

Modelling, control and optimization of a continuous distillation tower through fuzzy techniques

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Abstract: This paper applies a methodology for the design of a fuzzy controller applicable to continuous processes based on local fuzzy models and velocity linearizations. It has been applied to the implementation of a fuzzy controller for a continuous distillation tower. Continuous distillation towers can be subjected to variations in feed characteristics that cause loss of product quality or excessive energy consumption. Therefore, the use of a fuzzy controller is interesting to control process performance.

A dynamic model for continuous distillation was implemented and used to obtain data to develop the fuzzy controller at different operating points. The fuzzy controller was built by integration of linear controllers obtained for each linearization of the system. Simulation of the model with controller was used to validate the controller effectiveness under different scenarios.

The results showed that the fuzzy controller was able to keep the target output in the desired range for different inputs disturbances, changing smoothly from a predefined target output to another. The developed techniques are applicable to more complex distillation systems including more operating variables.

1. INTRODUCTION

This paper applies a methodology for the design of a fuzzy controller applicable to continuous processes. The controller is based on a fuzzy model describing the plant dynamics that is previously calculated using experimental or model data. In this case, the technique has been applied to the control of a continuous distillation tower.

Continuous distillation is one of the most used separation processes in the processing of large amount of products. This makes it a highly energy-consuming operation (Treybal, 1981). Usually, distillation towers are designed for a feed stream of constant characteristics and classic control is used to change the operation parameters in order to achieve the specifications required for the products. However, feed streams can be subjected to important variations in composition or energy state.

An efficient control system adaptable to different situations would assure product quality and minimize energy expenses (Ramanathan, 2001; Luyben, 1994). New approaches to control, other than classic linear controllers, can improve system performance. Although other techniques could be applied to control the distillation tower, such as nonlinear MPC or classic gain scheduled control, a Fuzzy Control approach as representative of Artificial Intelligence techniques has been chosen. In this way, the experience of

the plant operators can easily be included in the control design.

Control based on fuzzy modelling has been satisfactorily applied to many complex systems of different fields of science and engineering characterized by significant nonlinearities and/or noise. Any static or dynamic system that makes use of fuzzy sets is called a fuzzy system. In addition to their universal function approximation capabilities, fuzzy models resemble human reasoning processes, providing the readability of the obtained representations (Wang, 1997). Fuzzy models can therefore be validated by experts and incorporate additional qualitative or imprecise information that engineers or operators may have about the system.

A fuzzy model is usually automatically developed from experimental data by a rule extraction method based on genetic algorithms, neural networks, templates or clustering techniques (Babuska, 1996; Díez, 2007). However, in the case of distillation, the results provided by dynamic models are very accurate and can be used instead of experimental data. Therefore, a grey box approach, combining basic knowledge about the system and black box models, has been considered more suitable for this work. The final model built in this way consists of a set of local models (one for each rule) at different operating points, and it avoids fuzzy interpolation problems by means of velocity-based linearization (Leith, 2000; Díez, 2004).

The second section of this report briefly describes the equations of the dynamic distillation model and their implementation. The third and fourth sections present the theory of the fuzzy controller and the methodology used to build it for an example case. The fifth section shows the criteria used to choose the optimum variable to perform the control. Finally, in the last section, a discussion of the performance of the fuzzy controller for the example case is presented.

2. DYNAMIC MODEL FOR CONTINUOUS DISTILLATION

Distillation towers are multi-stage separation units conceived to separate feeds into streams of different composition. The separation is achieved when a vapour and a liquid stream are put in contact in the separation stages (trays) and tend to thermodynamic equilibrium. Accordingly, the more volatile components pass to the vapour phase and the less volatile to the liquid phase.

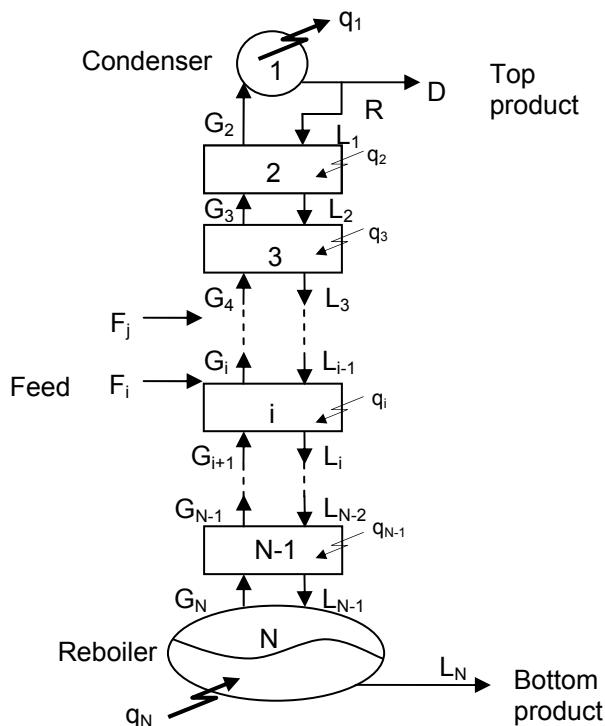


Fig. 1. Scheme of the continuous distillation tower from basic principles

Fig. 1 shows a schematic representation of the tower and the streams flows between different stages. The stages are numbered increasingly from top to bottom: 1 is the condenser, 2 to $N-1$ are the trays and N is the reboiler. Every stream is defined by their flow, composition (expressed as molar fractions of components j) and enthalpy. Thus, for the stage i the feed is defined by $F_i, z_{i,j}, h_{F_i}$, the output liquid by $L_i, x_{i,j}, h_{L_i}$ and the output vapour by $G_i, y_{i,j}, h_{G_i}$. In order to generate the vapour stream, a duty heat q_N is supplied to the reboiler. The vapour stream moves upwards exchanging components with the descending liquid stream and it is

progressively enriched in the most volatile components. Consequently, the liquid stream is enriched in the less volatile ones. To generate a saturated liquid stream, a heat duty q_1 must be removed from the vapour in the condenser. Part of the liquid stream is recycled back to the column, the rest constitute the top product of the column. This recirculation is determined by the reflux ratio R , defined in (1), which constitutes one of the main operation variables.

$$R = \frac{L_1}{D} \quad (1)$$

For every stage i , non-stationary balances of total amount of moles M_i , moles of each component j and energy can be established (2-4). These equations constitute the basis to obtain the dynamic model of the system (Treybal, 1981; Skovborg, 1992).

$$\frac{dM_i}{dt} = L_{i-1} - L_i + G_{i+1} - G_i + F_i \quad (2)$$

$$\frac{d(M_i x_{i,j})}{dt} = L_{i-1} x_{i-1,j} - L_i x_{i,j} + G_{i+1} y_{i+1,j} - G_i y_{i,j} + F_i z_{i,j} \quad (3)$$

$$\frac{d(M_i h_{L_i})}{dt} = L_{i-1} h_{L_{i-1}} - L_i h_{L_i} + G_{i+1} h_{G_{i+1}} - G_i h_{G_i} + F_i h_{F_i} + q_i \quad (4)$$

Substitution of (2) into (3) and (4) leads to the time derivatives of composition and enthalpy (5, 6).

$$\frac{dx_{i,j}}{dt} = M_i^{-1} \cdot \begin{bmatrix} L_{i-1} x_{i-1,j} + F_i z_{F,j} + \\ + G_{i+1,j} y_{i+1,j} + \\ + (G_{i,j} - G_{i+1,j} - L_{i-1} - F_i) x_{i,j} + \\ - G_{i,j} y_{i,j} \end{bmatrix} \quad (5)$$

$$\frac{dh_{L,i}}{dt} = M_i^{-1} \cdot \begin{bmatrix} L_{i-1} h_{L_{i-1}} + F_i h_{F,i} - L_i h_{L,i} + \\ + G_{i+1} h_{G_{i+1}} + q_i - G_i h_{G_i} + \\ - h_{L,i} (L_{i-1} + F_{i-1} - L_i + G_{i-1} - G_i) \end{bmatrix} \quad (6)$$

Assuming that there is enough contact time in each stage, the exiting phases will be close to equilibrium. So, using a thermodynamic model for the mixture, the vapour composition can be expressed as a function of the liquid composition. Besides, for a saturated liquid, the enthalpy and temperature depend exclusively on the liquid composition, hence we can express:

$$\frac{dh_{L,i}}{dt} = \sum_j \frac{\partial h_{L,i}}{\partial x_{j,j}} \cdot \frac{dx_{i,j}}{dt} \quad (7)$$

Our aim is to relate the stream flows and the molar fractions in the liquid that remain as the only state-variables of the problem. To have a unique vector W representing all the unknown internal flows, an index operator $n(i,k)$ is defined

for each stage i so that for indices $k=0$ to 4 stand for the streams F_i , G_i , L_{i-1} , G_{i+1} and L_i respectively.

$$n(i, k) = 4(i - 1) + k \quad (8)$$

Additionally, an operator $\text{sign}(k)$ is defined as (-1) for the output streams and (+1) for the input ones. Thus, combination of equations (5-7) yields to:

$$\sum_{k=1}^3 a_{n(i,k)}^e \cdot W_{n(i,k)} = q_i - a_{n(i,0)}^e \cdot W_{n(i,0)} \quad (9)$$

$$a_{n(i,k)}^e = h_{n(i,4)} - h_{n(i,k)} + \text{sign}(k) \sum_j \left[\frac{\partial h_L}{\partial x_j} \Big|_{x_{i,j}} \cdot (x_{n(i,k),j} - x_{i,j}) \right]$$

Additional equations are necessary to completely determine the stream flows. These equations can be obtained by expressing the liquid density as a function of the liquid composition and considering constant volume of liquid V_i as the liquid overflows in each stage:

$$\frac{dM_i}{dt} = V_i \frac{d\rho}{dt} = V_i \sum_j \frac{\partial \rho}{\partial x_j} \Big|_{x_{i,j}} \cdot \frac{dx_{i,j}}{dt} \quad (10)$$

Thus, the latter equation leads to following relationship:

$$\sum_{k=1}^4 a_{n(i,k)}^v \cdot W_{n(i,k)} = -a_{n(0,k)}^v \cdot W_{i,0} \quad (11)$$

$$a_{n(i,k)}^v = -1 + \frac{\text{sign}(k)}{\rho(x_{n(i,4)})} \sum_j \left[(x_{n(i,k),j} - x_{i,j}) \frac{\partial \rho}{\partial x_j} \Big|_{x_{i,j}} \right]$$

Equations (9, 11) constitute a non-linear system to solve the unknown internal flows as a function of the liquid composition. The coefficients of the system can be rearranged as a banded matrix. As this matrix is usually bad-conditioned, the Moore-Penrose pseudoinverse was used to solve the system of equations.

Therefore, the set of differential equations (5) can now be solved to obtain the evolution of the state-variables of the system. A multi-step stiff solver (Adams-Bashforth-Moulton PECE) was used to compute the tower dynamics as a function of feed, reflux ratio and reboiler heat duty.

3. VELOCITY-BASED FUZZY CONTROL OF A CONTINUOUS DISTILLATION PROCESS

A suitable modelling and identification of a system is essential for controller design. In our case, fuzzy identification adjusts those models to available data sets, and the local error of a number of local models that represent the

system in a region is preferred to the usual approach of minimizing the global prediction error (Díez, 2006).

The identification of fuzzy models needs to establish a model structure for a subsequent parameter identification step (Wang, 1997), as occurs in all identification procedures. This second identification step can be easily done, for example, by least mean squares if the system is linear in parameters (Babuska, 1996). This is our case, because input and output variables, and antecedent fuzzy membership functions A (interpretable as validity regions for local descriptions in a set of operating points) will be provided by experts, and rule consequents will be affine models following Takagi-Sugeno structure described by (12).

$$R_i: \text{If } x \text{ is } A_i \text{ then } y_i = a_i^T x + b_i, \quad i = 1, 2, \dots, K \quad (12)$$

where a_i is a vector of parameters, and b_i is a scalar. The model output y will be the convex combination of the consequents by means of its membership functions μ_i :

$$y = \frac{\sum_{i=1}^k \mu_i(x) y_i}{\sum_{i=1}^k \mu_i(x)} \quad (13)$$

Although the fuzzy model structure showed in (12) and (13) can give accurate results for prediction purposes, when the final goal is process control, a better approach overcoming interpolation problems caused by the terms b_i is the use of velocity-based models (Leith, 2000; Díez, 2004). These result from the substitution of (12) by its incremental form (14) where offset term is avoided.

$$R_i: \text{If } \Delta x \text{ is } A_i \text{ then } \Delta y_i = \Delta a_i^T \Delta x_i \quad i = 1, 2, \dots, K \quad (14)$$

A controller will be then designed for each rule, leading to the modelling and control structures outlined in Figure 2. System dynamics can be considered just taking past values of appropriate variables as new inputs.

In order to give theoretical framework for the proposed controller design technique, it is going to be applied as it was a classic gain scheduling controller (Leith, 2000). A possible approach to gain scheduling requires following the subsequent steps (Díez, 2004):

1. Determine a Linear Parameter Varying model of the non-linear system based on the linearization of the system at different operating points and integrating them, for example, by interpolation.
2. Calculate a Linear Controller for each linearization of the system and then integrate the controllers (for example, in the same way as the linearization), then obtaining a Non-Linear Controller.
3. Evaluate the control performance through simulation.

In our case, the proposal is that, once the operating points are given by the expert, a fuzzy model in the form of (13) with

rules as (14) will be tuned using experimental data. A model defined with this structure is a combination of purely linear (not affine) models, then matching the first step in the gain scheduling approach.

Next step will be the design of a linear (using classic techniques) controller for each rule consequent (i.e. each linearization or local model). The combination of all the controllers will be done using the same formula (13) used in the fuzzy modelling, but substituting local model output by local control action. In this way, a non-linear controller is designed as in the second step of the gain scheduling approach.

Performance can be tested via simulations, and a complete example is presented in the next section.

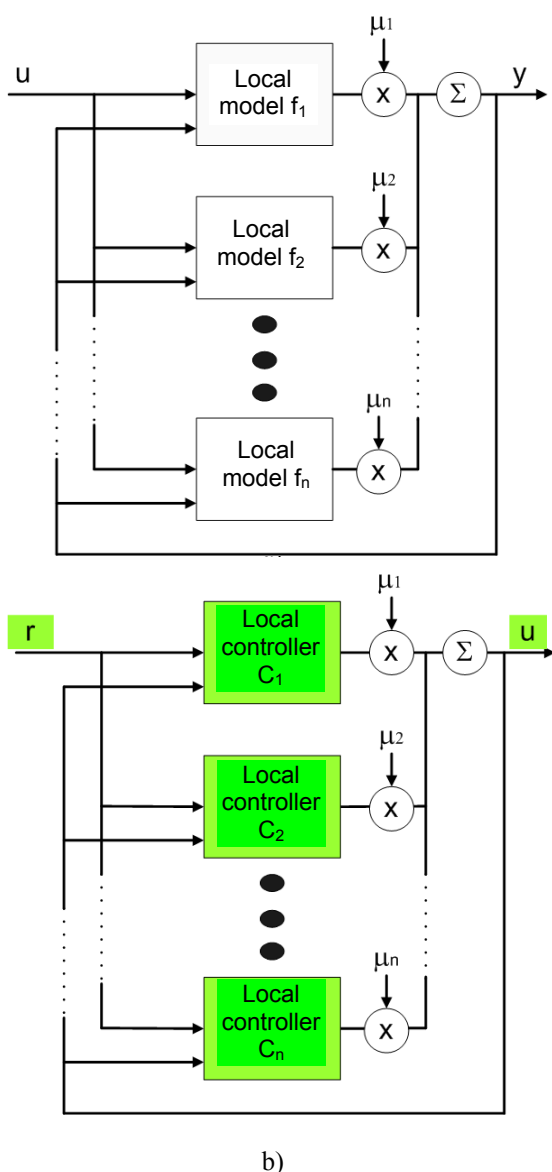


Fig.2. Outline of suggested structures for a) modelling and b) control.

4. MODEL RESULTS AND VALIDATION

The control system can be built using data from a real system. However, dynamic distillation models are accurate enough and can be used to generate data avoiding experimental cost. As an application example, a dynamic distillation model as described in section 2 has been used to obtain the necessary data to implement the fuzzy controller for a distillation of a mixture methanol-water. Parameters were: 10 trays, feed tray position = 6, tray volume of 0.064 m³ and condenser and reboiler volume of 0.4 m³. The same model was used to test the capability of the fuzzy controller under a set of scenarios.

The input disturbances applied were about ± 7.5 and $\pm 15\%$ for all inputs but R where the disturbances were $\pm 2.5\%$ and $\pm 5\%$ (bigger changes made the system unstable). The analysis of the results by experts yielded to the following conclusions:

- a positive increment in the inputs F and z_F causes a positive increment on x_{10} ,
- a positive increment in h_F , q_N , and R causes a negative increment on x_{10} ,
- a positive increment in z_F and q_N causes a positive increment on x_1 ,
- a positive increments in F , h_F and R causes a negative increment on x_1 ,
- the system response is similar to a first order system with a time constant of the system for each input at each working point for each output variable, and
- a set of six operating points (defined as combinations of F , z_F , h_F , q_N , and R) are the most common for system operation, and are summarized in Table 1.

Table 1. Operating points for the system under study.

Point	F (kmol/s)	z_F	h_F (kJ/kmol)	q_N (kJ/s)	R
1	0.217	0.36	11000	1878	1
2	0.217	0.46	11000	1878	1
3	0.217	0.26	11000	1878	1
4	0.217	0.36	20000	1878	1
5	0.310	0.36	11000	2744	1
6	0.110	0.36	11000	1011	1

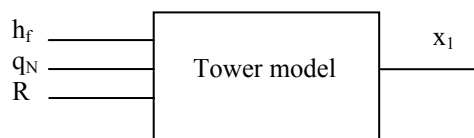


Fig. 3. Scheme of the continuous distillation tower from simple black box model point of view.

Then, a set of 5 models (one for each input/output pair) for each point (and then 30 systems for each output) was developed. As far as F and z_F are usually constant and that the most important output of the system is x_1 , (and the control of product composition x_1 will modify x_{10}), a simplified input-output view (including possible control variables as

inputs) of these small models is devised in Figure 3. This set of first order systems will be integrated with the fuzzy structure proposed in the previous section.

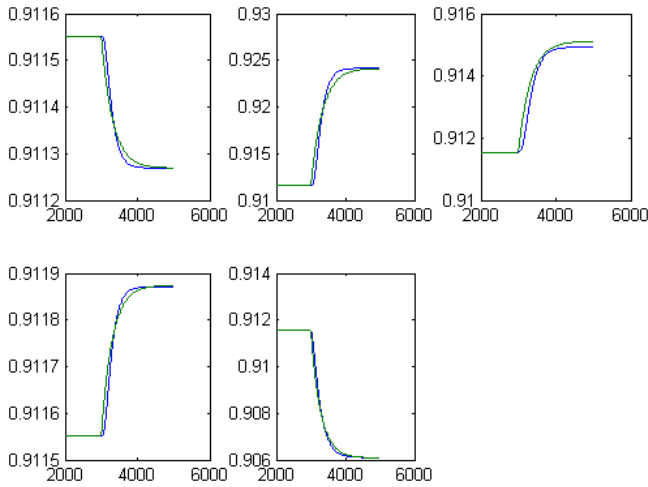


Fig. 4. Models of molar fraction x_I versus time (in seconds) at the operating point 5 (green: data, blue: model). From top left to bottom right, variations of x_I for disturbances in F , z_F , h_F , q , and R .

Equation (15) shows the G general single input single output linear discrete first order system (input u , output y , and instant k) placed at the consequent of each rule:

$$G = \frac{k}{z - \tau} \rightarrow y_k = \tau \cdot y_{k-1} + k \cdot u_k \quad (15)$$

That will be used in its incremental form:

$$\Delta y_k = \tau \cdot \Delta y_{k-1} + k \cdot \Delta u_k \quad (16)$$

Therefore, the fuzzy system considered 5 inputs (F , z_F , h_F , q_N , R) in order to indicate the operating point in what the tower is working, and 5 more inputs (ΔF , Δz_F , Δh_F , Δq_N , ΔR) to indicate the increment that it was applied to each input, and finally one more input to indicate the previous output (y_{k-1}). Rule selection was carried out for the first five inputs, and the other ones were used to calculate the value of the output. As the systems in this case just depend on one input, the coefficients for the others are zero. The behaviour obtained for the models was very good for the whole operation systems, being the modelling error lower than $5 \times 10^{-5} \%$. As an example, the model performance around the operating point 5 is shown in figure 4, but results are similar for the whole operating space.

Once the model was available, the controller could be designed. In this case it was a simple pole-zero for each rule consequent, following for its combination the structure of Figure 2.b. In fact, one controller was designed for each submodel and then the total model was built for all the controllers joined in a fuzzy system. The general incremental control equation is:

$$\Delta u_k = \frac{(p - \tau)}{k} \cdot \Delta y_k + \frac{(1 - p)}{k} \cdot \Delta r_k \quad (21)$$

where p is the chosen pole.

From a practical point of view, the controller can be applied to keep the target output in the desired range if unexpected disturbances occur in the feed. The controller must also be applied to set x_I at a value of interest, and changed it to another one if necessary. It can be reached operating on one of these: q_N , h_F or R . System feedback response is shown in Figure 5. The cases presented are those of possible variations of interest in x_I positive and negative steps, and the presented results make use of q_N as control variable. Results are similar in the whole operating space and also using h_F and R as control variables.

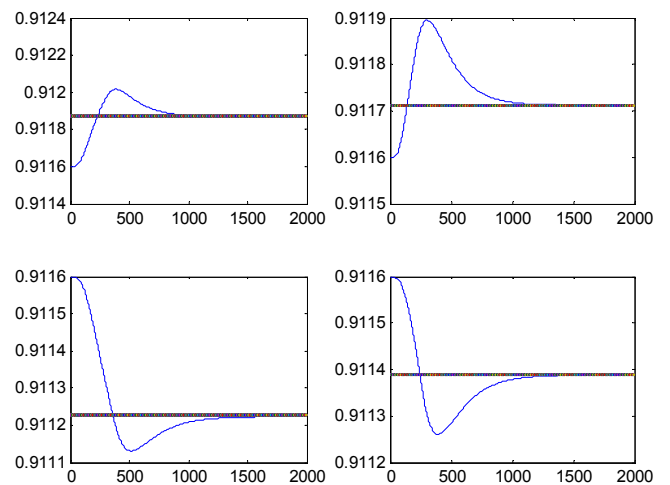


Fig. 5. Control of molar fraction x_I versus time (in seconds) for different positive and negative steps (7% and 15%) using q_N as control variable.

The advantage of having different control variables meeting the same requirements is that its effectiveness can be checked from different points of view (for example: economical, resources availability, etc.) and the decision on what control variable must be used can change from time to time depending on external factors.

5. OPTIMIZATION

As shown in Fig. 3 and described through all the text, the control can be performed by 3 different inputs. That allows the election of one of them attending to the chosen criteria.

These criteria could be imposed by two methods. One is the consideration of the availability of the resources needed to perform the control. The limits of each variable in the output level. For the h_F and q_N variables a heat source is needed, whereas a cooling source is needed for the R control input case.

As well as the limit of the q_N is the smallest one ($\pm 0.05\%$ in the example described here) it reduces the control possibilities by means of this variable. The limit of the control by the other two variables is bigger ($\pm 0.5\%$), allowing a big performance zone.

However, given that all the resources are available and the control could be built by any of the three control input, the selection criterion is, industrially attending, economic. As far as all the controls perform well, the cheaper one will be the chosen. Anyway, it is not only the cost of the application what should be studied, but also the benefits from production.

To apply heat to the stream feed or to the reboiler is more expensive than to subtract heat from the condenser, but the amount of product decreases when R input gets reduced.

The prices of fuel to get a heat source or the cooling water to perform the refrigeration must be updated from the market, as well as the price of the distillate to be sold. Therefore, not too many results can be exposed, but several conclusions can be drawn:

When control is applied by h_F or q_N , variables, the amount of product varies even if R is not modified, but this relation is no direct.

Control by R is always the cheaper one looking at application price.

The amount of product varies directly with R changes.

In all cases applied in the example case, control by R is the most profitable one, because the amount of product is not very different but the application price is much smaller than when control is performed by the other variables.

However, as said before, generalizations cannot be stated from these particular conclusions. In each case, the study must be performed again, due to price changes and amount of product obtained.

6. CONCLUSIONS

A velocity-based fuzzy control has been applied to a binary distillation simulated by a dynamic model. The necessary data were obtained using a dynamic distillation model that uses feed characteristics (enthalpy state), reflux ratio, and reboiler heat duty as inputs. The model was also useful to validate the efficiency of the fuzzy controller.

The final control structure consisted of a set of 5 local control models that were able to change the reflux ratio and heat duty either of the reboiler or of the feed stream in order to meet effectively the specified product quality for a wide range of feed variation. The results showed that the fuzzy controller was able to allow changes from a predefined output composition x_j to another, and the controller also was capable to keep the target output in the desired range. The developed

techniques are liable to be applied to more complex distillations. It was also seen that the variable to perform the control could be chosen in most of industrial cases, either by the availability of each one of the control variables or by economic reasons.

Future work includes a comparative study between the proposed technique and other applicable approaches, such as MPC. Additionally, the distillation of new distillation mixtures will be tested.

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