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A Model Reference Robust Control with Unknown High Frequency Gain Sign : General Case [1]

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Abstract: In this paper, we discuss the model reference robust control (MRRC) for plants with relative degree greater than one and without the knowledge of high frequency gain sign. Based on an appropriate monitoring function, a switching scheme is proposed so that after a finite number of switching, the tracking error converges to a residual set that can be made arbitrarily small by properly choosing some design parameters. Furthermore, if some initial states of the closed-loop system are zero, we show that at most one switching is needed.

1. INTRODUCTION

Model reference robust control (MRRC) was introduced by (Qu *et al.* 1994) as a new means of I/O based controller design for linear time invariant plants with nonlinear input disturbance and has been found useful in some flight controller design. In (Lin and Jiang 2004a), based on a transformation of system tracking error, tracking performance of the MRRC has been improved for plants with relative degree greater than one by using a new Lyapunov function.

Like most of the model following techniques, one of the fundamental requirements of the MRRC is that the high frequency gain (HFG) sign is known *a priori.* In (Lin and Jiang 2004b), a switching scheme was proposed to deal with plants with relative degree one and without the knowledge of HFG sign. The objective of this paper is to generalize the scheme to plants with relative degree greater than one for the MRRC.

The relaxation of the assumption of HFG sign has long been an attractive topic in control community. Several approaches have been proposed so far and most of them, however, are based on Nussbaum gain (Nussbaum 1983, Mudgett and Morse 1985). Related work may also be found in (Zhang *et al.* 2000) in backstepping design. The main disadvantage of the Nussbaum-type gain methods is that it lacks robustness. Besides, the transient behaviour may be unacceptable.

An alternative way is switching. In adaptive control, switching was first introduced by (Martensson 1985) and then was extended to more general cases by (Fu and Barmish 1986, Miller and Davison 1989, Miller and Davison 1991). The main idea of this kind of control is to design a switching law which may determine among a set of controller candidates when to switch from the current one to the next. It should be pointed out that robustness to disturbance is still a problem in (Martensson 1985, Fu and Barmish 1986, Miller and Davison 1989). In (Miller and Davison 1991), a switching method was proposed so that the tracking error may have an arbitrarily good transient and steady-state performance specifications given by designer in advance even when plant HFG sign is unknown. However, the price of this solution is that the control signal may be very large.

In this paper, we generalize our switching scheme in (Lin and Jiang 2004b) to plants with relative degree greater than one and without HFG sign. The main idea of the scheme is to construct a monitoring function to supervise the behaviour of the tracking error and then a switching control law is proposed. We show that after finite number of switching, the tracking error converges to a residual set that can be made arbitrarily small by properly choosing some design parameters. Furthermore, the input disturbance can be completely rejected without affecting the tracking performance.

2. PROBLEM FORMULATION AND PRELIMINARY RESULTS

Consider the following SISO linear time invariant plant

$$y = G_p(s)[u + d] = k_p(n_p(s)/d_p(s))[u + d]\,, \qquad (2.1)$$

where y and u are the system output and input, respectively, $G_p(s)$ is the plant transfer function with $d_p(s)$ and $n_p(s)$ being nomic polynomials of degree n and m, respectively, and d is an input disturbance. The reference model is given by

$$y_M = M(s)[r] = (k_M / d_M(s))[r]\,,\ k_M > 0\,, \qquad (2.2)$$

where $d_M(s)$ is a monic Hurwitz polynomial satisfying $\deg(d_M(s)) = n - m := n^*$ and r is any piecewise continuous, uniformly bounded reference signal.

We make the following assumptions:

(A1) *$G_p(s)$ is of minimum phase. The parameters of $G_p(s)$ are unknown but belong to a known compact set; the degree n and the relative degree n^* (>1) of $G_p(s)$ are known constants;*

(A2) *The sign of the high frequency gain $k_p(\neq 0)$ is unknown;*

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(A3) *The unmeasured disturbance* $d(t)$ *satisfies*

$$|d(t)| \le \bar{d}(t)\,,\ \forall t \ge 0\,, \tag{2.3}$$

where $\bar{d}(t)$ *is a known, piece-wise continuous and uniformly bounded function.*

In this paper, the control signal is of the following form:

$$u = \hat{\theta}^{\mathrm{T}}\omega + u_R\,, \tag{2.4}$$

where u_R is the nonlinear control to be designed to ensure that the tracking error

$$e := y - y_M \tag{2.5}$$

tends to a small residual set for $n^* > 1$, the constant vector $\hat{\theta} \in \mathbb{R}^{2n}$ will be defined below and ω, the regressor vector, is defined as $\omega := [v_1^{\mathrm{T}},\ y,\ v_2^{\mathrm{T}},\ r]^{\mathrm{T}}$, where v_1 and v_2 are generated by input /output filters according to

$$\begin{aligned} \dot{v}_1 &= \Lambda v_1 + b_\lambda u\,,\ v_1(0) = 0\,, \\ \dot{v}_2 &= \Lambda v_2 + b_\lambda y\,, v_2(0) = 0\,, \end{aligned} \tag{2.7}$$

where $\Lambda \in \mathbb{R}^{(n-1)\times(n-1)}$ is a matrix with $\det(sI - \Lambda)$ a Hurwitz polynomial and $b_\lambda \in \mathbb{R}^{n-1}$ is chosen such that $(\Lambda,\, b_\lambda)$ is a controllable pair. It is well known (Narendra and Annaswamy 1989) that under the above assumptions with $d(t) \equiv 0$, there exits a unique constant vector $\theta^* = [\theta_1^{*\mathrm{T}},\ \theta_0^*,\ \theta_2^{*\mathrm{T}},\ k^*]^{\mathrm{T}} \in \mathbb{R}^{2n}$, such that, modulo exponentially decaying terms due to initial conditions,

$$y = G_p(s)[\theta^{*\mathrm{T}}\omega] = M(s)[r] = y_M\,, \tag{2.8}$$

where $k^* = k_M / k_p$. Since the plant parameters are assumed to be uncertain, the constant vector $\hat{\theta}$ in (2.4) is defined as

$$\begin{aligned} \hat{\theta} &= [\hat{\theta}_1^{\mathrm{T}},\ \hat{\theta}_0,\ \hat{\theta}_2^{\mathrm{T}},\ \hat{k}]^{\mathrm{T}} \\ &:= \begin{cases} \hat{\theta}^+ = [(\hat{\theta}_1^+)^{\mathrm{T}} \quad \hat{\theta}_0^+ \quad (\hat{\theta}_2^+)^{\mathrm{T}} \quad k^+]^{\mathrm{T}}, & \text{if } k_p > 0, \\ \hat{\theta}^- = [(\hat{\theta}_1^-)^{\mathrm{T}} \quad \hat{\theta}_0^- \quad (\hat{\theta}_2^-)^{\mathrm{T}} \quad k^-]^{\mathrm{T}}, & \text{if } k_p < 0, \end{cases} \end{aligned} \tag{2.9}$$

which is a rough estimate of θ^* and is obtained from nominal plant. From (2.1)-(2.9), the error model, including the I/O filters, can be expressed as

$$e = M(s)[\tilde{\theta}^{\mathrm{T}}\omega + d_f + u_R]/k^* + \epsilon_t\,, \tag{2.10}$$

where ϵ_t decays exponentially due to non-zero initial conditions and

$$\begin{aligned} \tilde{\theta} &:= \hat{\theta} - \theta^*\,, \\ d_f &:= (1 - d_1(s))[d], \\ d_1(s) &:= \hat{\theta}_1^{\mathrm{T}} adj(s\mathrm{I} - \Lambda)b_\lambda\,. \end{aligned} \tag{2.11}$$

When $n^* > 1$, we can write (2.10) in the following form

$$e = M(s)L(s)[\tilde{\theta}^T\bar{\omega} + d_L + z_1]/k^* + \varepsilon_t\,, \tag{2.12}$$

where the Hurwitz polynomial

$$L(s) := s^{n^*-1} + \alpha_1 s^{n^*-2} + \cdots + \alpha_{n^*-1}\,, \tag{2.13}$$

is chosen such that $M(s)L(s)$ is a SPR function, $\bar{\omega}$ and d_L are defined as

$$\bar{\omega} := L^{-1}(s)[\omega]\,,\ d_L := L^{-1}(s)[d_f]\,, \tag{2.14}$$

and

$$z_1 := L^{-1}(s)[u_R]\,, \tag{2.15}$$

whose controllable canonical form is

$$\begin{aligned} \dot{z}_1 &= z_2\,, \\ \dot{z}_i &= z_{i+1}, \quad i = 2,\cdots,n^* - 2\,, \\ \dot{z}_{n^*-1} &= -\alpha_1 z_{n^*-1} - \alpha_2 z_{n^*-2} - \cdots - \alpha_{n^*-1}z_1 + u_R\,. \end{aligned} \tag{2.16}$$

For the sake of simplicity, let

$$M(s)L(s) = k_M/(s+\lambda)\,,\ \lambda > 0\,, \tag{2.17}$$

then (2.12) can be rewritten as

$$\dot{e} = -\lambda e + k_p(\tilde{\theta}^{\mathrm{T}}\bar{\omega} + d_L + z_1) + \varepsilon\,, \tag{2.18}$$

where ε decays exponentially.

The following lemma summarizes the main results when the sign of k_p is known.

Lemma 1: *Suppose the MRRC system satisfy the assumptions (A1) and (A3), and the sign of* k_p *is known. Let the control signal* u_R *be defined as*

$$\upsilon_1 := \begin{cases} \upsilon_1^+ = -\zeta e - \dfrac{\mu_1^+\left|\mu_1^+\right|^{\tau_1}}{\left|\mu_1^+\right|^{\tau_1+1} + \sigma_1^{\tau_1+1}} g_1^+, \text{if } k_p > 0, \\ \upsilon_1^- = \zeta e + \dfrac{\mu_1^-\left|\mu_1^-\right|^{\tau_1}}{\left|\mu_1^-\right|^{\tau_1+1} + \sigma_1^{\tau_1+1}} g_1^-, \ \text{if } k_p < 0, \end{cases}$$

$$\upsilon_2 = u_R := \begin{cases} u_R^+ = -\rho(z_1 - \upsilon_1^+) - e + \alpha_1 z_1 - \dfrac{\mu_2^+\left|\mu_2^+\right|^{\tau_2}}{\left|\mu_2^+\right|^{\tau_2+1} + \sigma_2^{\tau_2+1}} g_2^+, \text{if } k_p > 0, \\ u_R^- = -\rho(z_1 - \upsilon_1^-) + e + \alpha_1 z_1 + \dfrac{\mu_2^-\left|\mu_2^-\right|^{\tau_2}}{\left|\mu_2^-\right|^{\tau_2+1} + \sigma_2^{\tau_2+1}} g_2^-, \text{if } k_p < 0, \end{cases} \quad \text{if } n^* = 2,$$

$$\upsilon_2 := \begin{cases} \upsilon_2^+ = -\rho(z_1 - \upsilon_1^+) - e - \dfrac{\mu_2^+\left|\mu_2^+\right|^{\tau_2}}{\left|\mu_2^+\right|^{\tau_2+1} + \sigma_2^{\tau_2+1}} g_2^+, \text{if } k_p > 0, \\ \upsilon_2^- = -\rho(z_1 - \upsilon_1^-) + e - \dfrac{\mu_2^-\left|\mu_2^-\right|^{\tau_2}}{\left|\mu_2^-\right|^{\tau_2+1} + \sigma_2^{\tau_2+1}} g_2^-, \text{if } k_p < 0, \end{cases} \quad \text{if } n^* > 2,$$

$$\upsilon_i := \begin{cases} \upsilon_i^+ = -\rho(z_{i-1} - \upsilon_{i-1}^+) - (z_{i-2} - \upsilon_{i-2}^+) - \dfrac{\mu_i^+\left|\mu_i^+\right|^{\tau_i}}{\left|\mu_i^+\right|^{\tau_i+1} + \sigma_i^{\tau_i+1}} g_i^+, \text{if } k_p > 0, \\ \upsilon_i^- = -\rho(z_{i-1} - \upsilon_{i-1}^-) - (z_{i-2} - \upsilon_{i-2}^-) - \dfrac{\mu_i^-\left|\mu_i^-\right|^{\tau_i}}{\left|\mu_i^-\right|^{\tau_i+1} + \sigma_i^{\tau_i+1}} g_i^-, \text{if } k_p < 0, \end{cases}$$

$$i = 3,\cdots,n^* - 1\,,$$

$$\begin{aligned} u_R = \upsilon_{n^*} := &\begin{cases} u_R^+ = -\rho(z_{n^*-1} - \upsilon_{n^*-1}^+) - (z_{n^*-2} - \upsilon_{n^*-2}^+) + \\ u_R^- = -\rho(z_{n^*-1} - \upsilon_{n^*-1}^-) - (z_{n^*-2} - \upsilon_{n^*-2}^-) + \end{cases} \\ &+(\alpha_1 z_{n^*-1} + \cdots + \alpha_{n^*-1}z_1) - \frac{\mu_{n^*}^+\left|\mu_{n^*}^+\right|^{\tau_{n^*}}}{\left|\mu_{n^*}^+\right|^{\tau_{n^*}+1} + \sigma_{n^*}^{\tau_{n^*}+1}} g_{n^*}^+, \text{if } k_p > 0, \\ &+(\alpha_1 z_{n^*-1} + \cdots + \alpha_{n^*-1}z_1) - \frac{\mu_{n^*}^-\left|\mu_{n^*}^-\right|^{\tau_n}}{\left|\mu_{n^*}^-\right|^{\tau_{n^*}+1} + \sigma_{n^*}^{\tau_{n^*}+1}} g_{n^*}^-, \text{if } k_p < 0, \end{aligned} \tag{2.19}$$

where $\zeta \ge 0$, $\tau_j \ge 0$, $\sigma_j > 0$ $(j = 1,\cdots,n^*)$ *and* $\rho > 0$ *are design parameters, and*

$$g_1^\pm = \mathrm{BND}(\left|\tilde{\theta}^T\bar{\omega} + d_L\right|)\,,\ \mu_1^\pm = e g_1^\pm\,,$$

$$g_j^{\pm} = \mathrm{BND}(|\varsigma_{j-1}^{\pm}|),\ \mu_j^{\pm} = (z_{j-1} - \upsilon_{j-1}^{\pm})g_j^{\pm},\ j = 2,\cdots,n^*, \quad (2.20)$$

where $\mathrm{BND}(|\varsigma_{j-1}^{\pm}|)$ *is obtained by applying triangle inequality to* $|\dot{\upsilon}_{j-1}^{\pm}|$ *so that* ε *can be separated from* $|\dot{\upsilon}_{j-1}^{\pm}|$, *i.e.*

$$|\dot{\upsilon}_{j-1}^{\pm}| \le g_j^{\pm} + c_{j-1}\varepsilon^2 = \mathrm{BND}(|\varsigma_{j-1}^{\pm}|) + c_{j-1}\varepsilon^2, \quad (2.21)$$

with c_{j-1} *any positive constant. If the robust control is chosen as (2.4), then all the closed loop signals are uniformly bounded and* e *converges exponentially to a residual set whose radius can be made arbitrarily small.*

Proof. See (Lin and Jiang 2004a). ■

Remark 2.1: *The bounding function of a signal* f, *say,* $\mathrm{BND}(|f|)$ *is a known, continuous, nonnegative function that bounds the magnitude (or Euclidean norm) of* f. *Readers may refer to (Qu et al. 1994) for detail about the definition.*

Remark 2.2: *As will be shown in (A-1) of the Appendix A, the MRRC has to deal with* $|\dot{\upsilon}_{j-1}|$. *Let*

$$e_i := (z_{i-1} - \upsilon_{i-1}),\ i = 2,\cdots,n^*$$

$$u_j := -\frac{\mu_j |\mu_j|^{\tau_j}}{|\mu_j|^{\tau_j+1} + \sigma_j^{\tau_j+1}} g_j,\ j = 1,\cdots,n^*. \quad (2.22)$$

Here, for simplicity, we have dropped the superscript " ± ". Then, taking (2.19) and (2.22) into consideration, one has

$$\dot{\upsilon}_1 = \mp\zeta\dot{e} \pm \frac{\partial u_1}{\partial e_1}\dot{e}_1 \pm \frac{\partial u_1}{\partial g_1}\dot{g}_1,$$

$$\dot{\upsilon}_2 = -\rho\dot{e}_2 \mp \dot{e} + \frac{\partial u_2}{\partial e_2}\dot{e}_2 + \frac{\partial u_2}{\partial g_2}\dot{g}_2,$$

$$\dot{\upsilon}_i = -\rho\dot{e}_i - \dot{e}_{i-1} + \frac{\partial u_i}{\partial e_i}\dot{e}_i + \frac{\partial u_i}{\partial g_i}\dot{g}_i,\ i = 3,\cdots,n^*. \quad (2.23)$$

From (2.18), $\dot{e}$ *includes the term* ε. *Hence, we can see, step by step, that* $\dot{\upsilon}_1$, $\dot{\upsilon}_2$ *and* $\dot{\upsilon}_i$ *include* ε *also. Since* ε *is not available for measurement, as shown in (2.21), we must separate it by using triangle inequality.*

3. MAIN RESULTS

3.1 Signals to be switched

Since the sign of k_p is unknown, we have to redefine the control u_R and the vector $\hat{\theta}$ as

$$u_R := \begin{cases} u_R^+, & \text{if} \quad t \in \mathbb{T}^+, \\ u_R^-, & \text{if} \quad t \in \mathbb{T}^-, \end{cases} \quad (3.1)$$

and

$$\hat{\theta} = \begin{cases} \hat{\theta}^+, & \text{if} \quad t \in \mathbb{T}^+, \\ \hat{\theta}^-, & \text{if} \quad t \in \mathbb{T}^-, \end{cases} \quad (3.2)$$

respectively, and design a monitoring function to decide when $(u_R, \hat{\theta})$ will be switched from $(u_R^+, \hat{\theta}^+)$ to $(u_R^-, \hat{\theta}^-)$ and vice versa, where the sets $\mathbb{T}^+$ and $\mathbb{T}^-$ satisfy

$$\mathbb{T}^+ \cup \mathbb{T}^- = [0,\infty),\ \mathbb{T}^+ \cap \mathbb{T}^- = \phi, \quad (3.3)$$

and both $\mathbb{T}^+$ and $\mathbb{T}^-$ have the form

$$[t_k, t_{k+1}) \cup \cdots \cup [t_j, t_{j+1}). \quad (3.4)$$

Here, t_k or t_j denotes the switching time for $(u_R^+, \hat{\theta}^+)$ or $(u_R^-, \hat{\theta}^-)$, and will be defined later. Note that the difference between (2.19) and (3.1) is that if the sign of k_p is known, we need only one u_R and one $\hat{\theta}$ while if the sign of k_p is unknown, both $(u_R^+, \hat{\theta}^+)$ and $(u_R^-, \hat{\theta}^-)$ are needed. Since u_R is obtained recursively from υ_i^+ and υ_i^-, for $i = 1,\cdots,n^*-1$, in (2.19), both $k_p > 0$ and $k_p < 0$ in (2.19) should also *be replaced by* $t \in \mathbb{T}^+$ and $t \in \mathbb{T}^-$, respectively when HFG sign is unknown.

3.2 Monitoring function and switching law

For simplicity, in what follows we assume that

$$k_p \in [-\bar{k}_p, -\underline{k}_p] \cup [\underline{k}_p, \bar{k}_p],\ \underline{k}_p, \bar{k}_p > 0. \quad (3.5)$$

To proceed, we introduce the following lemma.

Lemma 2: *Suppose the sign of* k_p *has been correctly estimated for all* $t \ge \bar{t}_0$. *Let Lyapunov function*

$$V := \begin{cases} \frac{1}{2}e^2 + \frac{1}{2}k_p \sum_{i=1}^{n^*-1} (z_i - \upsilon_i^+)^2, \text{if} \quad k_p > 0 \\ \frac{1}{2}e^2 - \frac{1}{2}k_p \sum_{i=1}^{n^*-1} (z_i - \upsilon_i^-)^2, \text{if} \quad k_p < 0 \end{cases},\ t \ge \bar{t}_0. \quad (3.6)$$

Let the design parameters ζ *and* ρ *in (2.19) be chosen such that*

$$\gamma := \lambda + \underline{k}_p\zeta - c_\varepsilon > 0,\ \rho - a_i \ge \gamma, \quad (3.7)$$

where c_ε *is any positive constant satisfying the following triangle inequality*

$$\varepsilon e \le c_\varepsilon e^2 + \varepsilon^2 / c_\varepsilon, \quad (3.8)$$

λ *is defined by (2.17), and* a_i *is any positive constant. Then, the following inequality holds:*

$$\dot{V} \le -2\gamma V + |k_p|\sigma + \epsilon,\ \forall t \ge \bar{t}_0, \quad (3.9)$$

where ϵ *is a bounded, differentiable and exponentially decaying real function whose definition will be found in the following proof, and*

$$\sigma := \sum_{i=1}^{n^*} \sigma_i, \quad (3.10)$$

where σ_i *are defined by (2.19).*

Proof. See Appendix A. ■

The inequality (3.9) motivates us to consider the following differential equation:

$$\dot{\xi} = -2\gamma\xi + |k_p|\sigma + \epsilon,\ \xi(\bar{t}_0) = V(\bar{t}_0),\ t \ge \bar{t}_0. \quad (3.11)$$

Comparing (3.9) with (3.11) we have $\dot{V} \le \dot{\xi}$, $\forall t \ge \bar{t}_0$, which by using the Comparison Lemma (Filippov 1964, Th.7, p.214) and by noting that $\xi(\bar{t}_0) = V(\bar{t}_0)$ leads to

$$V \le \xi,\ \forall t \ge \bar{t}_0. \quad (3.12)$$

With no loss of generality, let

$$|\epsilon| \le c\exp(-2\delta t),\ t \ge 0, \quad (3.13)$$

where c and δ are unknown positive constants since ϵ is unknown. The solution of (3.9) thus satisfies

$$\begin{aligned} V(t) &\le \xi(t) \le \exp[-2\gamma(t-\bar{t}_0)]V(\bar{t}_0) \\ &\le \exp[-2\gamma(t-\bar{t}_0)]V(\bar{t}_0) + \sigma\bar{k}_p/2\gamma + c_\delta \exp(-2\delta t), t \ge \bar{t}_0, \end{aligned} \quad (3.14)$$

where $\bar{k}_p$ is defined by (3.5), and the constant c_δ in this section is defined as

$$c_\delta = \frac{c}{|\gamma - \delta|}, \quad (3.15)$$

where it is assumed that $\delta < \gamma$ since a less δ can only make (3.13) more conservative. However, since V is not available for measurement due to the uncertainty of k_p, let

$$\underline{V} := \frac{1}{2}e^2 + \frac{1}{2}\underline{k}_p \sum_{i=1}^{n^*-1} (z_i - \upsilon_i)^2 ,$$

$$\bar{V} := \frac{1}{2}e^2 + \frac{1}{2}\bar{k}_p \sum_{i=1}^{n^*-1} (z_i - \upsilon_i)^2 , \quad (3.16)$$

then from (3.14) and (3.16), the following relation holds:

$$\begin{aligned} \underline{V} \le V &\le \exp[-2\gamma(t-\bar{t}_0)]V(\bar{t}_0) + \sigma\bar{k}_p/2\gamma + c_\delta \exp(-2\delta t) \\ &\le \exp[-2\gamma(t-\bar{t}_0)]\bar{V}(\bar{t}_0) + \sigma\bar{k}_p/2\gamma + c_\delta \exp(-2\delta t), t \ge \bar{t}_0. \end{aligned} \quad (3.17)$$

Thus, we can define the monitoring function as

$$\psi_k(t) = \exp[-2\gamma(t-t_k)]\bar{V}(t_k) + \sigma\bar{k}_p/2\gamma + c_k \exp(-2\delta_k t),$$
$$\forall t \in [t_k, t_{k+1}),\ k = 0,1,\cdots;\ t_0 := 0, \quad (3.18)$$

where t_k is the switching time to be defined, δ_k is any monotonically decreasing positive sequence satisfying

$$\delta_k \to 0 \text{ as } k \to \infty, \quad (3.19)$$

and c_k is any monotonically increasing positive sequence satisfying

$$c_k \to \infty \text{ as } k \to \infty. \quad (3.20)$$

It is clear that we obtain $\psi_k(t)$ from (3.17) mainly by replacing both c_δ and δ by c_k and δ_k, respectively. From (3.16) and (3.18), for each t_k we always have

$$\underline{V}(t_k) < \psi_k(t_k). \quad (3.21)$$

Hence, we define the switching time of $(u_R, \hat{\theta})$ as follows:

$$t_{k+1} = \begin{cases} \min\{t : t > t_k, \underline{V}(t) \ge \psi_k(t)\}, \text{if the minimum exists} \\ +\infty, \text{otherwise.} \end{cases} \quad (3.22)$$

3.3 Main theorem

We now introduce the main result of this paper.

Theorem 1: *Suppose the MRRC system given by (2.12) satisfies the assumptions (A1)-(A3). Let $(u_R, \hat{\theta})$ be given by (3.1), (3.2), where u_R is obtained recursively by (2.19) with* $k_p > 0$ and $k_p < 0$ *being replaced by* $t \in \mathbb{T}^+$ and $t \in \mathbb{T}^-$, *respectively. Let the switching time of $(u_R, \hat{\theta})$ (from $(u_R^+, \hat{\theta}^+)$ to $(u_R^-, \hat{\theta}^-)$ and vice versa) be defined by (3.22) where the monitoring function is given by (3.18). Then,*

1) *$(u_R, \hat{\theta})$ will stop switching after a finite number of switching and all the closed loop system signals are uniformly bounded;*
2) *The tracking error e converges to a residual set that is proportional to $\sqrt{\sigma\bar{k}_p/\gamma}$, where σ, $\bar{k}_p$ and γ are defined by (3.10), (3.5) and (3.7), respectively.*

Proof. 1) By contradiction, suppose $(u_R, \hat{\theta})$ switches according to (3.22) without stopping. Then, after a finite number k of switching, $(u_R, \hat{\theta})$ must have a correct sign, *i.e.*, $u_R = u_R^+$, $\hat{\theta} = \hat{\theta}^+$ if $k_p > 0$ or $u_R = u_R^-$, $\hat{\theta} = \hat{\theta}^-$ if $k_p < 0$ and, at the same time, from (3.18), (3.19) and (3.20),

$$c_\delta < c_k,\ \exp(-\delta t) < \exp(-\delta_k t),\ t \ge t_k. \quad (3.23)$$

Note that we can make u_R^+ and u_R^- to be continuous (or piece-wise continuous) by properly choosing the signals $\upsilon_1^\pm, \cdots, \upsilon_{n^*}^\pm$ as shown in (Lin and Jiang 2004a). Thus, for any finite number of switching, the control signal u_R is piece-wise continuous and therefore, the solution of (2.12) exists and is continuous, which by noting (3.17) and (3.18), and by taking (3.23) into consideration, implies that

$$\underline{V}(t) \le V(t) < \psi_k(t),\ t \ge t_k, \quad (3.24)$$

where we have replaced $\bar{t}_0$ by t_k in (3.17). Combining (3.22), the above inequality shows that no switching is needed for all $t \ge t_k$, a contradiction. That is, after a finite number of switching, u_R will stop switching. Then according to Lemma 1, we have that the overall control u_R and all the signals of the close-loop system are uniformly bounded.

Furthermore, whatever which one of u_R^+ and u_R^- can finally be chosen, the other one is still uniformly bounded because of the finite number k of switching of $(u_R, \hat{\theta})$.

From (3.24) and (3.6), we have $e^2/2 < \psi_k(t)$; hence,

$$|e| < \sqrt{2\exp[-2\gamma(t-t_k)]\bar{V}(t_k) + \sigma\bar{k}_p/\gamma + 2(k+1)\exp(-2\delta_k t)},$$
$$t \ge t_k. \quad (3.25)$$

Since $\psi_k(t) \to \sigma\bar{k}_p/\gamma$ as $t \to \infty$, (3.25) shows that the tracking error e converges to a residual set that is proportional to $\sqrt{\sigma\bar{k}_p/\gamma}$. This completes the proof. ■

The following corollary shows a more interesting fact of our switching scheme.

Corollary 1: *if $\varepsilon = 0$, then at most one switching of $(u_R, \hat{\theta})$ is needed.*

Proof. From (A-9) in the Appendix A, $\varepsilon = 0$ implies that $\epsilon = 0$. Hence from (3.13) and (3.17) with $\bar{t}_0$ being replaced by t_k, we have

$$\underline{V}(t) \le V(t) \le \exp[-2\gamma(t-t_k)]\bar{V}(t_k) + \sigma\bar{k}_p/2\gamma,\ t \ge t_k. \quad (3.26)$$

Taking into account (3.18) it follows that for any finite $k \ge 0$,

$$\underline{V}(t) < \psi_k(t),\ \forall t \ge t_k. \quad (3.27)$$

From (3.22), if we correctly estimate the sign of k_p at $t_0 = 0$, no switching occurs; whereas, one switching is enough. ■

4. SIMULATION RESULTS

An example is given in this section by using Matlab/ Simulink toolbox. The relative degree two plant is

$$G_p(s) = k_p/(s^2 + as + b)\,,\; x(0) = [0.5\,,\; 0.5]^{\mathrm{T}}\,, \qquad (4.1)$$

where the plant parameters belong to the following compact set:

$$S = \{k_p, a, b : -2 \le k_p \le -0.5 \; or \; 0.5 \le k_p \le 2, \; 0.5 \le a \le 1.5, 0.5 \le b \le 1.5\}. \qquad (4.2)$$

Therefore, in view of (3.5), both $\underline{k}_p$ and $\overline{k}_p$ can be obtained. The reference model is

$$M(s) = 2/(s^2 + 6s + 5)\,. \qquad (4.3)$$

We choose $L(s) = s + 5$; hence, $M(s)L(s) = 2/(s+1)$ is a SPR function. From (2.17), we have $\lambda = 1$. The parameters of the I/O filters are $\Lambda = -10$ and $b_\lambda = 1$, the reference signal $r = \sin t$, the disturbance $d = \cos t + 0.5\cos y + y^2 \sin t$, and $\bar{d}(t) = 1.5 + y^2$. To obtain $u_R = \upsilon_2$, let $\tau_1 = 1$, $\tau_2 = 0$, $\sigma_1 = \sigma_2 = 2$, $\zeta = 8$, $\rho = 9$, then from (2.19),

$$\upsilon_1 = \upsilon_1^\pm = \mp\zeta e \mp \frac{\mu_1^\pm |\mu_1^\pm|}{|\mu_1^\pm|^2 + \sigma_1^2} g_1^\pm \qquad (4.4)$$

and

$$u_R = u_R^\pm = \upsilon_2^\pm := -\rho(z_1 - \upsilon_1^\pm) \mp e + \alpha_0 z_1 - \frac{\mu_2^\pm}{|\mu_2^\pm| + \sigma_2} g_2^\pm\,, \qquad (4.5)$$

where

$$\begin{aligned}
g_1^+ = g_1^- &= \mathrm{BND}(|\tilde{\theta}^T \varpi + d_L|) \\
&= \|\tilde{\theta}\| \sqrt{\bar{r}^2 + \bar{v}_1^2 + \bar{v}_1^2 + \bar{y}^2} + L^{-1}(s)[\bar{d}(y,t)], \\
|\dot{\upsilon}_1^\pm| &= \left| \frac{\partial \upsilon_1^\pm}{\partial e}\dot{e} + \frac{\partial \upsilon_1^\pm}{\partial g_1}\dot{g}_1 \right| \\
&\le \left| \frac{\partial \upsilon_1^\pm}{\partial e}[-\lambda e + k_p(\tilde{\theta}^T \varpi + d_f + z_1) + \varepsilon] + \frac{\partial \upsilon_1^\pm}{\partial g_1}\dot{g}_1 \right| \\
&\le \mathrm{BND}(\left|\frac{\partial \upsilon_1^\pm}{\partial e}\right|)\mathrm{BND}[-\lambda e + k_p(\tilde{\theta}^{\mathrm{T}} \varpi + d_f + z_1)] \\
&\quad + \left(\frac{\partial \upsilon_1^\pm}{\partial e}\right)^2 /2 + \mathrm{BND}(\left|\frac{\partial \upsilon_1^\pm}{\partial g_1}\right|)\mathrm{BND}(|\dot{g}_1|) + \varepsilon^2/2 \\
&:= \mathrm{BND}(|\varsigma_1^\pm|) + \varepsilon^2/2 = g_2^\pm + \varepsilon^2/2, \\
\mu_2^\pm &= (z_1 - \upsilon_1^\pm) g_2^\pm\,, \qquad (4.6)
\end{aligned}$$

where

$$\mathrm{BND}(\left|\frac{\partial \upsilon_1^\pm}{\partial e}\right|) = \zeta + \frac{2|e| g_1^3 \sigma_1^2}{((eg_1)^2 + \sigma_1^2)^2}\,,$$

$$\mathrm{BND}(\left|\frac{\partial \upsilon_1^\pm}{\partial g_1}\right|) = \frac{e^4 g_1^4 + 3e^2 g_1^2 \sigma_1^2}{((eg_1)^2 + \sigma_1^2)^2}\,. \qquad (4.7)$$

Let the nominal plant parameters k_p, a and b be -1, 1 and 1, respectively, and choosing $\hat{\theta} = 0$, hence, together with (4.2), we obtain that $\mathrm{BND}(\tilde{k}) = 2$, $\mathrm{BND}(\tilde{\theta}_0) = 113.5$, $\mathrm{BND}(\tilde{\theta}_1) = 5.5$ and $\mathrm{BND}(\tilde{\theta}_2) = 1061.5$. The monitoring function is given by (3.18) with $\gamma = 1.8$, $c_k = k$ and $\delta_k = 1/(k+1)$. The simulation results are shown in Fig.1 where we can see that after one switching of u_R from u_R^+ to u_R^-, the tracking error converges to a small residual set.

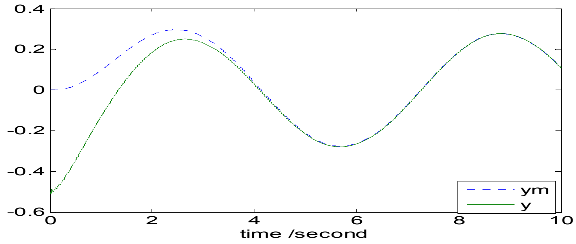

Fig. 1-1. Tracking error

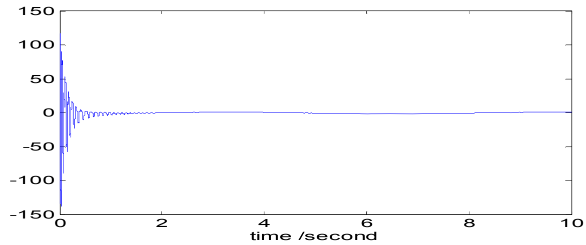

Fig. 1-2. Control signal u_R

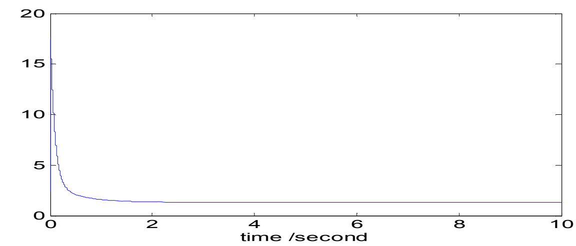

Fig. 1-3. Monitoring function φ_k

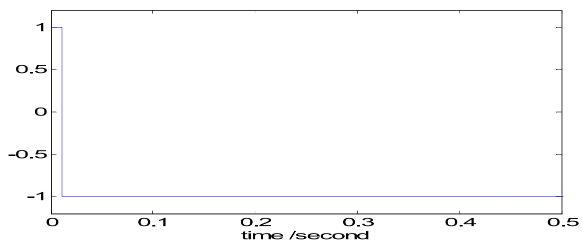

Fig. 1-4. Switching from "+" to "-"

5. CONCLUSION

In this paper, we have introduced a switching scheme for the controller design of MRRC systems with relative degree greater than one and without the knowledge of HFG sign. We have shown that for plants with relative degree greater than one our scheme can guarantee the tracking error converge to a residual set that can be made small by properly choosing design parameters σ_i, ζ and ρ. In particular, if some of the initial states of the closed-loop system are zero, we have shown that at most one switching is needed.

Appendix A. PROOF OF LEMMA 2

If k_p is greater than zero and has been correctly estimated for $t \ge \bar{t}_0$, from (3.6), for all $t \ge \bar{t}_0$, $\dot{V}$ satisfies

$$\begin{aligned}
\dot{V} &= -\lambda e^2 + k_p[(\tilde{\theta}^T \varpi + d_L)e + z_1 e] + e\varepsilon + k_p \sum_{i=1}^{n^*-1} (z_i - \upsilon_i)(\dot{z}_i - \dot{\upsilon}_i) \\
&= -\lambda e^2 + k_p e[(\tilde{\theta}^T \varpi + d_L) + \upsilon_1] + e\varepsilon + k_p(z_1 - \upsilon_1)(e + \upsilon_2 - \dot{\upsilon}_1)
\end{aligned}$$

$$+k_p\sum_{i=2}^{n^*-2}(z_i-\upsilon_i)[(z_{i-1}-\upsilon_{i-1})+\upsilon_{i+1}-\dot{\upsilon}_i]$$
$$+k_p(z_{n^*-1}-\upsilon_{n^*-1})[(z_{n^*-2}-\upsilon_{n^*-2})$$
$$+\underbrace{(-\alpha_1 z_{n^*-1}-\cdots-\alpha_{n^*-1}z_1)+u_R}_{\dot{z}_{n^*-1}}-\dot{\upsilon}_{n^*-1}], \quad \text{(A-1)}$$

where (2.18) and the following relationship have been used

$$(z_i-\upsilon_i)(\dot{z}_i-\dot{\upsilon}_i)=(z_i-\upsilon_i)(\upsilon_{i+1}-\dot{\upsilon}_i)+(z_i-\upsilon_i)(z_{i+1}-\upsilon_{i+1}), \quad \text{(A-2)}$$

in which we note that from (2.16), $\dot{z}_i=z_{i+1}$. Replacing (2.19) with $\upsilon_1=\upsilon_1^+$, $\upsilon_2=\upsilon_2^+$, $\upsilon_i=\upsilon_i^+$, $\mu_i=\mu_i^+$ and $u_R=u_R^+$ in (A-1) it follows that

$$\dot{V}=-\rho k_p\sum_{i=1}^{n^*-1}(z_i-\upsilon_i^+)^2+k_p[(\tilde{\theta}^T\varpi+d_L)e-\frac{|\mu_1^+|^{\tau_1+2}}{|\mu_1^+|^{\tau_1+1}+\sigma_1^{\tau_1+1}}]+e\varepsilon$$
$$-(\lambda+k_p\zeta)e^2+k_p\sum_{i=1}^{n^*-1}[-(z_i-\upsilon_i^+)\dot{\upsilon}_i^+-\frac{|\mu_{i+1}^+|^{\tau_{i+1}+2}}{|\mu_{i+1}^+|^{\tau_{i+1}+1}+\sigma_{i+1}^{\tau_{i+1}+1}}]. \quad \text{(A-3)}$$

From (2.20) and (2.21),

$$-(z_i-\upsilon_i^+)\dot{\upsilon}_i^+\le|z_i-\upsilon_i^+|(g_{i+1}^++c_i\varepsilon^2)\le|\mu_{i+1}^+|+c_i\varepsilon^2|z_i-\upsilon_i^+|. \quad \text{(A-4)}$$

By applying triangle inequality to the term $c_i\varepsilon^2|z_i-\upsilon_i^+|$, it follows that

$$c_i\varepsilon^2|z_i-\upsilon_i^+|\le a_i(z_i-\upsilon_i^+)^2+\frac{1}{a_i}c_i\varepsilon^4, \quad \text{(A-5)}$$

where a_i is any positive constant. Therefore,

$$-\rho(z_i-\upsilon_i^+)^2+c_i\varepsilon^2|z_i-\upsilon_i^+|\le-(\rho-a_i)(z_i-\upsilon_i^+)^2+\frac{1}{a_i}c_i\varepsilon^4$$
$$\le-\gamma(z_i-\upsilon_i^+)^2+d_i\varepsilon^4,\ i=1,\cdots,n^*-1, \quad \text{(A-6)}$$

where the term $-\rho(z_i-\upsilon_i^+)^2$ is given by (A-3), and the design parameters ρ, ζ and the constant a_i, are chosen such that (3.7) holds.

Now, using (A-6) and (3.7), and noting that the term $e\varepsilon$ satisfies (3.8), (A-3) can further be written as

$$\dot{V}\le-(\lambda+k_p\zeta-c_\varepsilon)e^2+k_p(|\mu_1^+|-\frac{|\mu_1^+|^{\tau_1+2}}{|\mu_1^+|^{\tau_1+1}+\sigma_1^{\tau_1+1}})$$
$$+\frac{1}{c_\varepsilon}\varepsilon^2-k_p[\rho\sum_{i=1}^{n^*-1}(z_i-\upsilon_i^+)^2-\sum_{i=1}^{n^*-1}\underbrace{c_i\varepsilon^2|z_i-\upsilon_i^+|}_{(A.5)}]$$
$$+k_p\sum_{i=1}^{n^*-1}(|\mu_{i+1}^+|-\frac{|\mu_{i+1}^+|^{\tau_{i+1}+2}}{|\mu_{i+1}^+|^{\tau_{i+1}+1}+\sigma_{i+1}^{\tau_{i+1}+1}})$$
$$\le-\gamma e^2-\gamma k_p\sum_{i=1}^{n^*-1}(z_i-\upsilon_i^+)^2+k_p\sum_{i=1}^{n^*}\sigma_i+\epsilon,\quad t\ge\bar{t}_0, \quad \text{(A-7)}$$

where the following inequalities have been used (Qu *et al.* 1994, p.2226):

$$k_p(|\mu_j^+|-\frac{|\mu_j^+|^{\tau_j+2}}{|\mu_j^+|^{\tau_j+1}+\sigma_j^{\tau_j+1}})\le k_p\sigma_j,\ j\in\{1,2,\cdots,n^*\}, \quad \text{(A-8)}$$

and ϵ is defined as

$$\epsilon:=\frac{1}{c_\varepsilon}\varepsilon^2+|k_p|\sum_{i=1}^{n^*}d_i\varepsilon^4, \quad \text{(A-9)}$$

which apparently is still an exponentially decaying function. If k_p is less than zero and has been correctly estimated for all $t\ge\bar{t}_0$, from (3.6), and similar to the above analysis for $k_p>0$, we can get

$$\dot{V}\le-\gamma e^2+\gamma k_p\sum_{i=1}^{n^*-1}(z_i-\upsilon_i^-)^2-k_p\sum_{i=1}^{n^*}\sigma_i+\epsilon,\ t\ge\bar{t}_0. \quad \text{(A-10)}$$

Combining (A-7) and (A-10), for both $k_p>0$ and $k_p<0$,

$$\dot{V}\le-\gamma e^2-\gamma|k_p|\sum_{i=1}^{n^*-1}(z_i-\upsilon_i)^2+|k_p|\sum_{i=1}^{n^*}\sigma_i+\epsilon$$
$$=-2\gamma\left(\frac{1}{2}e^2+\frac{1}{2}|k_p|\sum_{i=1}^{n^*-1}(z_i-\upsilon_i)^2\right)+|k_p|\sum_{i=1}^{n^*}\sigma_i+\epsilon$$
$$=-2\gamma V+|k_p|\sigma+\epsilon,\ \forall t\ge\bar{t}_0, \quad \text{(A-11)}$$

where σ and ϵ are given by (3.10) and (A-9), respectively. This completes the proof. ■

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