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Loop Filter Design for Phase-Locked Loops with Guaranteed Lock-in Range

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Abstract: Lock-in range is one of the key parameters which govern the dynamic performance of a phase-locked loop (PLL). For low-order PLLs, coarse formulas can be derived under certain assumptions and approximation for designing loop filters to achieve the performance requirement. However, it is difficult, if not impossible, to establish such relations for high-order PLLs. In this paper, we propose a new loop filter design which, in addition to satisfying the prescribed lock-in range specification, achieves several other performance requirements as well, such as small noise bandwidth and good transient response (small settling time, small overshoot). The proposed method is applicable to PLLs of any order.

1. INTRODUCTION

Over the last few decades, the PLL principle (Best, 2003; Gardner, 2005) has been proved to be very useful in a wide spread of engineering applications, such as carrier phase tracking, timing recovery, and servo control, etc (Hsieh and Huang, 1996). In order to cope with the increasingly tough performance requirements, high-order PLLs are desirable. Yet, the analysis and design of high-order PLLs are difficult (Carlosena and Manuel-Lazaro, 2007) and insufficient. For example, it is required that the loop filter of a PLL, in addition to stabilizing the loop, is low-pass so as to eliminate the high-frequency terms from the detector output. The importance of this requirement involves validity of the results derived based on linear models of the PLLs. However, current theory focuses only on lower-order PLLs; hence there lacks a useful method for the design of higher-order low-pass loop filters that guarantee loop stability as well as the other performances. On the other hand, lock-in range is one of the key parameters which govern the dynamic performance of a PLL. From the control's point of view, this particular range is closely related to the domain-of-attraction of a PLL. It can be evaluated through the nonlinear equation $\omega_L = K|F(j\omega_L)|$ where K is loop gain (Best, 2003). For the lower-order loop filters, the lock-in range could be derived, based on certain assumptions and approximation, in terms of the loop gain and the coefficients of the filter. However, to the best knowledge of the authors, there lacks such results for higher-order loop filters.

Motivated by the problems pointed out earlier, we propose a new loop filter design (applicable to any order) to deal with several performance requirements, such as noise bandwidth, transient response, and lock-in range. Trade-off among the conflict design objectives will be made via convex optimization over linear matrix inequalities (LMIs) (Boyd *et al*., 1994; Scherer *et al*., 1997; Gahinet *et al*., 1995) in conjunction with appropriate adjustment of certain design parameters. The paper is organized as follows. In Section 2, the preliminaries and problem statement are given. In Section 3, the design of PI form loop filters is presented. Section 4 shows the simulation results. Finally, Section 5 gives the conclusion.

The definition of the H_2 norm ($\|\bullet\|_2$) of a real-rational stable transfer function can be found in (Boyd *et al*., 1994). The symbol $\Re$ denotes the set of all real numbers. Throughout this paper, a signal in time domain and frequency domain are denoted by lower case and upper case, respectively.

2. PRELIMINARIES AND PROBLEM STATEMENT

2.1 *Basic Model of PLL*

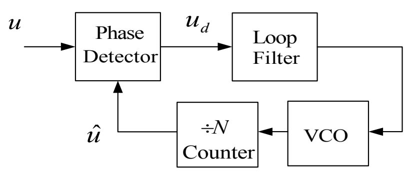

Fig.1 PLL schematic model

The PLL model used here is depicted in Fig. 1 (Best, 2003; Gardner, 2005), which consists of a phase detector, low-pass loop filter $F(s)$, and voltage controlled oscillator (VCO). The inputs to the phase detector are the two signals: the sum of the carrier and noise $n(t)$ that is stationary, Gaussian, bandpass and zero mean (Gardner, 2005), i.e.,

$$u(t) = \sqrt{2}A_d \sin(\omega_0 t + w_\theta(t)) + n(t),$$

and the VCO output

$$\hat{u}(t) = \sqrt{2}\cos(\omega_0 t + \hat{w}_\theta(t)).$$

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The phase detector produces, assuming the high frequency term is eliminated by the low-pass filter, the output signal

$$u_d(t) = A_d[\sin(w_\theta - \hat{w}_\theta) + n'(t)]$$

where n' represents the net effect caused by the noise n. For small phase errors and small input phase disturbance $w_n(t)$, the PLL can be further approximated by the linear model as depicted in Fig 2 (Gardner, 2005).

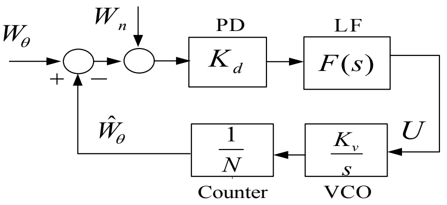

Fig. 2 PLL linear model approximation

2.2 *PROBLEM STATEMENT*

The design objectives are as follows.

(i) Closed-loop stability,

(ii) Perfect asymptotical tracking (i.e. $e(\infty)=0$) subject to the deterministic test signals $w_\theta(t)=t^k$, $k=0,1,2,\ldots,m$ with $w_n = 0$,

(iii) Good transient response (i.e., small settling time, small overshoot, etc),

(iv) Noise attenuation (assuming w_n to be white noise),

(v) Guaranteed lock-in range ω_L.

For (ii) and low-pass property requirement, PI form filters are considered. For (iii), placing the dominant poles in the conic sector region (see Fig. 3(a)) with smaller θ results in smaller percent overshoot. Similarly, placing the dominant poles in the semi-infinite vertical strip (see Fig. 3(b)) with larger h_1 results in smaller settling time.

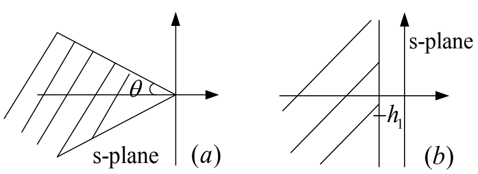

Fig.3 (a) conic sector (b) semi-infinite strip $(-\infty, -h_1)$

For (iv), note that the variance of VCO output phase is given by

$$\sigma_{\hat{w}_\theta}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|T_{\hat{w}_\theta w_n}(j\omega)\right|^2 \Phi_{w_n}(\omega)\, d\omega$$

where $T_{\hat{w}_\theta w_n}$ represents the transfer function from w_n to $\hat{w}_\theta$, and $\Phi_{w_n}(\omega)$ is the power spectral density (PSD) function of the noise w_n. If w_n is white Gaussian, i.e., $\Phi_{w_n}(\omega) = N_0$, then

$$\sigma_{\hat{w}_\theta}^2 = N_0 \left\|T_{\hat{w}_\theta w_n}\right\|_2^2 = 2N_0 B_n$$

where B_n (Hz) denotes the noise bandwidth of $T_{\hat{w}_\theta w_n}$. It is clear that small noise bandwidth B_n leads to small variance of the VCO output phase. In terms of the loop filter design, this can be achieved by minimizing the H_2 norm of the closed-loop system $T_{\hat{w}_\theta w_n}$ over all the stabilizing filters. Concerning (v), the lock-in range can be evaluated by solving the following nonlinear equation

$$\omega_L = K\left|F(j\omega_L)\right| \quad (1)$$

where $K = K_d K_v / N$. Fig. 4 gives a geometry interpretation of the solution to (1); that is, the intersection of the curve $K|F(j\omega)|$ and the straight line with unit slope. Obviously, filter F_3 has the largest lock-in range ω_3 among the three. This motivates us that, intuitively, one may choose a loop filter which has large dc gain and large first corner frequency for getting a large lock-in range.

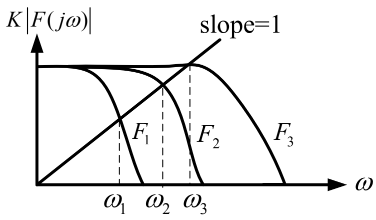

Fig.4 Geometry interpretation of the lock-in range

3. PI FORM LOOP FILTER DESIGN

In this section, a new method is proposed to design PI form loop filter for PLLs. First, we utilize the method presented in (Souza and Shaked, 1998; Chou *et al*., 2006) to transform the problem of designing a class of PI form filters into a static state feedback synthesis problem. Next, LMI constraint for guaranteed lock-in range is derived. Finally, a multi-objective state feedback synthesis technique is employed to find a loop filter that satisfies the design objectives mentioned in Section 2.2.

3.1 Problem Reformulation

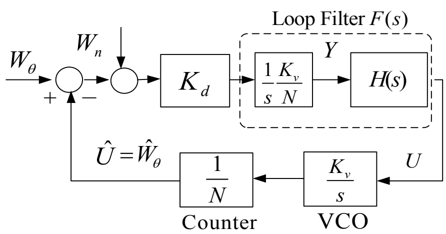

Fig.5 The reconstruction model of the PLL

Fig. 2 is redrawn as Fig. 5 with a m th-order loop filter loop filter $F(s)$ of the form

$$F(s)=\frac{1}{s}\frac{K_v}{N}H(s)=\sum_{i=0}^{m}\frac{f_i}{s^i} \tag{2}$$

In view of Fig. 5, the signal $Y(s)$ can be described by

$$Y(s)=\frac{1}{s}K\left(W_n(s)+W_\theta(s)-\hat{U}(s)\right) \tag{3}$$

Define state vector $\bar{\xi}(s)$ as follows:

$$\bar{\xi}(s)=\frac{1}{K}\left[Y(s)\ ,\frac{1}{s}Y(s)\ ,\cdots,\frac{1}{s^m}Y(s)\right]^T$$
$$=\left[\ \xi_1(s)\ ,\xi_2(s)\ ,\xi_3(s)\ ,\cdots\ ,\xi_{m+1}(s)\right]^T$$

and set $Z(s)=\hat{U}(s)$ and $\bar{Y}(s)=\bar{\xi}(s)$. Then (3) becomes

$$s\xi_1(s)=W_n(s)+W_\theta(s)-\hat{U}(s)$$

Moreover,

$$s\xi_i(s)=s\xi_{i-1}(s) \quad \text{for} \quad i=2,3,\cdots,m+1$$

With the notation defined above it is easy to check that the dynamic filter of the prescribed form (2) is converted into a static state feedback in the new coordinate, i.e., $\hat{U}(s)=\bar{F}_s\cdot\bar{Y}$ where $\bar{F}_s=KF_s$ with $F_s=[f_0\ \ \cdots\ \ f_m]$. Thus the original PI form filter design problem is equivalently transformed to a static state feedback control problem as illustrated in Fig. 6

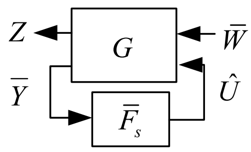

Fig.6 The equivalent state feedback model

with G described by

$$G\begin{cases} s\bar{\xi}(s)=A\bar{\xi}(s)+B_1\bar{W}(s)+B_2\hat{U}(s)\\ Z(s)=\ \ D_{12}\hat{U}(s)\\ \bar{Y}(s)=\ \bar{\xi}(s)\end{cases}$$

where $\bar{W}=\begin{bmatrix}W_n & W_\theta\end{bmatrix}^T$,

$$A=\left[\begin{array}{c|c}\mathbf{0}_{1\times m} & 0\\ \hline \mathbf{I}_m & \mathbf{0}_{m\times 1}\end{array}\right]_{(m+1)\times(m+1)},$$

$$B_1=\begin{bmatrix}1 & 0 & \cdots & 0\\ 1 & 0 & \cdots & 0\end{bmatrix}^T_{2\times(m+1)},\ B_2=\begin{bmatrix}-1 & 0 & \cdots & 0\end{bmatrix}^T_{1\times(m+1)},$$

$$D_{12}=1.$$

The resulting closed-loop system from $\bar{w}$ to z is as follows.

$$T_{z\bar{w}}\begin{cases} s\bar{\xi}(s)=(A+B_2\bar{F}_s)\bar{\xi}(s)+B_1\bar{W}(s)\\ Z(s)=\bar{F}_s\bar{\xi}(s)\end{cases}$$

For H_2 minimization from w_n to z (i.e., the noise band-width minimization problem), the closed-loop system matrix is given by

$$\begin{bmatrix} A+B_2\bar{F}_s & B_1\begin{bmatrix}1\\0\end{bmatrix}\\ \bar{F}_s & 0\end{bmatrix} \tag{4}$$

3.2 The LMI Formulation of Lock-in Range Performance

In this subsection, we derive a LMI condition under which the desirable lock-in range ω_g is guaranteed. Since the bode plot of any PI form filter starts from infinity at zero frequency and eventually decreases to a constant as the frequency goes to infinity, in view of Fig. 4, to have a larger lock-in range, the problem can be reformulated as finding a PI form filter $F(s)$ that satisfies $K\left|F(j\omega_g)\right|>\omega_g$. To this end, first, it can be verified that for any m th-order ($m\geq 1$) PI form filter $F(s)$, the expression $K^2\left|F(j\omega)\right|^2$ can be expanded as follows:

$$K^2\left|F(j\omega)\right|^2=\frac{K^2}{\omega^{2m}}\left[f_0^2\omega^{2m}+\sum_{i=1}^{m-1}(f_i^2-2f_{i-1}f_{i+1})\,\omega^{2m-2i}\right.$$
$$\left.+\sum_{j=1}^{m-3}2f_{j-1}f_{j+3}\omega^{2m-(2j+2)}+f_m^2\right]$$

Alternatively, it admits the following quadric form representation:

$$K^2\left|F(j\omega)\right|^2=\bar{F}_sP_m(\omega)\bar{F}_s^T$$

where $P_m(\omega)$ is a $(m+1)\times(m+1)$ symmetric matrix; specifically, for examples:

$$P_1(\omega)=\frac{1}{\omega^2}\begin{bmatrix}\omega^2 & 0\\ 0 & 1\end{bmatrix},\ P_2(\omega)=\frac{1}{\omega^4}\begin{bmatrix}\omega^4 & 0 & -\omega^2\\ 0 & \omega^2 & 0\\ -\omega^2 & 0 & 1\end{bmatrix}.$$

The matrices $P_m(\omega)$, $m=1,2,\cdots$, possess certain properties in common as will be formally stated in the following lemma.

Lemma 1: For any nonzero real number ω, the $(m+1)\times(m+1)$ matrices $P_m(\omega)$, $m=1,2,\cdots$, have the following properties.

(i) $P_1(\omega)$ is positive definite and rank($P_1(\omega)$)=2.

(ii) $P_m(\omega)$ is positive semi-definite and rank($P_m(\omega)$)=2 for $m=2,3,\cdots$.

Next, for a given positive integer m and nonzero real number ω, we perform the singular value decomposition (SVD) for $P_m(\omega)$ which reads

$$P_m(\omega) = S\Sigma S^T \tag{5}$$

where $S = [s_1, s_2, \cdots, s_{m+1}] \in \Re^{(m+1)\times(m+1)}$ is unitary satisfying $S^T S = I$, and

$$\Sigma = \begin{pmatrix} \Sigma_1 & 0 \\ 0 & 0_{(m-1)\times(m-1)} \end{pmatrix}$$

with $\Sigma_1 = diag(\sigma_1, \sigma_2)$ and $\sigma_1 \ge \sigma_2 \ge 0$. (Here we have used the rank property of Lemma 1).

Define the set Ω as follows:

$$\Omega = \left\{ F_s \in \Re^{1\times(m+1)} : (\bar{F}_s - F_c)M(\bar{F}_s - F_c)^T < 1 \text{ with } M = M^T > S\Lambda S^T \right\}$$

where

$$\Lambda = \begin{pmatrix} \Lambda_1 & 0 \\ 0 & 0_{(m-1)\times(m-1)} \end{pmatrix}$$

with $\Lambda_1 = diag(\varepsilon_1, \varepsilon_2)$ and reall that $\bar{F}_s = KF_s$.

Lemma 2: Given the positive values ω_g, $\varepsilon_1, \varepsilon_2$, c_1, c_2 with $c_1 + c_2 > 1$, and a positive integer m. Let $P_m(\omega_g) = S\Sigma S^T$ as determined in (5). Let $F_c = \bar{\beta} S^T$ where $\bar{\beta} = [\beta_1 \ \cdots \ \beta_{m+1}]$, and β_i is defined by

$$\beta_i = \frac{1}{\sqrt{\varepsilon_i}} + \frac{\sqrt{c_i}\,\omega_g}{\sqrt{\sigma_i}} \tag{6}$$

for $i = 1, 2$ and any positive value for $i = 3, \dots, m+1$. If a filter has its coefficient vector $F_s \in \Omega$, then the inequality $K|F(j\omega_g)| > \omega_g$ holds. Suppose further that together with this loop filter the resulting PLL is stable and there exists no solution to (1) for the frequency range $(0, \omega_g)$, then the lock-in range ω_L of the PLL is greater than ω_g.

Proof: Any vector $F_s \in \Omega$ can be rewritten as the form $\bar{F}_s = F_c + x^T$ where $\bar{F}_s = KF_s$ and

$$x \in \Omega_E = \left\{ x \in \Re^{(m+1)} : x^T M x < 1 \text{ with } M > S\Lambda S^T \right\}$$

Without loss of generality, we may assume $x^T = \bar{\gamma} S^T$ where $\bar{\gamma} = [\gamma_1 \ \cdots \ \gamma_{m+1}]$. It follows that $\bar{F}_s = F_c + x^T = (\bar{\beta} + \bar{\gamma})S^T$. Furthermore, $x^T M x < 1$ with $M > S\Lambda S^T$ implies that $\varepsilon_1\gamma_1^2 + \varepsilon_2\gamma_2^2 < 1$, which in turn implies that $|\gamma_i| < 1/\sqrt{\varepsilon_i}$, $i = 1, 2$. Finally,

$$K^2 \left|F(j\omega_g)\right|^2 = \bar{F}_s P_m(\omega_g) \bar{F}_s^T = \sum_{i=1}^{2} \sigma_i (\beta_i + \gamma_i)^2 \ge$$
$$\ge \sum_{i=1}^{2} \sigma_i \left(\beta_i - \frac{1}{\sqrt{\varepsilon_i}} \right)^2 = (c_1 + c_2)\cdot \omega_g^2 > \omega_g^2$$

The rest of the assertion follows from the definition of the lock-in range. This proof is complete. ■

Remark 1: Under the premise of Lemma 2 and the assumption $M > 0$, the task of finding a loop filter coefficient vector $F_s \in \Omega$ is equivalent to finding the matrices $N \in \Re^{1\times(m+1)}$, $M = M^T \in \Re^{(m+1)\times(m+1)}$ satisfying the following LMIs:

$$M > S\Lambda S^T, \tag{7}$$

$$\begin{pmatrix} 1 & N - F_c M \\ (N - F_c M)^T & M \end{pmatrix} > 0. \tag{8}$$

In the affirmative case, $F_s = NM^{-1}/K \in \Omega$.

Remark 2: While F_c can be interpreted as the center of an ellipsoid, the lengths of the axes of the ellipsoid are related to M. The smaller M is, the larger the set Ω. Hence $\varepsilon_1, \varepsilon_2$ are usually chosen as small positive values so as to increase the chance of finding a solution.

3.3 LMI Design of the Loop Filter

In this subsection, we incorporate some other performances into consideration. Specifically, the problem is to design a PI form loop filter so that the closed-loop is stable with good transient response and guaranteed lock-in range. In addition, the noise bandwidth is as small as possible. In an attempt to solve the problem we consider to formulate it as the following optimization problem:

$$\text{Minimize } \nu \tag{9}$$

over $N \in \Re^{1\times(m+1)}$, $M = M^T \in \Re^{(m+1)\times(m+1)}$, $Q \in \Re$, $\nu \in \Re$, satisfying: (7), (8),

$$\begin{cases} \begin{pmatrix} He(AM + B_2 N) & B_1 \\ B_1^T & -I \end{pmatrix} < 0 \\ Trace(Q) < \nu \\ \begin{pmatrix} M & N^T \\ N & Q \end{pmatrix} > 0 \end{cases},$$

$$\begin{pmatrix} \sin(\theta)\cdot He(AM + B_2 N) & \cos(\theta)\cdot Sh(AM + B_2 N) \\ (\cos(\theta)\cdot Sh(AM + B_2 N))^T & \sin(\theta)\cdot He(AM + B_2 N) \end{pmatrix} < 0,$$

$$2h_1 M - He(AM + B_2 N) < 0.$$

where the notation $He(A) = A + A^T$, $Sh(A) = A - A^T$ are used. Denote the optimal solution by (N_{opt}, M_{opt}, Q_{opt}, ν_{opt}).

Theorem 1: Given the loop gain K, the positive values ω_g, the parameters θ, h_1, ε_1, ε_2, c_1, c_2 with $c_1+c_2>1$, a positive integer m. If the optimization problem (9) is solvable, there exists a loop filter of the form

$$F(s)=\sum_{i=0}^{m}\frac{f_i}{s^i}$$

with $F_s=[f_0,\cdots,f_m]=(N_{opt}M_{opt}^{-1})/K$, such that

(i) the resulting PLL is stable,

(ii) the noise bandwidth $B_n<\frac{v_{opt}^2}{2}$,

(iii) the closed-loop poles lie within the intersection of $(-\infty,-h_1)$ and the conic sector with parameter θ (ref. Fig. 3),

(iv) $K|F(j\omega_g)|>\omega_g$.

Suppose further that there exists no solution to (1) for the frequency range $(0,\omega_g)$, then the lock-in range ω_L of the resulting PLL is greater than ω_g.

Proof: The results follow from standard multi-objective state feedback synthesis technique in (Boyd *et al.*, 1994). ■

For simplicity of the loop filter design, let $c_i=\sigma_i$ for all i. Then (6) becomes

$$\beta_i=\frac{1}{\sqrt{\varepsilon_i}}+\omega_g \tag{10}$$

A conceptual algorithm based on iteratively carrying out the optimization problem (9) is presented as follows.

Algorithm 1: Given the loop gain K, the desired guaranteed lock-in range ω_g, and the desired order m of the loop filer.

Step 1: Select initial value for F_c and compute β_i by $F_c=\bar{\beta}S^T$ (adjust S in (5) if necessary), such that $\beta_i>\omega_g$ is satisfied for $i=1,2$.

Step 2: Calculate ε_i by (10) and determine matrix Λ. Perform (9) without the regional pole placement constraints to obtain closed-loop poles $\lambda_i(1)$ and gain $\bar{F}_s(1)$.

Step 3: Update F_c by $\bar{F}_s(1)$. Select θ as small as possible and h_1 as large as possible with reference to $\lambda_i(1)$. Perform (9) to get $\bar{F}_s(2)$. (note: If (9) has no solution, then relax the values h_1 and θ).

Step 4: Update F_c by the current $\bar{F}_s$. Select θ as small as possible and h_1 as large as possible with reference to the closed-loop poles in the previous iteration. Perform (9) to get new $\bar{F}_s$. Repeat this step until the objectives are satisfied or there is no significant progress on the performances.

Note that Step 1 of Algorithm 1, β_i must be positive (see (10)). This can be achieved by adjusting the columns of S in (5) (by adding a minus sign to the corresponding column). To ensure $\beta_i>\omega_g$ for $i=1,2$, this can be done by increasing the magnitude of F_c since $|\beta_i|$ is proportional to $|F_c|$.

4. SIMULATION RESULTS

The parameters chosen for the PLL are showed in TABLE 1. The lock-in range is about ±30% of the FRF (Gardner, 2005; Carlosena and Manuel-Lazaro, 2007), i.e., $15\cdot10^3$ rad/sec. The abbreviation FRF means the Free Running Frequency of the oscillator. Furthermore, we assume that the standard deviation of the input Gaussian noise $n(t)$ is 0.015.

TABLE 1. THE PARAMETERS OF THE PLL

VCO	Divider ratio N	PD gain K_d	Lock-in Range
$K_v=10^4$	1	7	~30%FRF
FRF= $50\cdot10^3$			

(Frequencies in rad/sec)

TABLE 2 (A) and (B) show the transfer functions of PI loop filters and the design parameters during the iterations. In TABLE 1 (A), the value inside each bracket denotes the number of iteration. For example, the symbol $F_c(0)$ denotes the value initially chosen for F_c.

TABLE 2 (A). THE PARAMETERS OF THE ITERATIONS

Loop order	Design parameters
2	$F_c(0)=[1.8\quad 32]$, $F_c(1)=[1.504\quad 26.746]$ $F_c(2)=[1.505\quad 37.723]$, $\bar{F}_s(\infty)=[1.5\quad 37.609]$, $h_1(1\sim\infty)=25$
3	$F_c(0)=[1.6\quad 40\quad 200]$, $F_c(1)=[1.58\quad 39.58\quad 197.88]$ $F_c(2)=[1.58\quad 46.29\quad 319.68]$, $F_c(3)=[1.58\quad 54.21\quad 463.74]$ $F_c(4)=[1.58\quad 63.29\quad 645.18]$, $\bar{F}_s(\infty)=[1.5\quad 60.57\quad 617.46]$ $h_1(1)=11$, $h_1(2)=11$, $h_1(3)=16$, $h_1(4)=20$, $h_1(5\sim\infty)=20$

(The scale for F_c is 10^4. $\theta=\pi/6$ for all iterations)

TABLE 2 (B). THE RESULTING FILTERS

Loop order	Loop Filter
2	$F(s)=\frac{0.21s+5.37}{s}$
3	$F(s)=\frac{0.21s^2+8.65s+88.21}{s^2}$

TABLE 3. THE PERFORMANCE INDICES

Loop order	Settling time	Noise Bandwidth B_n	Lock-in Range
2	$0.256\cdot10^{-3}$	$3.76\cdot10^3$	$15\cdot10^3$
3	$0.253\cdot10^{-3}$	$3.77\cdot10^3$	$15\cdot10^3$

Settling time (sec), B_n (Hz), and Lock-in Range (rad/sec)

Several performance indices are evaluated for the resulting PLLs as shown in TABLE 3. For the lock-in range, our design is to meet 30%FRF exactly in order to obtain smaller noise bandwidth.

Next we use the software **MATLAB's SIMULINK** Version 7.3 to simulate the resulting PLLs. A more realistic system model (Gardner, 2005) as shown in Fig. 7, which can be approximated by that in Fig. 2, is used.

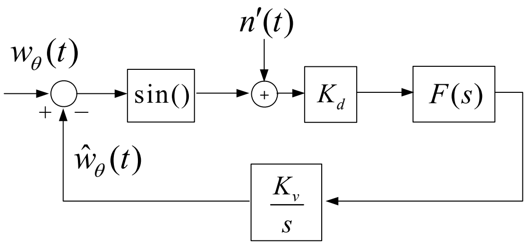

Fig.7 The simulation model of PLL

Fig. 8 show the VCO input responses of the second-order and third-order PLLs when the instantaneous $15 \cdot 10^3$ rad/s frequency step (FRF+30%, which is in the margin of acquisition range exactly) is applied. Both show that the resulting PLLs work well in the lock-in range and track the reference input signal within 0.5 ms. The transients of the resulting PLLs are similar.

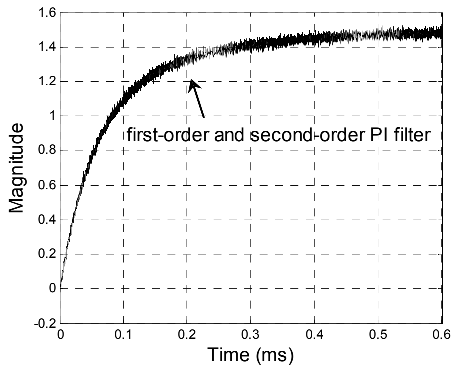

Fig.8 The signal of VCO input when reference input has frequency offset $15 \cdot 10^3$ rad/sec.

Fig. 9 show the VCO input responses of the resulting PLLs when the instantaneous $15.9 \cdot 10^3$ rad/s frequency step (out of the lock-in range) is encountered. Pull-out phenomenon occurs for both PLLs. As expected, the loop can get locked again as long as it is within the lock-in range. It is observed that the PLLs with the first-order PI loop filters exhibit shorter pull-in time than that with the second-order PI filter, but they almost get locked simultaneously (about 4 ms).

5. CONCLUSIONS

We have presented a new loop filter design for PLLs with particular emphasis on guaranteed lock-in range. Despite that there is no analytic formula for computing the lock-in range of high-order PLLs, a sufficient condition for finding a filter to meet this requirement is derived in terms of LMI constraint. In addition to satisfying the desired constraint, the proposed method considered to trade-off the other objectives such as small noise bandwidth and good transient response via multi-objective control techniques. In comparison with the existing results, the proposed method is simple and applicable to PLL of any order. Simulation results show that the resulting PLLs work very well without cycle slips when the frequency offset of the reference signal is within the lock-in range.

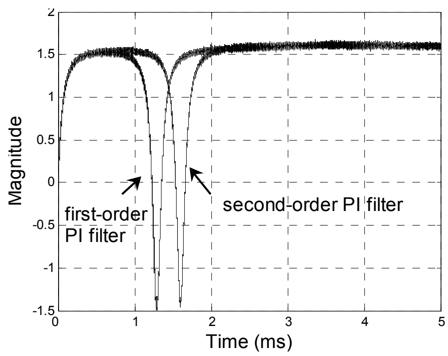

Fig.9 The signal of VCO input when reference input has frequency offset $15.9 \cdot 10^3$ rad/sec.

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