)\mathrm{d}s$$

$$= \tau \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^{\mathrm{T}} & Z_{22} \end{bmatrix} \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix} - \int_{t-\tau}^{t} \begin{bmatrix} z(s) \\ \dot{z}(s) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^{\mathrm{T}} & Z_{22} \end{bmatrix} \begin{bmatrix} z(s) \\ \dot{z}(s) \end{bmatrix} \mathrm{d}s$$

$$\dot{V}_4(z(t)) = 2\dot{z}^{\mathrm{T}}(t)Rf(z(t)) = 2[-Cz(t) + Af(z(t)) + Bf(z(t-\tau))]^{\mathrm{T}} Rf(z(t))$$

Using the Leibniz-Newton formula, the following equation is true for any matrix M with appropriate dimensions

$$0 = 2\eta^{\mathrm{T}}(t)M[z(t) - z(t-\tau) - \int_{t-\tau}^{t} \dot{z}(s)\mathrm{d}s] \tag{8}$$

where $\eta(t) = [z^{\mathrm{T}}(t)\ \ z^{\mathrm{T}}(t-\tau)\ f^{\mathrm{T}}(z(t))\ \ f^{\mathrm{T}}(z(t-\tau))]^{\mathrm{T}}$

On the other hand, it is clear from (5) that

$$f_i(z_i(t)[f_i(z_i(t)) - k_i z_i(t)] \le 0, \quad i = 1,2\cdots n \tag{9}$$

and

$$f_i(z_i(t-\tau))[f_i(z_i(t-\tau)) - k_i z_i(t-\tau)] \le 0, \quad i = 1,2\cdots n \tag{10}$$

Thus, for any $T = diag(t_1,t_2,\ldots,t_n) \ge 0$ and $S = diag(s_1,s_2,\ldots,s_n) \ge 0$, it follows from (9) and (10) that

$$\begin{aligned} 0 \le & -2\sum_{i=1}^{n} t_i f_i(z_i(t))[f_i(z_i(t)) - k_i z_i(t)] \\ & -2\sum_{i=1}^{n} s_i f_i(z_i(t-\tau))[f_i(z_i(t-\tau)) - k_i z_i(t-\tau)] \\ & = 2z^{\mathrm{T}}(t)KTf(z(t)) - 2f^{\mathrm{T}}(z(t))Tf(z(t)) \\ & +2z^{\mathrm{T}}(t-\tau)KSf(z(t-\tau)) - 2f^{\mathrm{T}}(z(t-\tau))Sf(z(t-\tau)) \end{aligned} \tag{11}$$

Adding the term on the right side of (8),(11) to $\dot{V}(z(t))$ yields

$$\dot{V}(z(t)) \le \frac{1}{\tau}\int_{t-\tau}^{t} \eta^{\mathrm{T}}(t,s)\Phi_0\eta(t,s)\mathrm{d}s \tag{12}$$

where

$$\Phi_0 = \begin{bmatrix} \Phi_{11} + \tau C^{\mathrm{T}}Z_{22}C & \Phi_{12} & \Phi_{13} - \tau C^{\mathrm{T}}Z_{22}A & \Phi_{14} - \tau C^{\mathrm{T}}Z_{22}B & \Phi_{15} & \Phi_{16} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} \\ * & * & \Phi_{33} + \tau A^{\mathrm{T}}Z_{22}A & \Phi_{34} + \tau A^{\mathrm{T}}Z_{22}B & \Phi_{35} & \Phi_{36} \\ * & * & * & \Phi_{44} + \tau B^{\mathrm{T}}Z_{22}B & \Phi_{45} & \Phi_{46} \\ * & * & * & * & \Phi_{55} & \Phi_{56} \\ * & * & * & * & * & \Phi_{66} \end{bmatrix}$$

and $\eta(t,s) = [z^{\mathrm{T}}(t), z^{\mathrm{T}}(t-\tau), f^{\mathrm{T}}(z(t)), f^{\mathrm{T}}(z(t-\tau)), z^{\mathrm{T}}(s), \dot{z}^{\mathrm{T}}(s)]^{\mathrm{T}}$

and other parameters are defined in Theorem 1 and $*$ denotes the symmetric terms in a symmetric matrix.

By Schur complements, the matrix inequality $\Phi < 0$ is equivalent to $\Phi_0 < 0$, then $\dot{V}(z(t)) < -\varepsilon\|z(t)\|^2$ for a sufficiently small $\varepsilon > 0$ such that system (3) is asymptotically stable.

IV. EXAMPLE

In this section, one example is given to show the effectiveness of the theorem presented in this paper. The LMI is solved by the LMI-Toolbox in Matlab. See Nemirovskii (1995).

Example 1: Consider a delayed neural networks (1) with Xu, Lam, Ho and Zou (2005).

$C = diag(1.2769, 0.6231, 0.9230, 0.4480)$

$$A = \begin{bmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -0.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix}$$

$k_1 = 0.1137, k_2 = 0.1279, k_3 = 0.7994, k_4 = 0.2368.$

Table 1

Calculated upper bounds of τ for Example 1

Methods	τ
Cao and zhou (1998), Zhang, Ma, Xu (2001), Liao and Chen (2002), Arik (2003)	failed
Xu, Lam, Ho and Zou (2005)	1.4224
Park,and Cho (2007)	1.9320
Hua, Long and Guan (2006) Liu and Rees (2007a)	3.5841
This paper	3.6828

It is clear that our condition has improved the existing results both theoretically and numerically.

V. CONCLUSION

This paper has investigated the delay-dependent stability problem of delayed neural networks. By constructing a new augmented Lyapunov functional and employing free-weighting matrices, a less conservative delay-dependent stability criterion expressed in terms of LMI has been presented. A numerical example is given to demonstrate the reduced conservativeness of the proposed results.

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