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A Diagnostic Model For Identifying Parametric Faults

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Abstract: This paper considers a new approach to fault detection and isolation (FDI) for systems modeled as an interconnection of subsystems that are each subject to parametric faults. The paper develops i) the concept of a *diagnostic model* that parameterizes all possible subsystem faults, ii) an off-line scheme for identification of the diagnostic model, iii) a parity equation that results in a residual that is a linear function of the change in the *diagnostic parameters* and iv) a fault isolation scheme that does not require a recursive least squares type identifier.

1. INTRODUCTION

This paper considers a new approach to fault detection and isolation (FDI) for systems modeled by an interconnection of subsystems that are each subject to parametric faults. Each subsystem is modelled as a transfer function and a fault occurs in a subsystem when one or more of its transfer function coefficients change. The coefficients subject to failure are called the diagnostic parameters.

Existing approaches, (Simani *et al.*, 2003),(Gertler, 1998), use an on-line identifier that usually takes the form of a recursive least squares filter to estimate the parameters, θ, of a transfer function. The overall system is modeled by a transfer function, $P(z, \theta)$, with the input, r, and parametric faults represented by changes in the diagnostic parameter vector, γ.

The diagnostic parameters influence the system parameters through the nonlinear function, $\theta = \varphi(\gamma)$. At a particular time instant, measurements of the output, y, (corrupted by noise, v), and measurements of the input, r, are used by the on-line identifier to generate the estimate, $\hat{\theta}$. An inverse mapping, $\hat{\gamma} = g(\hat{\theta})$, where $\varphi(g()) = I$, is used to compute $\hat{\gamma}$. Faults are then detected and isolated based on the difference, $\Delta\gamma = \gamma - \hat{\gamma}$. One issue with this approach is the requirement to know $\varphi()$ a-priori. If it is based on a simplified linear model then there may be significant uncertainty in $\varphi()$. Another issue is the effect of measurement noise, v. Even if $\varphi()$ is known exactly there will be errors in the estimate, $\hat{\theta}$, as a result of the noise, v.

We take a different approach. First, we formalize the nonlinear function, $\varphi()$, for an interconnection of subsystems subject to parametric faults. For the class of all possible parametric faults including multiple faults, we show that $\theta = Q\rho$ where ρ is a vector with elements that are multi-linear in γ, and Q is a matrix that depends on the interconnection topology of the subsystems but is otherwise independent of ρ and γ.

We also develop a diagnostic model that governs the mapping among the system input, the system output and subsystem faults. . The notion of a diagnostic model is new. The diagnostic model characterizes the evolution of a *feature vector* and *influence matrix* as the diagnostic parameters change. We assume $\varphi()$ is not known a-priori and therefore the diagnostic model needs to be identified. The identification problem reduces to that of identifying the multilinearity matrix, Q, from measurements of r and y. It need only be identified once. However the identification procedure requires the ability to change γ over some range. An important consideration is the choice of the order of the identified model since there is uncertainty in the physical system that may take the form of unmodeled dynamics, disturbances, noise and nonlinear effects such as friction, deadzone and saturation.

The FDI scheme uses measurements of the system input, r, and output, y, to detect and isolate failures without the need for an on-line recursive parameter identifier. A parity equation is derived and used to form a residual that is a linear function of the change in the *diagnostic parameters*. A fault is detected whenever the moving average of the residual energy exceeds a threshold value. Once detected, the isolation scheme uses on a pattern classification paradigm and a Bayes decision strategy where the maximum correlation between the measured residual and a number of residual estimates is generated by a set of failure hypotheses

The paper first lays the mathematical foundation for the diagnostic model. Then we show how the diagnostic model may be identified using measurements of the system input and output. Next we show how faults may be isolated using on-line measurements and a decision making strategy that is based on the identified diagnostic model. The complete methodology including identification of the diagnostic model and on-line fault detection and isolation was evaluated and verified on a physical system that consisted of a computer controlled servo system.

2. MATHEMATICAL MODEL

The system is assumed to be a linear time invariant system described by,

$$\boldsymbol{D}(z^{-1})y(k) = \boldsymbol{N}(z^{-1})r(k) + \boldsymbol{v}(k) \tag{2.1}$$

$$\begin{aligned} \boldsymbol{D}(z^{-1}) &= 1 - \boldsymbol{a}_1 z^{-1} - \boldsymbol{a}_2 z^{-2} - \boldsymbol{a}_3 z^{-3} - \ldots - \boldsymbol{a}_L z^{-L} \\ \boldsymbol{N}(z^{-1}) &= \boldsymbol{b}_0 + \boldsymbol{b}_1 z^{-1} + \boldsymbol{b}_2 z^{-2} + \boldsymbol{b}_3 z^{-3} + \ldots + \boldsymbol{b}_L z^{-L} \end{aligned} \tag{2.2}$$

where z^{-1} is a unit delay operator, $y(k)$ is the scalar output, $r(k)$ is the scalar input, $\boldsymbol{v}(k)$ is the totality of measurement noise and disturbances. The noise, $\boldsymbol{v}$(k), is assumed to have a rational spectra and uncorrelated with the input, $\boldsymbol{r}$(k),

$$\boldsymbol{v}(z) = G_v(z)\boldsymbol{\xi}(z) \tag{2.3}$$

where $\boldsymbol{\xi}$ represents zero mean white noise if $\boldsymbol{v}$ is a stochastic process or $\boldsymbol{\xi}$ represents a Kronecker delta function if $\boldsymbol{v}$ is a deterministic process such as a bias or sinusoidal. The *feature vector*, $\boldsymbol{\theta}$, is defined as a $(2L+1)\times 1$ vector of the coefficients of $\boldsymbol{N}(q^{-1})$ and $\boldsymbol{D}(q^{-1})$,

$$\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \ldots & \boldsymbol{a}_L & \boldsymbol{b}_0 & \boldsymbol{b}_1 & \cdots & \boldsymbol{b}_L \end{bmatrix}^T \tag{2.4}$$

The feature vector, $\boldsymbol{\theta}$, and the transfer function, $\boldsymbol{T}(z) = \boldsymbol{N}(z)/\boldsymbol{D}(z)$, are equivalent representations. The mathematical model may also be represented in θ-form,

$$y(k) = \psi^T(k)\boldsymbol{\theta} + \boldsymbol{v}(k) \tag{2.5}$$

where $\psi(k)$ is the *regressor* or *data vector*,

$$\psi(k) = \left[y(k-1)\ y(k-2)\ .\ y(k-L)\ r(k)\ r(k-1)\ .\ r(k-L) \right]^T \tag{2.6}$$

3. PARAMETERIC FAULT MODEL

The overall system consists of an interconnection of subsystems, $G_i(z)$, $i = 1, 2, \ldots n_f$. Each subsystem, $G_i(z)$, is a transfer function that may represent a physical entity such as a sensor, actuator, controller or other system component that is subject to parametric faults. Each subsystem may be driven by additive noise or disturbance input. The numerator and denominator coefficients of subsystem, $G_i(z)$, form a $(q_i \times 1)$ vector, γ^i. The *diagnostic parameter,* γ, is a $(q\times 1)$ vector that augments the coefficients of all subsystems, γ^i ,1,2, n_f,

$$\gamma = \begin{bmatrix} \gamma^1 & \gamma^2 & \gamma^3 & \ldots & \gamma^{n_f} \end{bmatrix}^T = \begin{bmatrix} \gamma_1 & \gamma_2 & \ldots & \gamma_q \end{bmatrix}^T$$

3.1 Illustrated example

Consider the discrete model of a sampled data servo control system. It consists of subsystems that include a PID controller, a PWM amplifier, position sensor, velocity sensor and the open loop motor dynamics**.** Let's say there are four subsystems subject to failure: the controller, position sensor, velocity sensor and the amplifier. Then the diagnostic parameter, γ, is given by the 5×1 vector,

$\gamma = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & \gamma_5 \end{bmatrix}^T$, where,

$$\begin{aligned} &\gamma^1 = \gamma_1 = K_s,\quad \gamma^2 = \gamma_2 = K_v, \\ &\gamma^3 = \gamma_3 = K_a,\quad \gamma^4 = \begin{bmatrix} k_p & k_I \end{bmatrix}^T = \begin{bmatrix} \gamma_4 & \gamma_5 \end{bmatrix}^T \end{aligned} \tag{3.3}$$

If the controller is not subject to failure then the diagnostic parameter vector, γ, is given by,

$$\gamma = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix}^T = \begin{bmatrix} K_s & K_v & K_a \end{bmatrix}^T \tag{3.4}$$

The feature vector, θ, is a 6×1 vector given by,

$$\theta = \begin{bmatrix} -\left[\gamma_2\gamma_3 k_1 k_d - \alpha - 2\right] \\ -\left[k_p k_1 k_2 \gamma_1\gamma_3 + 2\alpha + 1 - 2k_1 k_d \gamma_2\gamma_3\right] \\ -\left[k_1 k_2\left(k_i - k_p\right)\gamma_1\gamma_3 + k_1 k_d \gamma_2\gamma_3 - \alpha\right] \\ 0 \\ k_p k_1 k_2 \gamma_1\gamma_3 \\ k_1 k_2\left(k_i - k_p\right)\gamma_1\gamma_3 \end{bmatrix} \tag{3.5}$$

The parameter γ is assumed to be measured. It is assumed that the feature vector, θ, is multilinear in γ. Define a set Υ formed of the monomials of $\{\gamma_i\ i = 1,2,...,q\}$ of the order 0,1,2,..,q, that is, Υ, is a set formed of unity and the products of $\{\gamma_i\ i = 1,2,...,q\}$ taken one at-a-time, two at-a-time and so on till q at-a-time given by

$$\Upsilon = \left\{1\ \gamma_1 \cdot \gamma_i \cdot \gamma_1\gamma_2 \cdot \gamma_i\gamma_j \cdot \gamma_1\gamma_2\gamma_3 \cdot \gamma_i\gamma_j\gamma_k \cdot \gamma_1\gamma_2\gamma_3\gamma_4 .. \gamma_q\right\} \tag{3.6}$$

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The feature vector θ is multilinear in γ and the map $\gamma \to \theta$ takes the form

$\theta = Q\rho$

ρ is a Mx1 vector formed of the elements of Υ,

$$\rho = \begin{bmatrix} \rho_1 & \rho_2 & \rho_3 & . & \rho_M \end{bmatrix}^T,\quad \rho_i \in \Gamma \tag{3.7}$$

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and Q is a (2L+1)xM matrix. It does not depend upon the parameter, γ and depends only upon the system parameters, which do not vary with time. The matrix, Q depends upon the topology of the system: the interconnection of the functional units forming the system. The dimension of ρ has an upper bound $M \le 2^q$. However, in practice M is much smaller its upper bound.

Example: Consider the example in Section 3.1. The multilinear relationship, $\theta = Q\rho$, is given by,

$$Q = \begin{bmatrix} \alpha+2 & 0 & -k_1 k_d \\ -2\alpha-1 & -k_p k_1 k_2 & 2k_1 k_d \\ \alpha & -k_1 k_2\left(k_i - k_p\right) & -k_1 k_d \\ 0 & 0 & 0 \\ 0 & k_p k_1 k_2 & 0 \\ 0 & k_1 k_2\left(k_i - k_p\right) & 0 \end{bmatrix} \quad \rho = \begin{bmatrix} 1 \\ \gamma_1\gamma_3 \\ \gamma_2\gamma_3 \end{bmatrix} \tag{3.8}$$

where ρ is a subset of the elements of Υ given by (3.6).

4. **THE DIAGNOSTIC MODEL**

The idea is to derive a diagnostic model that characterizes the interaction among the system input, $r(k)$, the error term, $v(k)$, and the changes in the diagnostic parameter vector, $\Delta\gamma = \gamma - \gamma_0$ about a nominal value, γ_0. At time instant, k, the diagnostic parameter vector, $\gamma(k)$, influences the feature vector, $\theta(k)$. The input $r(k)$ affects the data vector, $\psi(k)$, and the noise term, $v(k)$, corrupts the output, $y(k)$. The diagnostic model is a map, $\Im : \left[r(k)\ v(k)\ \gamma(k)\right] \Rightarrow y(k)$, that may be described by the recursive model,

$$\rho(k) = \rho^0 + \Delta\rho(k)$$
$$y(k) = \psi^T(k)Q\rho(k) + v(k) \tag{4.1}$$

where ρ^0 is the nominal value of ρ^0, and $\Delta\rho(k)$ denotes the variation in the multilinear diagnostic parameter. Let $\rho_i^{(1)}$, $\rho_{ij}^{(2)}$, $\rho_{ijk}^{(3)}$,.... $\rho_{1,2..n_f}^{(n_f)}$ denote the first, second, third and up to n_f-th partial of ρ with respect to Υ as given by,

$$\rho_i^{(1)} = \frac{\partial\rho}{\partial\gamma^i},\ \rho_{ij}^{(2)} = \frac{\partial^2\rho}{\partial\gamma^i\partial\gamma^j},\ \dots,\ \rho_{1,2\dots n_f}^{(n_f)} = \frac{\partial^{n_f}\rho}{\partial\gamma^1\partial\gamma^2\dots\partial\gamma^{n_f}} \tag{4.2}$$

The associated Jacobian matrix, such as $\rho_{ijk}^{(3)}$, may be obtained by replacing those elements of $\rho(k)$ by 1 if they contain γ^i, γ^j, γ^k, and by zero otherwise. Now consider the following M hypotheses where $M = 2^q$.
Using (4.1) and (4.2) the expression for $y(k)$ becomes

$$y(k) = \psi^T(k)Q\left(\rho^0 + \Delta\rho(k)\right) + v(k) \tag{4.3}$$

Substituting for $\Delta\rho(k)$ we obtain the diagnostic model,

$$y(k) = \psi^T(k)Q\wp(k)\Delta\Upsilon(k) + v(k) \tag{4.4}$$

where,

$$\Delta\Upsilon = \left[1\ \Delta\gamma_1\ .\ \Delta\gamma_1\Delta\gamma_2\ .\ \Delta\gamma_1\Delta\gamma_2\Delta\gamma_3\ .\ \prod_{i=1}^{q}\Delta\gamma_i\right]^T \tag{4.5}$$

and the matrix, $\wp$, is the Jacobian of ρ with respect to all the members in the set, Υ,

$$\wp(k) = \left[\rho^0\ \rho_1^{(1)}(k)\ \rho_{ij}^{(2)}(k)\ \rho_{ijk}^3(k)\ \rho_{12\dots q}^{(q)}(k)\right]^T \tag{4.6}$$

The diagnostic model is characterized completely by Q.

5. IDENTIFICATION OF THE DIAGNOSTIC MODEL

Identification of the diagnostic model reduces to identifying Q using measurements of y, r and γ. The diagnostic model need be identified only once but it requires access to γ. This is a constraint of the methodology. In the absence of a-priori knowledge of Q, adjustment of the elements of γ is required in the identification stage. Once the data is collected, the elements of Q are identified using a least squares formulation.To reduce the effects of noise and disturbances both the input and the output are filtered. Applying the filtering operation to both sides of (2.5) we get,

$$y_f(k) = \psi_f^T(k)Q\wp(k)\Delta\Upsilon(k) + \xi(k) \tag{5.1}$$

where ξ is white noise and $\psi_f(k)$ is the filtered data vector formed from the filtered input and the filtered output,

5.2. Perturbed parameter experiments

Substituting $\Delta\Upsilon(k)$ from (4.11) into (5.4) we get,

$$y_f(k) = \psi_f^T(k)\left[\theta^0 + \sum_i \theta_i^{(1)}\Delta\gamma_i + \sum_i\sum_j \theta_{ij}^{(2)}\Delta\gamma_i\Delta\gamma_j + \sum_i\sum_j\sum_k \theta_{ijk}^{(3)}\Delta\gamma_i\Delta\gamma_j\Delta\gamma_k\cdots\right] + \xi(k) \tag{5.2}$$

where $\theta^0 = Q\rho^0,\ \theta_i^{(1)} = Q\rho_i^{(1)},\ \theta_{ij}^{(2)} = Q\rho_{ij}^{(2)},\ \theta_{ijk}^{(3)} = Q\rho_{ijk}^{(3)},..$ are $2L\times 1$ vectors representing the partial derivatives as defined in (4.2). The criterion for identifying Q is that the mean-squared error of the identified diagnostic model, $\hat{y}(k)$, be less than some specified ε for all perturbations of $\Delta\gamma$ in a given range, δ,

$$\min_Q\left\{\left\|y(k) - \hat{y}(k)\right\|\right\} \le \varepsilon \quad \forall \quad \|\Delta\gamma\| \le \delta \tag{5.3}$$

To identify Q, a series of experiments is performed for particular values of γ and an associated data record, $\{y_f(k-i)\quad r_f(k-1)\}$, $i = 0, 1, 2, \dots N - 1$, is obtained. In general the experiments consists of perturbing the physical parameters, γ, one-at-a time, two-at-a-time, three-at-a-time and so on until all of the parameters have been perturbed. If ρ contains no product terms then the parameters need be perturbed one-at-time. If it contains a double product, $\gamma_i\gamma_j$, then two parameters γ_i and γ_j need be perturbed and so on.

Consider the example of section 3. We will choose ρ to contain all combinations of the parameters, γ_1, γ_2, and γ_3 instead of using the mathematical model to define the vector as, $\rho^T = [1\quad \gamma_1\gamma_3\quad \gamma_2\gamma_3]$, to emphasize the case when the structure as well as the model are unknown.

The vector ρ is

$$\rho^T = [1\quad \gamma_1\quad \gamma_2\quad \gamma_3\quad \gamma_1\gamma_2\quad \gamma_1\gamma_3\quad \gamma_2\gamma_3\quad \gamma_1\gamma_2\gamma_3]$$

The model is

$$y_f(k) = \psi_f^T(k)\left[\theta^0 + \sum_{i=1}^{3}\theta_i^{(1)}\Delta\gamma_i + \sum_{i=1}^{3}\sum_{j=1}^{3}\theta_{ij}^{(2)}\Delta\gamma_i\Delta\gamma_j + \theta_{123}^{(3)}\Delta\gamma_1\Delta\gamma_2\Delta\gamma_3\right] + v(k) \tag{5.4}$$

The number of experiments to be performed is M=8. In the first experiment no parameters is perturbed and N input and output data record is obtained.

$$\mathbf{y}^0(k) = \boldsymbol{\psi}_0^T(k)\theta^0 + v(k) \tag{5.5}$$

In second experiment γ is varied one at-a-time and we

$$\mathbf{y}^i(k) - \boldsymbol{\psi}_i^T(k)\theta^0 = \boldsymbol{\psi}_i^T(k)\theta_i^{(1)}\Delta\gamma_i + v(k)\quad i = 1,2,3 \tag{5.10}$$

In third experiment γ is varied two-at-a-time and we get

$$\mathbf{y}^{ij}(k) - \boldsymbol{\psi}_{ij}^T(k)\left(\theta^0 + \sum_{i=1}^{3}\theta_i^{(1)}\Delta\gamma_i\right) = \boldsymbol{\psi}_{ij}^T(k)\theta_{ij}^{(2)}\Delta\gamma_i\Delta\gamma_j + v(k) \tag{5.11}$$

In fourth experiment all elements of γ is varied. We get

$$\mathbf{y}^{123}(k)-\boldsymbol{\psi}_{123}^{T}(k)\left(\theta^{0}+\sum_{i=1}^{3}\theta_{i}^{(1)}\Delta\gamma_{i}+\sum_{i=1}^{3}\sum_{j=1}^{3}\theta_{ij}^{(2)}\Delta\gamma_{i}\Delta\gamma_{j}\right) \quad (5.6)$$
$$=\boldsymbol{\psi}_{123}^{T}(k)\theta_{123}^{(3)}\Delta\gamma_{1}\Delta\gamma_{2}\Delta\gamma_{3}+v(k)$$

where $\mathbf{y}^{i}(k)$, $\mathbf{y}^{ij}(k)$, and $\mathbf{y}^{123}(k)$ are Nx1 vector, $\theta_{i}^{(1)}$, $\theta_{ij}^{(2)}$ .and $\theta_{123}^{(3)}$ are 6x1 vector and $\boldsymbol{\psi}_{i}^{T}(k)$, $\boldsymbol{\psi}_{ij}^{T}(k)$, and $\boldsymbol{\psi}_{123}^{T}(k)$ is Nx6 matrix. The estimates of, $\theta_{i}^{(1)}$, $\theta_{ij}^{(2)}$ .and $\theta_{123}^{(3)}$, are obtained recursively using SVD based least-squares approach. After estimating them, the columns of the Q matrix are obtained recursively.

6. FAULT ISOLATION

6.1. Assumptions

We assume that at a given time instant, k, only one subsystem may change, i.e., $\gamma^{j}(k)$ may change while $\gamma^{i}(k), i\neq j$ do not change, and the diagnostic parameters remain constant during the execution of the fault diagnostics algorithm.

6.2. Parity equation

The residual, $e_f(k)$, is generated by the parity equation,

$$e_f(k)=y_f(k)-\psi_f^{T}(k)\theta^{0} \quad (6.1)$$

where θ^{0} is the nominal value of the feature vector. The residual has the property,

$$e_f(k)\begin{cases}=\xi(k) & no\ fault \quad \theta^{0}=\theta\\ \neq\xi(k) & fault \quad \theta^{0}\neq\theta\end{cases} \quad (6.2)$$

6.3. Model of the residual

Denote the nominal value of θ, ρ and γ by $\theta^{0}=Q\rho^{0}$, ρ^{0} and γ^{0}. Then,

$$e_f(k)=\psi_f^{T}(k)Q\left[\rho(k)-\rho^{0}\right]=\psi_f^{T}(k)Q\Delta\rho(k)$$
(6.3)

If hypothesis H_j and Assumptions (i) and (ii) hold then,

$e_f(k)=\Phi^{j}(k)\Delta\gamma^{j}(k)+\xi(k)$ (6.4)

where $\Phi^{j}(k)=\psi_f^{T}(k)\Omega^{j},\quad \Omega^{j}=Q\rho_j^{(1)}=\theta_j^{(1)}$ (6.5)

A record of N samples of $\{e_f(k-i): i=0,\ 1,\ ...,\ N-1\}$ is employed to isolate a fault at time instant, k. N is chosen sufficiently large to attenuate the effect noise and sufficiently small to ensure a timely diagnosis. Since $\gamma(k)$ remains constant during the diagnosis interval, $[k-N+1,\ \ k]$, the parameters $\{\theta(k-i),\ \ \gamma(k-i),\ \ \rho(k-i)\}$ also remains constant. Using (6.3) we may write,

$$\mathbf{z}(k)=\boldsymbol{\Phi}^{j}(k)\Delta\gamma^{j}(k)+\xi(k) \quad (6.6)$$

where, $\mathbf{z}(k)=\left[e(k-1)\ e(k-2)\ .\ e(k-N+1)\right]^{T}$

$$\boldsymbol{\Psi}_{\mathbf{f}}^{\mathbf{T}}(k)=\left[\psi_f(k)\ \psi_f(k-1)\ .\ \psi_f(k-N+1)\right]^{T}$$

$$\xi(k)=\left[\xi(k)\ \xi(k-1)\ .\ \xi(k-N+1)\right]^{T}$$

and $\mathbf{z}$ is a *residual* vector, $\Delta\gamma^{j}$ is a $q_j\times 1$ vector, $\boldsymbol{\Phi}^{j}(k)=\Psi_{\mathbf{f}}^{\mathbf{T}}(k)\Omega^{j}$ is a $N\times q_j$ matrix called the *residual influence matrix* to distinguish it from the influence matrix, Ω^{j}. $\boldsymbol{\Phi}^{j}\Delta\gamma^{j}$ is the estimate of the residual and ξ the $N\times 1$ *noise* vector. Assumptions 1) and 2) in Section 6.1 imply the residual, $\mathbf{z}(k)$, is affine in the change in the diagnostic parameter, $\Delta\gamma^{j}(k)$.

6.4. The fault isolation scheme

The detection problem amounts to selecting the correct hypothesis, H_0: *no fault* or H_1: *fault*, given measurements of $z(k)$ defined by,

$$\begin{aligned}H_0:&\quad \mathbf{z(k)}=\xi(k)\\ H_1:&\quad \mathbf{z(k)}=\boldsymbol{\Phi}^{j}(k)\Delta\gamma^{j}(k)+\xi(k)\end{aligned} \quad (6.7)$$

There are a variety of well known and feasible algorithms for solving the detection problem in the literature. We will focus on the fault isolation scheme. Once a fault is detected, the residual is predicted under different hypotheses and then correlated with the measured residual to classify the fault. There are n_f hypotheses of the form,

$$H_i:\quad \mathbf{z(k)}=\boldsymbol{\Phi}^{i}(k)\Delta\gamma^{i}(k)+\xi(k)\quad i\ =\ 1,\ 2,\ ...\ n_f \quad (6.8)$$

If ξ is a zero mean Gaussian random variable, then the Bayes strategy suggests the *most likely* hypothesis, H_i, is the one that satisfies,

$$\min_{j}\left\|\mathbf{z}(k)-\boldsymbol{\Phi}^{j}(k)\Delta\gamma^{j}(k)\right\|^{2} \quad (6.9)$$

where $\|x\|^{2}=x^{T}\Sigma^{-1}x$ and Σ is the covariance matrix of ξ. Equation (6.10) says that the distance between the vectors $\mathbf{z}$ and the hyperplane generated by the columns of the residual influence matrix $\boldsymbol{\Phi}^{j}(k)\Delta\gamma^{j}(k)$ must be a minimum. Since the fault is unknown, $\Delta\gamma^{j}(k)$, is unknown. Therefore a composite hypothesis testing scheme is used in which we substitute the unknown $\Delta\gamma^{j}(k)$ by its least-squares estimate,

$$\Delta\gamma^{j}(k)=\left(\boldsymbol{\Phi}^{j}(k)\right)^{\dagger}\mathbf{z}(k) \quad (6.10)$$

$$\left\|\mathbf{z}(k)-\boldsymbol{\Phi}^{j}(k)\Delta\gamma^{j}(k)\right\|^{2}=\left\|\mathbf{z}(k)\right\|^{2}\left(1-\cos^{2}\varphi_j(k)\right) \quad (6.11)$$

$$\cos^{2}\varphi_j(k)=\frac{\mathbf{z}^{T}(k)\boldsymbol{\Psi}_f^{T}(k)\Omega^{j}\Sigma^{-1}(\boldsymbol{\Psi}_f^{T}(k)\Omega^{j})\ \mathbf{z}(k)}{\left\|\mathbf{z}(k)\right\|^{2}} \quad (6.12)$$

$$\cos\phi_j(k)=\frac{\langle\mathbf{z}(k),\boldsymbol{\Psi}_f^{T}(k)\Omega_j\rangle}{\left\|\mathbf{z}(k)\right\|\left\|\boldsymbol{\Psi}_f^{T}(k)\Omega_j\right\|} \quad (6.13)$$

If a fault has been detected then hypothesis, H_j, signifying a change in $\gamma^{j}(k)$, is asserted true where j is given by,

$$j = \arg\left\{\begin{matrix}\max \\ i\end{matrix}\left\{ \cos^2 \varphi_i(k)\right\}\right\} \quad (6.14)$$

The case of isolating a the change in a single diagnostic parameter, γ_j, has a simpler interpretation than the case of simultaneous changes in several elements of the vector, $\Delta\gamma^j$. The j^{th} subsystems is faulty if the angle between the residual vector **z** and the vector $\mathbf{\Phi}^j$ is minimum for $i = j$ indicating that the vectors are maximally aligned in a plane.

7. EVALUATION ON A PHYSICAL SYSTEM

The diagnostic model identification and FDI scheme was implemented and tested on the DC servo system. The motor was driven by a PWM amplifier. A tachogenerator and quardrature position encoder provided measurements of angular velocity and position. The control input to the PWM amplifier, u, is generated by a DAC on the target PC. The scaled velocity sensor voltage, v, is applied to the input of an ADC and the position sensor is interfaced to an incremental position decoder on the target PC. A host PC and target PC were used as part of a rapid prototyping system that included MATLAB, Simulink, Real Time Workshop, MS Visual C++ and xPC Target. The target PC boots a real-time kernel which permits feedback and signal processing algorithms to be downloaded from the host PC and executed in real time. The host PC and target PC communicate through a communication channel used for downloading compiled code from the host PC and exchanging commands and data.

With the initial choice of ρ the identified value of Q had full rank and none of the columns had negligible energy. Hence all of the elements of ρ were retained. However if the range of the diagnostic parameters, γ, was restricted to a very small region in neighbourhood of the nominal value, γ^0, a third order model was adequate with $\rho^T = [1 \quad \gamma_1\gamma_3 \quad \gamma_2\gamma_3]$. For different model orders, the time and frequency response of the identified model was analyzed over the range of diagnostic parameter variations

The model was identified by performing experiments in which γ_1, γ_2 and γ_3 were perturbed. The probing input was a square wave of 0.5 Hz and the sample frequency was 100Hz and the data records contained 1000 sample points. The set of experiments with diagnostic parameter values used in the identification phase is called the *training set*. The diagnostic parameters used in the validation test are termed the *validation set*. For different model orders the mean-squared error between the identified model output and the actual physical system using blind test input. For different model orders the mean-squared error between the identified model output and the actual physical system output using blind test input are listed in Table 7.1. The performance of the conventional scheme with a model order of 3 was compared. . The system was identified for the nominal values, $\gamma_i = 1$, i = 1, 2, 3 and validated when one of the diagnostic parameter varies and all others are fixed. In this case the identified model is not robust to parameter variations. Faults were injected by changing the diagnostic parameters stepwise in small increments. The sample period selected as 6 ms. The isolability measures are listed in Table 7.2. It can be seen that all faults are isolable, however, K_a versus K_s has poor isolability.

For comparison, the performance of the conventional scheme with a model order of 3 is given in Fig. 7.2 (a) when γ_1 changes. The system was identified for the nominal values, $\gamma_i = 1$, i = 1, 2, 3 and validated when one of the diagnostic parameter varies and all others are fixed. In this case the identified model is not robust to parameter variations. The performance of a 10^{th} order model identified using the scheme outlined in this paper is shown in Fig. 7.2 (b) when γ_1 in the physical system changes with the test set within the range of the training set. It can be seen that a very good fit is achieved. Fig. 7.2 (c) is similar to 7.2 (b) but with the test set outside the range of the training set. The actual and the estimated faults are shown in Fig. 7.4 where each of three faults in sequence increase in a stepwise fashion to 1.5 times the nominal value and then decreases suddenly back to the nominal value of unity. The implementation indicates that the methodology is able to capture incipient faults and sudden faults.

8. CONCLUSIONS

This paper presents a new approach both for modelling parametric faults and isolating parametric faults using the diagnostic model. The diagnostic model eliminates the need for on-line parameter identification to isolate parametric faults. The basic idea is that parametric faults in particular subsystems are isolated on the basis of their propagation to the parameters of the overall system. This requires knowledge of the influence matrix or Jacobian of the system parameters with respect to the diagnostic parameters. We assume this relation is unknown and is identified off-line, greatly simplifying the on-line calculations required for fault detection and isolation. The scheme for identifying the diagnostic model and isolating parametric faults in real time was successfully implemented, verified and tested on a DC servo control system.

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Table 7.1. Maximum and average model error

Order	3	10	25
Max Error	0.1371	0.0880	0.0601
Avg Error	0.0139	0.0106	0.0101

Table 7.2. Worst case of isolability.

γ_i	γ_j	$max(\lvert cos\ \varphi_{ij} \rvert)$
K_v	K_s	0.6255
K_v	K_a	0.8005
K_s	K_a	0.7817

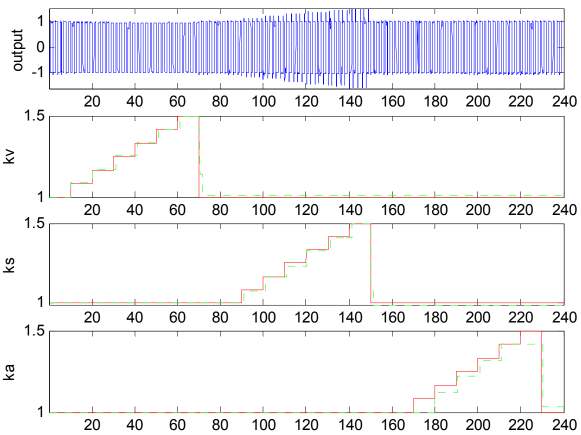

Fig.7.4. Implementation of fault isolation showing the faults, (solid lines) and the fault estimates (dotted lines).

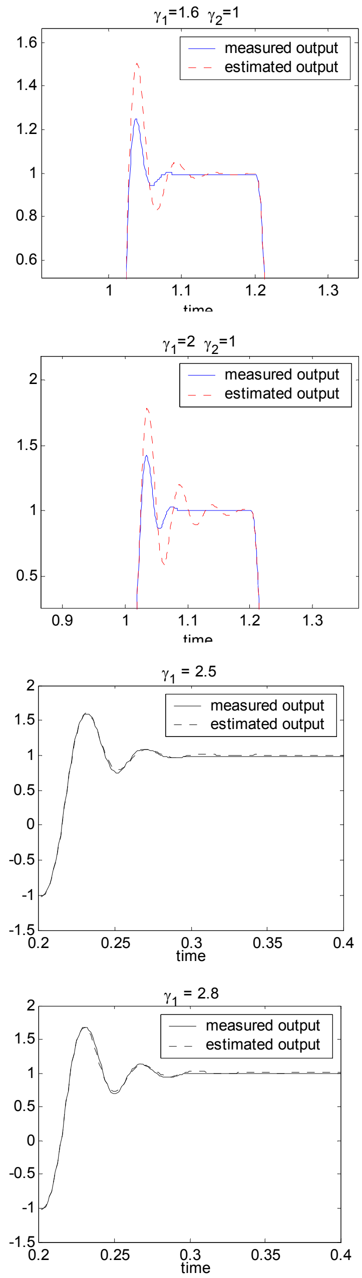

Fig. 7.2. Comparison of the physical system with the output when γ_1 in the physical system changes. The top two figures a 3^{th} order diagnostic model and bottom two are 10^{th} order model.