Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Stability Analysis of Multi-input and Multi-output Networked Control Systems

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Abstract: The stability of multi-input and multi-output networked control systems (MIMO NCS) is considered in this paper. A less conservative delay dependent stability criterion for MIMO NCS is proposed in terms of linear matrix inequality. The new result is obtained by taking the relationship between the delay upper bounds into account in Lyapunov functional, estimating the upper bound of the derivative of Lyapunov functional without ignoring the additional useful terms and introducing the new free-weighting matrices. The resulting criterion is also extended to the stability analysis for MIMO NCS with time-varying structured uncertainties. Numerical example demonstrates that the proposed criteria are less conservative than the existing ones.

1. INTRODUCTION

Networked control systems (NCS) where the control loops are closed over a shared communication network have attracted significant attention in recent years (Antsaklis and Baillieul, 2004; Bushnell, 2001; Tipsuwan and Chow, 2003). The components such as sensors, controllers and actuators exchange information across the shared communication network in NCS. So compared with the traditional control systems using point-to-point architecture, NCS have the advantages of high flexibility, low cost, easy maintenance and reconfiguration. Consequently, NCS have great potential in a broad range of areas such as modern manufacturing plants, vehicles, and aircrafts, etc.

The change of communication architecture from point-to-point to shared network brings about some new problems which complicate the analysis and design of NCS (Zhang, Branicky, and Phillips, 2001). When the controlled plant is a multi-input and multi-output system, the sensors, controllers and actuators of the NCS are usually distributed over a large physical area. Moreover, packet switched networks can only carry limited information in a packet due to packet size constraints. Therefore, it is impossible to put the data of the multi-input and multi-output networked control systems (MIMO NCS) into one packet. The measurement and control data are often broken into multiple packets to be transmitted. Another problem is that the network-induced delay is inevitable when data are transmitted over the shared communication network. Depending on the adopted communication protocol, the network-induced delay may be constant or time-varying. In addition, the network is an unreliable transmission link. Some packets may be lost during transmission due to external disturbance. We assume that there is no data packet dropout in this paper for simplicity.

Many efforts have been made to the stability analysis of MIMO NCS in the last few years. The NCS with multiple-packet transmission are modelled as an asynchronous dynamical system without considering the effect of network-induced delay in Zhang, Branicky, and Phillips (2001). Lian, Moyne, and Tilbury (2001) analyze and model a MIMO NCS with multiple time delays. Kim, Lee, Kwon, and Park (2003) propose a method to obtain a maximum allowable delay bound of the MIMO NCS in terms of linear matrix inequality. Based on the delay obtained through the proposed method, a scheduling method is also presented. In Yan, Huang, Wang, and Zhang (2007) and Yan, Huang, Wang, and Zhang (2006 b), the network-induced delay is assumed to be constant. The MIMO NCS are described as a system with multiple time delays. The delay dependent robust stability criteria and delay independent robust stability criteria are presented respectively. He, Wu, and She (2006) propose delay dependent stability criteria for linear systems with multiple constant delays. It is well known that in practical applications, the delays are usually time-varying. He, Wang, Xie, and Lin (2007) present delay dependent stability criteria for system with a time-varying delay. Yan, Huang, Wang, and Zhang (2006 a) propose delay dependent robust stability criteria for MIMO NCS with multiple time-varying delays in terms of linear matrix inequality. However, the obtained results are still conservative.

In this paper, new delay dependent stability criteria for MIMO NCS are proposed by constructing new Lyapunov functional and using new method to estimate the upper bound of the derivative of Lyapunov functional. Compared with the existing results, the proposed ones are less conservative.

2. PROBLEM FORMULATION

Consider the MIMO NCS shown as Fig.1. The controlled plant dynamics can be described by

$$\begin{cases} \dot{x}_p(t) = A_p x_p(t) + B_p u_p(t) \\ y_p(t) = C_p x_p(t) \end{cases} \tag{1}$$

978-1-1234-7890-2/08/$20.00 © 2008 IFAC 10.3182/20080706-5-KR-1001.2310

where $x_p(t) \in R^{n_p}$, $u_p(t) \in R^m$, $y_p(t) \in R^r$ are the plant state, input and output vector, respectively. A_p , B_p , C_p are known constant matrices of appropriate dimensions.

The controller dynamics can be described by

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c u_c(t) \\ y_c(t) = C_c x_c(t-\tau_c) + D_c u(t-\tau_c) \end{cases} \tag{2}$$

where $x_c(t) \in R^{n_c}$, $u_c(t) \in R^r$, $y_c(t) \in R^m$ are the controller state, input and output vector, respectively. A_c , B_c , C_c , D_c are known constant matrices of appropriate dimensions. $0 \le \tau_c \le \tau_{\max}$ is the computational delay of controller. It can be neglected if the computational rate of controller is fast enough.

Every output of the plant is measured by a sensor and is separately lumped into one packet to be transmitted. Based on the received data, the controller calculates the control commands and transmits them to the actuators in separate packet. Owing to the effect of the communication network, there exists the following relationship between the inputs and outputs of the controller and the plant:

$$\begin{cases} u_c^j(t) = y_p^j(t-\tau_{sc}^j), j = 1, 2, \cdots, r \\ u_p^i(t) = y_c^i(t-\tau_{ca}^i), i = 1, 2, \cdots, m \end{cases} \tag{3}$$

where $\tau_{sc}^j, j = 1, 2, \cdots, r$ is network-induced delay from sensor to controller; $\tau_{ca}^i, i = 1, 2, \cdots, m$ is network-induced delay from controller to actuator. Since each data packet is routed on different network paths, the network-induced delay is usually different from each other.

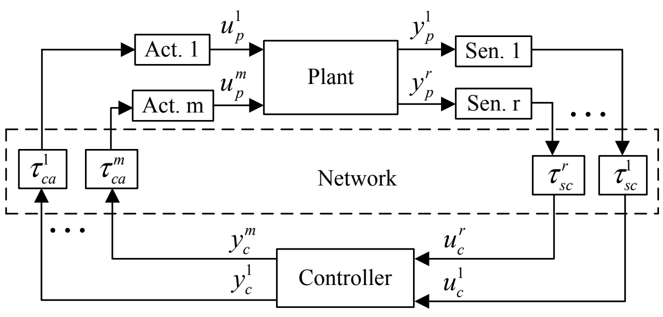

Fig.1. Schematic diagram of MIMO NCS

From (1)-(3), we can obtain

$$u_c(t) = E_1 x_p(t-\tau_{sc}^1) + E_2 x_p(t-\tau_{sc}^2) + \cdots + E_r x_p(t-\tau_{sc}^r) = \sum_{j=1}^{r} E_j x_p(t-\tau_{sc}^j)$$

$$u_p(t) = \sum_{i=1}^{m} F_i x_c(t-\tau_{ca}^i-\tau_c) + \sum_{i=1}^{m}\sum_{j=1}^{r} G_{ij} x_p(t-\tau_{ca}^i-\tau_{sc}^j-\tau_c)$$

where the j line of E_j is C_p^j , and the other lines are all zero vectors. The i line of F_i is C_c^i , and the other lines are all zero vectors. The i line of G_{ij} is $D_c^i E_j$,and the other lines are all zero vectors.

Set the augmented vector $x(t) = \left[x_p^T(t), x_c^T(t) \right]^T$, the MIMO NCS can be modelled as a system with multiple time delays:

$$\dot{x}(t) = Ax(t) + \sum_{j=1}^{r} A_j x(t-\tau_1^j) + \sum_{i=1}^{m} B_i x(t-\tau_2^i) + \sum_{i=1}^{m}\sum_{j=1}^{r} C_{ij} x(t-\tau_3^{ij}) \tag{4}$$

where

$$A = \begin{bmatrix} A_p & 0 \\ 0 & A_c \end{bmatrix}, A_j = \begin{bmatrix} 0 & 0 \\ B_c E_j & 0 \end{bmatrix}, B_i = \begin{bmatrix} 0 & B_p F_i \\ 0 & 0 \end{bmatrix}, C_{ij} = \begin{bmatrix} B_p G_{ij} & 0 \\ 0 & 0 \end{bmatrix},$$

$0 \le \tau_1^j = \tau_{sc}^j \le \tau_{sc,\max}^j = \tau_{1,\max}^j$,

$0 \le \tau_2^i = \tau_{ca}^i + \tau_c \le \tau_{ca,\max}^i + \tau_{c,\max} = \tau_{2,\max}^i$,

$0 \le \tau_3^{ij} = \tau_{sc}^j + \tau_{ca}^i + \tau_c \le \tau_{sc,\max}^j + \tau_{ca,\max}^i + \tau_{c,\max} = \tau_{3,\max}^{ij}$.

For simplicity, we shall make the following assumptions: $A_{r+i} = B_i$, $A_{m+ir+j} = C_{ij}$, $\tau_j = \tau_1^j$, $\tau_{r+i} = \tau_2^i$, $\tau_{m+ir+j} = \tau_3^{ij}$, $n = m + r + mr$, $i = 1, \cdots, m$, $j = 1, \cdots, r$. In addition the network-induced delay is often time-varying, so the nominal MIMO NCS can be written as:

$$\begin{cases} \dot{x}(t) = Ax(t) + \sum_{i=1}^{n} A_i x(t-\tau_i(t)), t > 0 \\ x(t) = \phi(t), t \in [-\tau, 0] \end{cases} \tag{5}$$

where A , A_i , $i = 1, \cdots, n$ are known real constant matrices with appropriate dimensions. The time delay $\tau_i(t)$ is time-varying continuous function that satisfies $0 \le \tau_i(t) \le \tau_i \le \tau$, $\dot{\tau}_i(t) \le \mu_i \le \mu < 1$, $i = 1, \cdots, n$. Without loss of generality, we assume that $\tau_1 \le \tau_2 \le \cdots \le \tau_n \le \tau$. $\phi(t)$ is a vector-valued initial continuous function of $-\tau \le t \le 0$.

In practice, it is almost hard to get an exact model of a system due to the complexity of the system, environmental noises, uncertain and/or time-varying parameters, etc. Therefore, the MIMO NCS with time-varying structured uncertainties can be described by

$$\begin{cases} \dot{x}(t) = (A + \Delta A(t))x(t) + \sum_{i=1}^{n} (A_i + \Delta A_i(t))x(t-\tau_i(t)), t > 0 \\ x(t) = \phi(t), t \in [-\tau, 0] \end{cases} \tag{6}$$

where the time-varying structured uncertainties $\Delta A(t), \Delta A_i(t)$ are of the following forms

$$\Delta A(t) = DF(t)E \, , \Delta A_i(t) = D_i F_i(t) E_i \, , i = 1, \cdots, n \, .$$

D , E , D_i , E_i are known real constant matrices with appropriate dimensions, $F(t)$, $F_i(t)$ are unknown real time-varying matrices with Lebesgue measurable elements bounded by $F^T(t)F(t) \le I$, $F_i^T(t)F_i(t) \le I$, $\forall t, i = 1, \cdots, n$.

Before presenting the main results, we give the following lemma, which will be used in the sequel.

Lemma 1 *Xie, (1996).* Given matrices $Q = Q^T$, H , E and $R = R^T > 0$ of appropriate dimensions, for all F satisfying $F^T F \le R$, $Q + HFE + E^T F^T H^T < 0$ if and only if there exists some $\lambda > 0$ such that $Q + \lambda H H^T + \lambda^{-1} E^T R E < 0$.

3. STABILITY ANALYSIS OF MIMO NCS

In this section, we establish some new delay dependent stability criteria for MIMO NCS. First we present the stability criterion of the nominal MIMO NCS.

Theorem 1. Given scalars $0 \le \tau_i \le \tau$, $\mu_i \le \mu < 1, i = 1, \cdots, n$, for all time-varying delays $0 \le \tau_i(t) \le \tau_i \le \tau$, $\dot{\tau}_i(t) \le \mu_i \le \mu < 1$, the nominal MIMO NCS is asymptotically stable if there exist matrices $P = P^T > 0$, $Q_i = Q_i^T > 0$, $R_i = R_i^T > 0$, $Z_{i1} = Z_{i1}^T > 0$, $Z_{i2} = Z_{i2}^T > 0$, $U_{kj} = U_{kj}^T > 0$, $k = 1, \cdots, n-1$, $j = k+1, \cdots, n$ and any appropriately dimensioned matrices

$$H^i = \begin{bmatrix} H_1^i \\ \vdots \\ H_{2n+1}^i \end{bmatrix}, S^i = \begin{bmatrix} S_1^i \\ \vdots \\ S_{2n+1}^i \end{bmatrix}, M^i = \begin{bmatrix} M_1^i \\ \vdots \\ M_{2n+1}^i \end{bmatrix}, N^{kj} = \begin{bmatrix} N_1^{kj} \\ \vdots \\ N_{2n+1}^{kj} \end{bmatrix}$$

such that the following LMI holds:

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ * & \Pi_{22} & 0 \\ * & * & \Pi_{33} \end{bmatrix} < 0 \qquad (7)$$

where

$$\Pi_{11} = \Phi_1 + \Phi_2 + \Phi_2^T,$$

$$\Phi_1 = \begin{bmatrix} \hat{\Phi}_1 & PA_1 & 0 & \cdots & PA_n & 0 \\ * & -(1-u_1)Q_1 & 0 & \cdots & 0 & 0 \\ * & * & -R_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \cdots & -(1-u_n)Q_n & 0 \\ * & * & * & \cdots & * & -R_n \end{bmatrix},$$

$$\hat{\Phi}_1 = PA + A^T P + \sum_{i=1}^{n} Q_i + \sum_{i=1}^{n} R_i ,$$

$$\Phi_2 = \Big[\sum_{i=1}^{n} (H^i + M^i), -H^1 + S^1, -S^1 - M^1 + \sum_{j=2}^{n} N^{1j}, \cdots, -H^k + S^k, -S^k - M^k - \sum_{i=1}^{k-1} N^{ik} + \sum_{j=k+1}^{n} N^{kj}, \cdots, -H^n + S^n, -S^n - M^n - \sum_{i=1}^{n-1} N^{in} \Big],$$

$$\Pi_{12} = \big[\tau_1 H^1, \tau_1 S^1, \tau_1 M^1, \cdots, \tau_n H^n, \tau_n S^n, \tau_n M^n, (\tau_2 - \tau_1) N_{12}, \cdots, (\tau_n - \tau_1) N_{1n}, \cdots, (\tau_{k+1} - \tau_k) N_{k(k+1)}, \cdots, (\tau_n - \tau_k) N_{kn}, \cdots, (\tau_n - \tau_{n-1}) N_{(n-1)n} \big],$$

$$\Pi_{13} = [A, A_1, 0, \cdots, A_n, 0]^T W ,$$

$$W = \left[\sum_{i=1}^{n} \tau_i (Z_{i1} + Z_{i2}) + \sum_{k=1}^{n-1} \sum_{j=k+1}^{n} (\tau_j - \tau_k) U_{kj} \right],$$

$$\Pi_{22} = -diag\{ \tau_1 Z_{11}, \tau_1 Z_{11}, \tau_1 Z_{12}, \cdots, \tau_n Z_{n1}, \tau_n Z_{n1}, \tau_n Z_{n2}, (\tau_2 - \tau_1) U_{12}, \cdots, (\tau_n - \tau_1) U_{1n}, \cdots, (\tau_{k+1} - \tau_k) U_{k(k+1)}, \cdots, (\tau_n - \tau_k) U_{kn}, \cdots, (\tau_n - \tau_{n-1}) U_{(n-1)n} \},$$

$$\Pi_{33} = -W .$$

Proof. Choose a Lyapunov functional candidate to be

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) + V_4(x_t) + V_5(x_t)$$

where

$$V_1(x_t) = x^T(t) P x(t) , V_2(x_t) = \sum_{i=1}^{n} \int_{t-\tau_i(t)}^{t} x^T(s) Q_i x(s) ds ,$$

$$V_3(x_t) = \sum_{i=1}^{n} \int_{t-\tau_i}^{t} x^T(s) R_i x(s) ds ,$$

$$V_4(x_t) = \sum_{i=1}^{n} \int_{-\tau_i}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) (Z_{i1} + Z_{i2}) \dot{x}(s) ds d\theta ,$$

$$V_5(x_t) = \sum_{k=1}^{n-1} \sum_{j=k+1}^{n} \int_{-\tau_j}^{-\tau_k} \int_{t+\theta}^{t} \dot{x}^T(s) U_{kj} \dot{x}(s) ds d\theta ,$$

$P = P^T > 0$, $Q_i = Q_i^T > 0$, $R_i = R_i^T > 0$, $Z_{i1} = Z_{i1}^T > 0$, $Z_{i2} = Z_{i2}^T > 0$, $U_{kj} = U_{kj}^T > 0$ are matrices to be determined. From the Leibniz-Newton formula, the following equations are true for any matrices H^i , S^i , M^i , N^{kj} with appropriate dimensions:

$$\sum_{i=1}^{n} 2\zeta^T(t) H^i \left[x(t) - x(t - \tau_i(t)) - \int_{t-\tau_i(t)}^{t} \dot{x}(s) ds \right] = 0 \quad (8)$$

$$\sum_{i=1}^{n} 2\zeta^T(t) S^i \left[x(t - \tau_i(t)) - x(t - \tau_i) - \int_{t-\tau_i}^{t-\tau_i(t)} \dot{x}(s) ds \right] = 0 \quad (9)$$

$$\sum_{i=1}^{n} 2\zeta^T(t) M^i \left[x(t) - x(t - \tau_i) - \int_{t-\tau_i}^{t} \dot{x}(s) ds \right] = 0 \quad (10)$$

$$\sum_{k=1}^{n-1} \sum_{j=k+1}^{n} 2\zeta^T(t) N^{kj} \left[x(t - \tau_k) - x(t - \tau_j) - \int_{t-\tau_j}^{t-\tau_k} \dot{x}(s) ds \right] = 0 \quad (11)$$

where

$$\zeta(t)=\left[x^T(t),x^T(t-\tau_1(t)),x^T(t-\tau_1),\cdots,x^T(t-\tau_n(t)),\right.$$
$$\left. x^T(t-\tau_n)\right]^T.$$

Calculate the derivative of $V(x_t)$ along the solutions of the nominal MIMO NCS

$$\dot V_1(x_t)=x^T(t)\left[PA+A^TP\right]x(t)+2x^T(t)P\sum_{i=1}^{n}A_ix(t-\tau_i(t)),$$

$$\dot V_2(x_t)=\sum_{i=1}^{n}\left[x^T(t)Q_ix(t)-(1-\dot\tau_i(t))x^T(t-\tau_i(t))Q_ix(t-\tau_i(t))\right]$$
$$\le\sum_{i=1}^{n}\left[x^T(t)Q_ix(t)-(1-\mu_i)x^T(t-\tau_i(t))Q_ix(t-\tau_i(t))\right],$$

$$\dot V_3(x_t)=\sum_{i=1}^{n}\left[x^T(t)R_ix(t)-x^T(t-\tau_i)R_ix(t-\tau_i)\right],$$

$$\dot V_4(x_t)=\dot x^T(t)\sum_{i=1}^{n}\tau_i(Z_{i1}+Z_{i2})\dot x(t)-$$
$$\sum_{i=1}^{n}\int_{t-\tau_i}^{t}\left[\dot x^T(s)(Z_{i1}+Z_{i2})\dot x(s)\right]ds.$$

Adding the left hand side of (8)-(10) into $\dot V_4(x_t)$ yields:

$$\dot V_4(x_t)=\dot x^T(t)\sum_{i=1}^{n}\tau_i(Z_{i1}+Z_{i2})\dot x(t)$$
$$-\sum_{i=1}^{n}\int_{t-\tau_i(t)}^{t}\left[\zeta^T(t)H^i+\dot x^T(s)Z_{i1}\right]Z_{i1}^{-1}$$
$$\times\left[(H^i)^T\zeta(t)+Z_{i1}\dot x(s)\right]ds$$
$$-\sum_{i=1}^{n}\int_{t-\tau_i}^{t-\tau_i(t)}\left[\zeta^T(t)S^i+\dot x^T(s)Z_{i1}\right]Z_{i1}^{-1}$$
$$\times\left[(S^i)^T\zeta(t)+Z_{i1}\dot x(s)\right]ds$$
$$-\sum_{i=1}^{n}\int_{t-\tau_i}^{t}\left[\zeta^T(t)M^i+\dot x^T(s)Z_{i2}\right]Z_{i2}^{-1}$$
$$\times\left[(M^i)^T\zeta(t)+Z_{i2}\dot x(s)\right]ds$$
$$+\sum_{i=1}^{n}\int_{t-\tau_i(t)}^{t}\zeta^T(t)H^iZ_{i1}^{-1}(H^i)^T\zeta(t)ds$$
$$+\sum_{i=1}^{n}\int_{t-\tau_i}^{t-\tau_i(t)}\zeta^T(t)S^iZ_{i1}^{-1}(S^i)^T\zeta(t)ds$$
$$+\sum_{i=1}^{n}\int_{t-\tau_i}^{t}\zeta^T(t)M^iZ_{i2}^{-1}(M^i)^T\zeta(t)ds$$
$$+\sum_{i=1}^{n}2\zeta^T(t)H^i\left[x(t)-x(t-\tau_i(t))\right]$$
$$+\sum_{i=1}^{n}2\zeta^T(t)S^i\left[x(t-\tau_i(t))-x(t-\tau_i)\right]$$
$$+\sum_{i=1}^{n}2\zeta^T(t)M^i\left[x(t)-x(t-\tau_i)\right].$$

$$\dot V_5(t)=\dot x^T(t)\sum_{k=1}^{n-1}\sum_{j=k+1}^{n}(\tau_j-\tau_k)U_{kj}\dot x(t)$$
$$-\sum_{k=1}^{n-1}\sum_{j=k+1}^{n}\int_{t-\tau_j}^{t-\tau_k}\dot x^T(s)U_{kj}\dot x(s)ds.$$

Adding the left hand side of (11) into $\dot V_5(x_t)$ yields:

$$\dot V_5(x_t)=\dot x^T(t)\sum_{k=1}^{n-1}\sum_{j=k+1}^{n}(\tau_j-\tau_k)U_{kj}\dot x(t)$$
$$-\sum_{k=1}^{n-1}\sum_{j=k+1}^{n}\int_{t-\tau_j}^{t-\tau_k}\left[\zeta^T(t)N^{kj}+\dot x^T(s)U_{kj}\right]U_{kj}^{-1}$$
$$\times\left[(N^{kj})^T\zeta(t)+U_{kj}\dot x(s)\right]ds$$
$$+\sum_{k=1}^{n-1}\sum_{j=k+1}^{n}\int_{t-\tau_j}^{t-\tau_k}\zeta^T(t)N^{kj}U_{kj}^{-1}(N^{kj})^T\zeta(t)ds$$
$$+\sum_{k=1}^{n-1}\sum_{j=k+1}^{n}2\zeta^T(t)N^{kj}\left[x(t-\tau_k)-x(t-\tau_j)\right].$$

Therefore,

$$\dot V(x_t)=\dot V_1(x_t)+\dot V_2(x_t)+\dot V_3(x_t)+\dot V_4(x_t)+\dot V_5(x_t)$$
$$\le\zeta^T(t)\left[\Pi_{11}-\Pi_{12}\Pi_{22}^{-1}\Pi_{12}^T-\Pi_{13}\Pi_{33}^{-1}\Pi_{13}^T\right]\zeta(t)$$
$$-\sum_{i=1}^{n}\int_{t-\tau_i(t)}^{t}\left[\zeta^T(t)H^i+\dot x^T(s)Z_{i1}\right]Z_{i1}^{-1}$$
$$\times\left[(H^i)^T\zeta(t)+Z_{i1}\dot x(s)\right]ds$$
$$-\sum_{i=1}^{n}\int_{t-\tau_i}^{t-\tau_i(t)}\left[\zeta^T(t)S^i+\dot x^T(s)Z_{i1}\right]Z_{i1}^{-1}$$
$$\times\left[(S^i)^T\zeta(t)+Z_{i1}\dot x(s)\right]ds$$
$$-\sum_{i=1}^{n}\int_{t-\tau_i}^{t}\left[\zeta^T(t)M^i+\dot x^T(s)Z_{i2}\right]Z_{i2}^{-1}$$
$$\times\left[(M^i)^T\zeta(t)+Z_{i2}\dot x(s)\right]ds$$
$$-\sum_{k=1}^{n-1}\sum_{j=k+1}^{n}\int_{t-\tau_j}^{t-\tau_k}\left[\zeta^T(t)N^{kj}+\dot x^T(s)U_{kj}\right]U_{kj}^{-1}$$
$$\times\left[(N^{kj})^T\zeta(t)+U_{kj}\dot x(s)\right]ds.$$

Since $Z_{i1}>0$, $Z_{i2}>0$, $U_{kj}>0$, then

$$-\sum_{i=1}^{n}\int_{t-\tau_i(t)}^{t}\left[\zeta^T(t)H^i+\dot x^T(s)Z_{i1}\right]Z_{i1}^{-1}$$
$$\times\left[(H^i)^T\zeta(t)+Z_{i1}\dot x(s)\right]ds<0,$$
$$-\sum_{i=1}^{n}\int_{t-\tau_i}^{t-\tau_i(t)}\left[\zeta^T(t)S^i+\dot x^T(s)Z_{i1}\right]Z_{i1}^{-1}$$
$$\times\left[(S^i)^T\zeta(t)+Z_{i1}\dot x(s)\right]ds<0,$$

$$-\sum_{i=1}^{n}\int_{t-\tau_i}^{t}\left[\zeta^T(t)M^i+\dot{x}^T(s)Z_{i2}\right]Z_{i2}^{-1}\times\left[(M^i)^T\zeta(t)+Z_{i2}\dot{x}(s)\right]ds<0,$$

$$-\sum_{k=1}^{n-1}\sum_{j=k+1}^{n}\int_{t-\tau_j}^{t-\tau_k}\left[\zeta^T(t)N^{kj}+\dot{x}^T(s)U_{kj}\right]U_{kj}^{-1}\times\left[(N^{kj})^T\zeta(t)+U_{kj}\dot{x}(s)\right]ds<0.$$

Applying Schur complement to (7) gives $\Pi_{11}-\Pi_{12}\Pi_{22}^{-1}\Pi_{12}^T-\Pi_{13}\Pi_{33}^{-1}\Pi_{13}^T<0$, which implies $\dot{V}(x_t)<0$. Then, the nominal MIMO NCS is asymptotically stable. □

As the method of this paper takes into account the relationship between the upper bounds of the delays in the Lyapunov functional and estimates the upper bound of the derivative of the Lyapunov functional without ignoring the useful information, the obtained stability criterion has the potential to be less conservative.

Next, we extend the above result to the MIMO NCS with time-varying structured uncertainties.

Theorem 2. Given scalars $0\le\tau_i\le\tau$, $\mu_i\le\mu<1$, $i=1,\cdots,n$, for all time-varying delays $0\le\tau_i(t)\le\tau_i\le\tau$, $\dot{\tau}_i(t)\le\mu_i\le\mu<1$ and all admissible uncertainties, the MIMO NCS with time-varying structured uncertainties is robustly asymptotically stable if there exist matrices $P=P^T>0$, $Q_i=Q_i^T>0$, $R_i=R_i^T>0$, $Z_{i1}=Z_{i1}^T>0$, $Z_{i2}=Z_{i2}^T>0$, $U_{kj}=U_{kj}^T>0$, $k=1,\cdots,n-1$, $j=k+1,\cdots,n$, scalars $\varepsilon>0$, $\varepsilon_i>0$ and any appropriately dimensioned matrices

$$H^i=\begin{bmatrix}H_1^i\\ \vdots\\ H_{2n+1}^i\end{bmatrix},S^i=\begin{bmatrix}S_1^i\\ \vdots\\ S_{2n+1}^i\end{bmatrix},M^i=\begin{bmatrix}M_1^i\\ \vdots\\ M_{2n+1}^i\end{bmatrix},N^{kj}=\begin{bmatrix}N_1^{kj}\\ \vdots\\ N_{2n+1}^{kj}\end{bmatrix}$$

such that the following LMI holds:

$$\begin{bmatrix}\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14}\\ * & \Xi_{22} & 0 & 0\\ * & * & \Xi_{33} & \Xi_{34}\\ * & * & * & \Xi_{44}\end{bmatrix}<0 \qquad (12)$$

where

$$\Xi_{11}=\Phi_1+diag\left\{\varepsilon E^TE,\varepsilon_1E_1^TE_1,0,\cdots,\varepsilon_nE_n^TE_n,0\right\}+\Phi_2+\Phi_2^T,$$

$\Xi_{12}=\Pi_{12}$, $\Xi_{13}=\Pi_{13}$,

$$\Xi_{14}=P\begin{bmatrix}D & D_1 & \cdots & D_n\\ 0 & 0 & \cdots & 0\\ 0 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 0\\ 0 & 0 & \cdots & 0\end{bmatrix},$$

$\Xi_{22}=\Pi_{22}$, $\Xi_{33}=\Pi_{33}$, $\Xi_{34}=W\left[D,D_1,\cdots,D_n\right]$, $\Xi_{44}=-diag\left\{\varepsilon I,\varepsilon_1I,\cdots,\varepsilon_nI\right\}$, Φ_1, Φ_2, Π_{12}, Π_{13}, Π_{22}, Π_{33}, W are defined in (7) and I denotes identity matrix with an appropriate dimension.

Proof. Replacing A, A_i, $i=1,\cdots,n$ in (7) with $A+DF(t)E$, $A_i+D_iF_i(t)E_i$ respectively, (7) is equivalent to

$$\begin{bmatrix}\bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{\Pi}_{13}\\ * & \bar{\Pi}_{22} & 0\\ * & * & \bar{\Pi}_{33}\end{bmatrix}+\begin{bmatrix}\bar{D}\\ 0\\ WD\end{bmatrix}F\left[\bar{E}\quad 0\quad 0\right]+\begin{bmatrix}\bar{E}^T\\ 0\\ 0\end{bmatrix}F^T\left[\bar{D}^T\quad 0\quad D^TW^T\right]<0$$

where

$\bar{\Pi}_{11}=\bar{\Phi}_1+\Phi_2+\Phi_2^T$,

$$\bar{\Phi}_1=\begin{bmatrix}\hat{\Phi}_1 & PA_1+PD_1F_1E_1 & 0 & \cdots & PA_n+PD_nF_nE_n & 0\\ * & -(1-u_1)Q_1 & 0 & \cdots & 0 & 0\\ * & * & -R_1 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ * & * & * & \cdots & -(1-u_n)Q_n & 0\\ * & * & * & \cdots & * & -R_n\end{bmatrix},$$

$\hat{\Phi}_1=PA+A^TP+\sum_{i=1}^{n}Q_i+\sum_{i=1}^{n}R_i$,

$\bar{\Pi}_{12}=\Pi_{12}$,

$\bar{\Pi}_{13}=\left[A,A_1+D_1F_1E_1,0,\cdots,A_n+D_nF_nE_n,0\right]^TW$,

$\bar{\Pi}_{22}=\Pi_{22}$, $\bar{\Pi}_{33}=\Pi_{33}$,

$\bar{D}=\left[D^TP,0,0,\cdots,0,0\right]^T$, $\bar{E}=\left[E,0,0,\cdots,0,0\right]$.

By lemma 1, a sufficient condition guaranteeing (7) is that there exists a positive number $\varepsilon>0$ such that

$$\begin{bmatrix}\bar{\Pi}_{11}+\varepsilon\bar{E}^T\bar{E} & \bar{\Pi}_{12} & \bar{\Pi}_{13}\\ * & \bar{\Pi}_{22} & 0\\ * & * & \bar{\Pi}_{33}\end{bmatrix}+\varepsilon^{-1}\begin{bmatrix}\bar{D}\\ 0\\ WD\end{bmatrix}\begin{bmatrix}\bar{D}\\ 0\\ WD\end{bmatrix}^T<0.$$

Applying the Schur complement, the above inequality can be written as

$$\begin{bmatrix} \overline{\Pi}_{11} + \varepsilon \overline{E}^T \overline{E} & \overline{\Pi}_{12} & \overline{\Pi}_{13} & \overline{D} \\ * & \overline{\Pi}_{22} & 0 & 0 \\ * & * & \overline{\Pi}_{33} & WD \\ * & * & * & -\varepsilon I \end{bmatrix} < 0.$$

Repeating the same procedures as above for n times more, we can obtain (12). □

4. NUMERICAL EXAMPLE

In this section, a numerical example is given to demonstrate the improvement of the method proposed in this paper.

Example. Consider the following MIMO NCS with time-varying structured uncertainties in Yan, Huang, Wang, and Zhang (2006 a):

$$\dot{x}(t) = \begin{bmatrix} -2+0.3\cos t & 0 \\ 0 & -1+0.2\sin t \end{bmatrix} x(t) + \begin{bmatrix} -1+0.2\cos t & 0 \\ -1 & -1+0.3\sin t \end{bmatrix} x(t-\tau \sin t)$$

The uncertainties can be described by

$$\Delta A(t) = DF(t)E\,,\ \Delta A_1(t) = D_1 F_1(t) E_1$$

with

$$D = E = \begin{bmatrix} \sqrt{0.3} & 0 \\ 0 & \sqrt{0.2} \end{bmatrix}, D_1 = E_1 = \begin{bmatrix} \sqrt{0.2} & 0 \\ 0 & \sqrt{0.3} \end{bmatrix},$$

$$F(t) = \begin{bmatrix} \cos t & 0 \\ 0 & \sin t \end{bmatrix}.$$

There exists one time delay in this example, so the Lyapunov functional is $V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t)$. When the derivatives of the time delay μ are 0.5 and 0.9, the upper bounds on the time delay obtained by the method presented in this paper are 1.0136, 0.8224, respectively. However, the results obtained by the method in Yan, Huang, Wang, and Zhang (2006 a) are 0.9932, 0.6994, respectively. For the delay-derivative-free stability criterion, the Lyapunov functional is $V(t) = V_1(t) + V_3(t) + V_4(t)$. The upper bound on the time delay obtained by the method presented in this paper is 0.8932. However, the result obtained by the method in Yan, Huang, Wang, and Zhang (2006 a) is 0.6358. Detailed results are summarized in Table 1. It is obvious that the method proposed in this paper has better results than the existing ones.

Table 1. Upper bound of delay for different μ

μ	0.5	0.9	delay-derivative-free
Yan *et al.*(2006 a)	0.9932	0.6994	0.6358
Theorem 2	1.0136	0.8224	0.8932

5. CONCLUSIONS

In this paper, the stability of MIMO NCS is discussed. Less conservative delay dependent stability criteria are proposed by introducing a new Lyapunov functional and considering the useful information when estimating the upper bound of the derivative of the Lyapunov functional. Numerical example demonstrates that the criteria presented in this paper are an improvement over the previous ones.

6. ACKNOWLEDGEMENT

This work was supported by National Basic Research Program of China under Grant No. 2007CB714006.

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