

## Information Based Fault Diagnosis

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**Abstract:** Fault detection and isolation, (FDI) of parametric faults in dynamic systems will be considered in this paper. An active fault diagnosis (AFD) approach is applied. The fault diagnosis will be investigated with respect to different information levels from the external inputs to the systems. These inputs are disturbance inputs, reference inputs and auxiliary inputs. The diagnosis of the system is derived by an evaluation of the signature from the inputs in the residual outputs. The changes of the signatures from the external inputs are used for detection and isolation of the parametric faults.

**Keywords:** Active fault diagnosis, closed-loop systems, parametric faults, signal information.

### 1. INTRODUCTION

Fault diagnosis in dynamic systems can be derived in mainly two different ways, either by using the well known passive approach, Basseville and Nikiforov [1993], Chen and Patton [1998], Gertler [1998], Gustafsson [2000], Willsky [1976], and the more recent active approach, Campbell and Nikoukhah [2004], Kerestecioglu [1993], Niemann [2006], Nikoukhah [1998], Nikoukhah et al. [2000], Zhang [1989]. In the passive approach, the diagnosis is based only on measurement signals and the control signals. A consequence of this is that faults can only be detected and/or isolated when the system is "disturbed" by external inputs as e.g. disturbances, reference changes etc. In the active approach, an auxiliary input (test signal) is injected into the system. The diagnosis is then derived by considering the signature from the auxiliary input in the outputs from the system or in residual outputs. The active approach will in general allow a fast detection compared with a passive approach. Further, it will also be possible to detect and isolate faults all the time, because the auxiliary input can be applied all the time.

Both approaches has some disadvantages. In the passive approach, the effect from the parametric faults in the residual outputs is not well described, it will depend on the inputs to the system. The diagnosis is based on changes in the residuals, but these changes are more or less unknown. In the active approach, it is a central element that the input to the system is well defined. Therefore, it is also well defined how the residual generator and the associated stochastic test should be optimized. The disadvantage is that an external input is injected into the system. This will also have an effect on the controlled outputs with a performance reduction as the consequence.

There is an alternative between these two approaches that can be very useful. Instead of using auxiliary inputs, it will in many cases be possible to use the information from the existing external inputs in connection with the diagnosis. The advantages with such an approach is that no new signals are injected into the system with a performance degradation as the result. Further, the residual generator and the associated stochastic test can

be optimized with respect to the information about the inputs. These inputs can e.g. be disturbance including periodic components, constants components or other known time depending components. A reference input is also an external input that can be used in connection with fault diagnosis. The diagnosis is based on the known or partly known signature in the residual vector from the external input. These type of systems can be called self-excited systems. Examples of self-excited systems are mechanical systems including rotating elements as e.g. a wind turbine, momentum wheels etc. Constant external inputs on systems can be found as e.g. constant momentum or force on a system is the gravity force.

The main issue in this paper is to investigate the application of external inputs in connection with active fault diagnosis. The derived results can be used for both continuous-time as well as for discrete-time systems. The applied CUSUM method is given in a discrete-time version in this paper, but a continuous-time version can also be derived.

### 2. SYSTEM SET-UP

Let a general system be given by:

$$\Sigma_{P,\theta} : \begin{cases} e = G_{ed}(\theta)d + G_{eu}(\theta)u \\ y = G_{yd}(\theta)d + G_{yu}(\theta)u \end{cases} \quad (1)$$

where  $d \in \mathcal{R}^r$  is a disturbance signal vector,  $u \in \mathcal{R}^m$  the control input signal vector,  $e \in \mathcal{R}^q$  is the external output signal vector to be controlled, and  $y \in \mathcal{R}^p$  is the measurement vector. Further,  $\theta$  is a vector given by

$$\theta = (\theta_1, \dots, \theta_i, \dots, \theta_k)^T \quad (2)$$

including parametric faults/variations in the system.  $\theta_i$  is the  $i$ 'th parametric fault. Let  $\theta_i = 0$ ,  $i = 1, \dots, k$  represent the fault free case.

Further, let the system be controlled by a stabilizing feedback controller given by:

$$\Sigma_C : \{ u = Ky \} \quad (3)$$

### 3. PRELIMINARY RESULTS

#### 3.1 Coprime Factorization

Let a coprime factorization of the nominal system  $G_{yu}(0)$  from (1) and the stabilizing controller  $K$  from (3) be given by:

$$\begin{aligned} G_{yu} &= NM^{-1} = \tilde{M}^{-1}\tilde{N}, \quad N, M, \tilde{N}, \tilde{M} \in \mathcal{RH}_\infty \\ K &= UV^{-1} = \tilde{V}^{-1}\tilde{U}, \quad U, V, \tilde{U}, \tilde{V} \in \mathcal{RH}_\infty \end{aligned} \quad (4)$$

where the eight matrices in (4) must satisfy the double Bezout equation given by, see Tay et al. [1997]:

$$\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} = \begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix} \begin{pmatrix} M & U \\ N & V \end{pmatrix} = \begin{pmatrix} M & U \\ N & V \end{pmatrix} \begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix} \quad (5)$$

#### 3.2 FDI Set-up

Based on the system  $\Sigma_{P,\theta}$  given by (1) and the feedback controller  $\Sigma_C$  given by (3), the following block diagram shown in Fig. 1 can be derived.

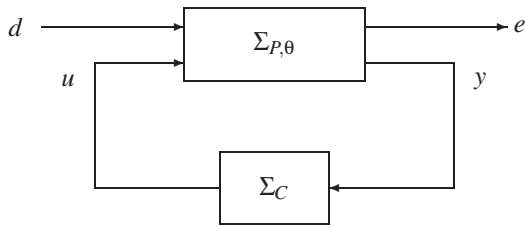


Fig. 1. Feedback control system.

Now, including also a residual vector in connection with the feedback control system in Fig. 1. A residual vector  $\varepsilon$  can be given by, Frank and Ding [1994]

$$\varepsilon = \tilde{M}y - \tilde{N}u \quad (6)$$

Let's use the feedback controller described by coprime factors, i.e.  $K = \tilde{V}^{-1}\tilde{U}$ , in the block diagram. Further, include also an auxiliary input vector  $\eta$  in between the two controller coprime matrices in feedback controller. The block diagram in Fig. 1 including  $\eta$  and  $\varepsilon$  is shown in Fig. 2. The two signal vectors,  $\eta$  and  $\varepsilon$ , will be applied in connection with the active fault diagnosis.

Based on the feedback system in Fig. 2, it is possible to give the transfer functions from the two input vectors  $d, \eta$  to the two output vectors  $e, \varepsilon$ . This is given by:

$$\begin{pmatrix} e \\ \varepsilon \end{pmatrix} = \begin{pmatrix} P_{ed} & P_{e\eta} \\ P_{\varepsilon d} & P_{\varepsilon\eta} \end{pmatrix} \begin{pmatrix} d \\ \eta \end{pmatrix} \quad (7)$$

where  $P$  is given by Niemann and Stoustrup [2002]:

$$\begin{aligned} P_{ed} &= G_{ed}(\theta) + G_{eu}(\theta)U(V - G_{yu}(\theta)U)^{-1}G_{yd}(\theta) \\ P_{e\eta} &= G_{eu}(\theta)(M - U(V - G_{yu}(\theta)U)^{-1}(N - G_{yu}(\theta)M)) \\ P_{\varepsilon d} &= (V - G_{yu}(\theta)U)^{-1}G_{yd}(\theta) \\ P_{\varepsilon\eta} &= -(V - G_{yu}(\theta)U)^{-1}(N - G_{yu}(\theta)M) \end{aligned} \quad (8)$$

Note that the transfer function from the input vector  $\eta$  to the residual vector  $\varepsilon$  is equal to the dual YJBK transfer function, Niemann [2003], Tay et al. [1997]. The dual YJBK parameterization gives the parameterization of all systems stabilized by

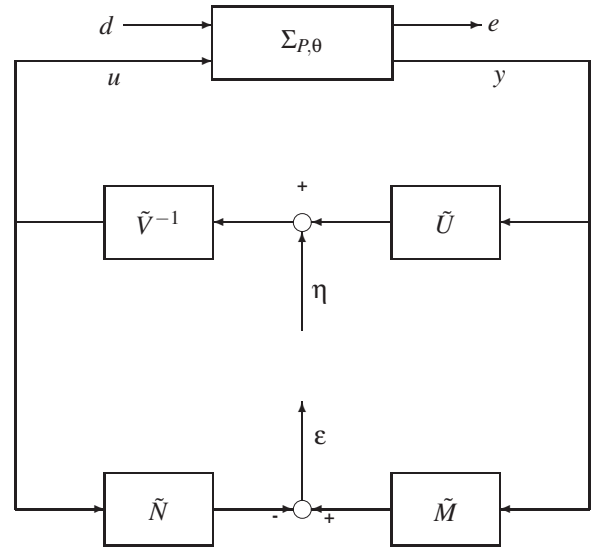


Fig. 2. Controller structure including residual vector  $\varepsilon$  and the external input vector  $\eta$ .

a fixed feedback controller in terms of the stable dual YJBK transfer function. The dual YJBK transfer function is a control relevant quantity. If a parameter change is not visible in the dual YJBK transfer function, then it will not have an influence on the control performance. The dual YJBK transfer function is normally denoted  $S$ . We will use  $S$  instead of  $P_{e\eta}$  in the rest of this paper.

It is possible to rewrite  $S = P_{e\eta}$  given by (8) when the system  $\Sigma_{P,\theta}$  is described as a linear fractional transformation (LFT) of  $\theta$ , see Niemann [2003].

#### 3.3 Passive Fault Diagnosis

It has been shown in Frank and Ding [1994] that it is possible to parameterize all residual generators by using the YJBK parameterization. All residual vectors  $\varepsilon_q$  for the  $\Sigma_{P,\theta}$  given by (1) can be described by

$$\varepsilon_q = Q_{FD,O}(\tilde{M}y - \tilde{N}u) = Q_{FD,O}\varepsilon \quad (9)$$

where  $Q_{FD,O}$  is a stable and proper filter of suitable order.  $Q_{FD,O}$  needs to be designed such that the residual signal/vector  $\varepsilon_q$  satisfies the following conditions for fault detection, Saberi et al. [2000]:

$$\begin{aligned} \varepsilon_q &= 0 \text{ for } \theta = 0, \forall(d, u) \\ \varepsilon_q &\neq 0 \text{ for } \theta \neq 0, \forall(d, u) \neq (0, 0) \end{aligned} \quad (10)$$

Equivalent, definitions for fault isolation can also be derived, Saberi et al. [2000]. Depending on how many faults that can occur simultaneously, different conditions can be given, Saberi et al. [2000]. It is also important to point out that it is not always possible to design  $Q_{FD,O}$  such that it is possible to obtain exact fault detection and fault isolation. Instead, different forms of approximative fault detection and/or fault isolation need to be considered, Frank and Ding [1994].

#### 3.4 Active Fault Diagnosis

The system given by (7) will now be applied in connection with AFD. Let the system be given by:

$$\Sigma_{FD} : \begin{cases} e = P_{ed}(\theta)d + P_{e\eta}(\theta)\eta \\ \varepsilon = P_{\varepsilon d}(\theta)d + S(\theta)\eta \end{cases} \quad (11)$$

The internal vectors will be applied for AFD. The system  $\Sigma_{FD}$  is shown in Fig. 3.

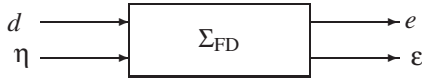


Fig. 3. The system set-up for active fault diagnosis.

The first important observation of  $\Sigma_{FD}$  is that  $S(\theta)$  is zero in the fault free case, i.e.

$$S(0) = 0 \quad (12)$$

It is clear from this first observation, that  $S(\theta)$  is very important in connection with AFD. Following the definition of *fault signature* for additive faults in Massoumnia [1986],  $S(\theta)$  will be named as the *fault signature matrix* in connection with parametric faults, Niemann [2005].

The fault diagnosis based on the active approach considered in Niemann [2005], Niemann and Poulsen [2005, 2006], Poulsen and Niemann [2007] is based on using simple periodic auxiliary inputs. The fault diagnosis is then derived by an investigation of the residual vector with respect to a signature from the auxiliary input. This is done by considering the residual vector at the same frequency applied for the periodic auxiliary input. However, it is also possible to use other types of auxiliary inputs than periodic signals. This has e.g. been done in the approach used in Campbell and Nikoukhah [2004], Kerestecioglu [1993], Zhang [1989].

#### 4. INFORMATION BASED FAULT DIAGNOSIS

As pointed out in the introduction, one of the disadvantages with AFD is that an auxiliary input is injected into the system which will in general reduce the performance of the system. However, it will in many cases be possible to apply the AFD concept without an injection of an auxiliary input. The reason is that the controlled system has a number of external inputs that can be applied in connection with AFD.

Including an auxiliary input, the external inputs to a system can be derived into four different types of inputs depending on the level of information. These four inputs are: unknown white disturbance/noise  $d_u$ , partly known disturbance input  $d_p$ , complete known reference input  $r$  and auxiliary input  $\eta$  explicit designed for AFD. Introducing the these four different inputs in (11) gives the following description for  $\Sigma_{FD}$ :

$$\Sigma_{FD} : \begin{cases} e = P_{ed_u}(\theta)d_u + P_{ed_p}(\theta)d_p + P_{er}(\theta)r + P_{e\eta}(\theta)\eta \\ \varepsilon = P_{\varepsilon d_u}(\theta)d_u + P_{\varepsilon d_p}(\theta)d_p + P_{\varepsilon r}(\theta)r + S(\theta)\eta \end{cases} \quad (13)$$

The system  $\Sigma_{FD}$  given by (13) is then modified as shown in Fig. 4.

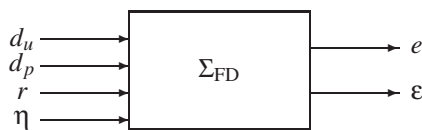


Fig. 4. The system set-up for information based fault diagnosis.

The information levels related to the four different inputs are described in the following and related to fault diagnosis.

#### Unknown disturbance input $d_u$

The disturbance input  $d_u$  is assumed to be unknown white noise. If the disturbance input  $d_u$  is not white noise, a noise model can be included in the system set-up. The colored noise  $d_u$  is then given by:

$$d_u = H\bar{d}_u$$

where  $\bar{d}_u$  is white noise and  $H$  is a stable filter.  $H$  is included in the system model.

Fault diagnosis based on  $d_u$  is the standard passive fault diagnosis, where the residual vector  $\varepsilon$  or  $\varepsilon_q$  is evaluated with respect to change in the mean value or the variance. Statistical test methods as e.g. CUSUM (cumulative sum) or ML (maximum likelihood) test, see Basseville and Nikiforov [1993], Gustafsson [2000] for further details, can be applied.

One of the disadvantages with this method is that a parametric fault given by  $\theta_i$  might not necessary give a change in the mean value or the variance of the residual vector. This need to be investigated for all possible parametric faults.

#### Partly known disturbance input $d_p$

The input  $d_p$  is the disturbance input that is partly known. Some informations about the disturbance input  $d_p$  are known. This can e.g. be the disturbance include a specific frequency or a limited number of frequencies, the disturbance is only active in a limited frequency range, the amplitude of the disturbance is given or is bounded or the disturbance include a constant component.

Especially the case where the disturbance include one or a few known frequencies is relevant in a large number of systems including rotating components. A wind turbine is disturbed by the shadow from the tower. This is a disturbance with a known frequency. The disturbance in a gear box will also be related directly to the velocities of the axes in the box.

Fault diagnosis in connection with rotating components can in many cases be derived by evaluation of the signatures in the residuals from periodic disturbances related to the rotation velocity. A constant disturbance with unknown amplitude will also be useful to detect and isolate parametric faults.

For a more detailed analysis of this case, let's consider the residual vector as function of  $d_p$  given by:

$$\varepsilon = P_{\varepsilon d_p}(\theta)d_p \quad (14)$$

where  $P_{\varepsilon d_p}(0)$  is non-zero in the fault free case. This means that diagnosis based on  $d_p$  is based on detecting of deviation away from the nominal residual vector. Let's analyze the case where the disturbances include a periodic part or the disturbances include a constant part. Only the SISO case will be considered in the following, but it will be possible to extend the results to the MIMO case.

The disturbance  $d_p$  can be modelled by

$$d_p = a_\omega \cos(\omega_0 t) + \xi \quad (15)$$

where  $a_\omega$  is constant,  $\omega_0$  is the frequency for the periodic disturbance and  $\xi$  is the remaining part of the disturbance. It is assumed that  $\xi$  is a Gaussian disturbance. The frequency  $\omega_0$  as well as the effect of  $a_\omega$  might either be known or estimated from the normal situation. Considering the periodic part of the disturbance in (15) gives the following residual vector:

$$\varepsilon = |P_{\varepsilon d_p}(\theta)|a_\omega \cos(\omega_0 t + \phi) + v \quad v = P_{\varepsilon d_p}(\theta)\xi \quad (16)$$

In the normal situation the noise component in the residuals  $\varepsilon$  will be white noise. This can be obtained either by a filter on the residual signal or by incorporate a Kalman filter in the coprime factorization.

In order to detect if the signature of the periodic signal is changed due to faults in the residual, the following two signals are formed:

$$c = \varepsilon \cos(\omega_0 t) \quad s = \varepsilon \sin(\omega_0 t)$$

where according to (16)

$$c = |P_{ed_p}(\theta)| \frac{a_\omega}{2} (\cos(\phi) + \cos(2\omega_0 t + \phi)) + v \cos(\omega_0 t)$$

$$s = |P_{ed_p}(\theta)| \frac{a_\omega}{2} (-\sin(\phi) + \sin(2\omega_0 t + \phi)) + v \sin(\omega_0 t)$$

The signal  $c$  (equivalent for  $s$ ) consist of three signals, a constant signal  $|P_{ed_p}(\theta)| \frac{a_\omega}{2} \cos(\phi)$  a time varying signal  $|P_{ed_p}(\theta)| \frac{a_\omega}{2} \cos(2\omega_0 t + \phi)$  with zero mean value and a disturbance signal  $v \cos(\omega_0 t)$ .

From this it is clear that in the fault free situation

$$c \in \bar{N}(|P_{ed_p}(0)| \frac{a_\omega}{2} \cos(\phi), \frac{1}{2} \sigma_0^2)$$

$$s \in \bar{N}(-|P_{ed_p}(0)| \frac{a_\omega}{2} \sin(\phi), \frac{1}{2} \sigma_0^2)$$

where  $\bar{N}$  indicates the Gaussian distribution with time average mean and variance. It is assumed that the time average mean:

$$\begin{bmatrix} m_c \\ m_s \end{bmatrix} = |P_{ed_p}(\theta)| \frac{a_\omega}{2} \begin{bmatrix} \cos \phi(\theta) \\ -\sin \phi(\theta) \end{bmatrix} \quad (17)$$

and variance  $\sigma_0$  can be determined or estimated from the normal situation.

The detection can be implemented as a CUSUM detection (which in discrete time is) given by

$$z(t+1) = \max(0, z(t) + \left[ \frac{\delta}{\sigma_1} - \frac{1}{2} \gamma \right]) \quad (18)$$

where

$$\delta = \begin{bmatrix} c \\ s \\ -c \\ -s \end{bmatrix} - \begin{bmatrix} m_c \\ m_s \\ m_c \\ m_s \end{bmatrix}, \quad \sigma_1 = \frac{\sigma_0}{\sqrt{2}} + |P_{ed_p}(0)| \frac{a_\omega}{2}$$

The  $H_0$  is accepted if  $z_t$  is smaller than the threshold, i.e.

$$z(t) \leq h$$

where the inequality is to be understood element wise. The tuning parameters in this CUSUM detector is  $h$ , which is related to the average length between false detections,  $\gamma$  which forms a typical lower limit of changes to be detected. The latter quantity is off course related to the lower limits of detection for the individual parameter changes.

Now, consider the case where  $d_p$  include a constant part.  $d_p$  can then be modelled by

$$d_p = a_0 + \xi \quad (19)$$

where  $a_0$  is unknown but constant and  $\xi$  is the remaining part of the disturbance. The constant part of the residual vector  $\varepsilon$  is given by

$$\varepsilon = |P_{ed_p}(\theta)| a_0 + v \quad v = P_{ed_p}(\theta) \xi \quad (20)$$

Note that there is no phase information available when a constant is applied. This gives that the CUSUM test can only be based on changes in the mean value of the residual vector  $\varepsilon$ . This gives two CUSUM tests, one for positive change and one for negative change of the mean value of  $\varepsilon$ .

### Reference input $r$

The reference input  $r$  is known exact, but it will not in general be possible to change or modify  $r$ . The reference input is designed with respect to the controlled output  $e$ . The transfer function from  $r$  to the residual vector is given by:

$$\varepsilon = P_{\varepsilon r}(\theta) r \quad (21)$$

In the construction of the residual vector  $\varepsilon$ , the effect from all known signals are decoupled in the nominal case. This mean that the transfer function from  $r$  to  $\varepsilon$  is zero in nominal case, i.e.

$$P_{\varepsilon r}(\theta) = 0, \text{ for } \theta = 0$$

The diagnosis is then derived by an investigation of the residual signal with respect to signatures from the reference input. In the case when the reference input is either a periodic, a constant input or can be described as linear combination of these, the methods described above can be applied directly. An alternative to this is to use dedicated filters on the residual signal that will give a response when a signature from the reference input occur in the residual vector.

### Auxiliary input $\eta$

The auxiliary input vector  $\eta$  is free to design with respect to optimize the fault diagnosis. This mean that the amplitude, the input direction, the frequency in periodic inputs etc. are free to select/design with respect to optimizing the fault diagnosis. As pointed out before, one consequence of using an auxiliary input is that the input will in general also affect the controlled output  $e$  with a performance reduction as the result. However, the freedom in the design of  $\eta$  can also be applied to minimize or reduce the effect from  $\eta$  on the controlled output. Some preliminary results has been derived in Niemann and Poulsen [2005, 2006] with respect to minimize the time to detect and also minimize the effect from  $\eta$  on the controlled output  $e$ .

Instead of using a general criterion for the optimization of the auxiliary input, it is possible to design  $\eta$  more directly with respect to remove the effect completely from all or some of the controlled outputs. Let's start with considering the transfer function from  $\eta$  to the controlled output  $e$  given by

$$e = P_{e\eta}(\theta) \eta \quad (22)$$

Now, assume that the considered system is a MIMO system. In the SISO case, it is not possible to decoupling based on using specific input directions of the auxiliary input  $\eta$ . For designing an auxiliary input  $\eta$  such that it will not affect all or some of the controlled output  $e$ , let's include a stable pre-filter in (22), i.e.

$$e = P_{e\eta}(\theta) Q_{FD,I} \eta_q \quad (23)$$

Decoupling is obtained if:

$$P_{e\eta}(\theta) Q_{FD,I} = 0, \text{ for } \theta = 0 \quad (24)$$

$$S(\theta) Q_{FD,I} \neq 0, \text{ for } \theta \neq 0$$

The last condition will guarantee that the transfer function from  $\eta$  to the residual vector is non-zero in the faulty case. The first condition in (24) can only be satisfied if

$$q < m \quad (25)$$

When the above condition is satisfied, a non-zero stable pre-filter  $Q_{FD,I}$  can be designed. The design can be derived by using the method described in Saberi et al. [2000], where design of post-filters has been considered in connection with passive fault diagnosis. In connection with the design of  $Q_{FD,I}$ , it is



not necessary to design the pre-filter such that the decoupling is satisfied for all frequencies. In the case where the auxiliary inputs is based on simple periodic signals, it is only necessary that the decoupling is satisfied at the specific frequency or frequencies. This simplifies the design of the pre-filter.

The condition given above will not be satisfied in all cases with the result that exact decoupling of the auxiliary input in the controlled output cannot be obtained. However, it is still possible to obtain decoupling in a number of controlled outputs. It will always be possible to obtain an exact decoupling of  $m - 1$  controlled outputs. The decoupling can then be obtained for the most critical outputs in  $e$ .

### 5. EXAMPLE

Let's consider a sampled version of a simple second order system given by

$$G(s) = \frac{k}{s^2 + 2\zeta\psi s + \psi^2} = \frac{1}{s^2 + 0.2s + 1}$$

influenced by stochastic disturbances. Variations in the three parameters  $k$ ,  $\zeta$  and  $\psi$  will be considered as parametric faults in the system.

In discrete time (with sampling period  $T_s = 0.01 \text{ sec}$ ) and in state space the system is given by

$$x(t+1) = Ax(t) + Bu(t) + v(t) \quad (26)$$

$$y(t) = Cx(t) + e(t) \quad (27)$$

where the process noise consists of unknown and partly known disturbances:

$$v(t) = Bd_u(t) + Bd_p(t)$$

The stochastic noise processes are zero mean white noise sequences and

$$\text{Var} \left\{ \begin{bmatrix} d_u(t) \\ e(t) \end{bmatrix} \right\} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.01 \end{bmatrix}$$

The process noise (unknown as well as partial known) is here an input disturbance, but the methods are by no means restricted to this type. The control is based on a state estimate obtained by means of a stationary Kalman filter and the control is an ordinary LQ controller which aim at minimizing the objective function

$$J = \mathbf{E} \left\{ \sum_{t=0}^{\infty} x^T(t)Qx(t) + u^T(t)Ru(t) \right\} \quad Q = I_2 \quad R = 0.2$$

This design results in a controller given by:

$$V = \frac{z^2 - 1.931z + 0.9332}{z^2 - 1.957z + 0.9581} \quad U = \frac{-0.2664z + 0.2661}{z^2 - 1.957z + 0.9581}$$

and a model parameterized through

$$N = \frac{5.05z + 5.046}{z^2 - 1.957z + 0.9581} 10^{-5} \quad M = \frac{z^2 - 1.998z + 0.998}{z^2 - 1.957z + 0.9581}$$

A simple analysis of this closed system results in  $\sigma_0^2 = 1.2e^{-3}$  and

$$|P_{ed_p}(0)| \frac{a_\omega}{2} = 0.0553 \quad \begin{bmatrix} m_c \\ m_s \end{bmatrix} = \begin{bmatrix} 0.0530 \\ 0.0156 \end{bmatrix}$$

Notice that  $m_c$  and  $m_s$  can be estimated from the normal situation or be determined by means of closed-loop analysis.

The parameters in the CUSUM detector was chosen to be:

$$\gamma = 0.01 \quad h = 2$$

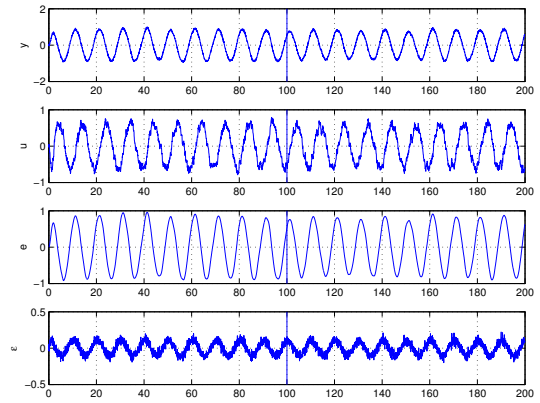


Fig. 5. The signals ( $y, u, e, \epsilon$ ) for a fault in  $k$  at  $t=100 \text{ sec}$ .

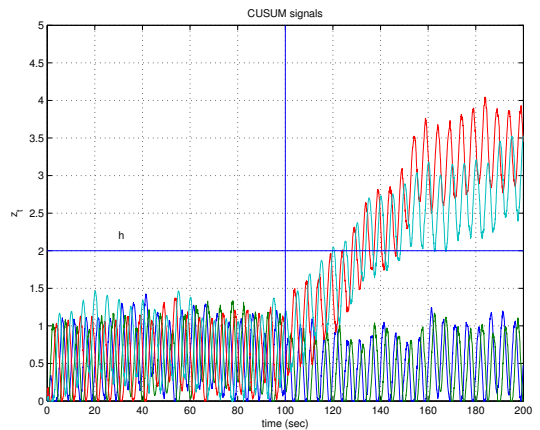


Fig. 6. The cusum signals ( $z$ ) for a fault in  $k$  at  $t=100 \text{ sec}$ .

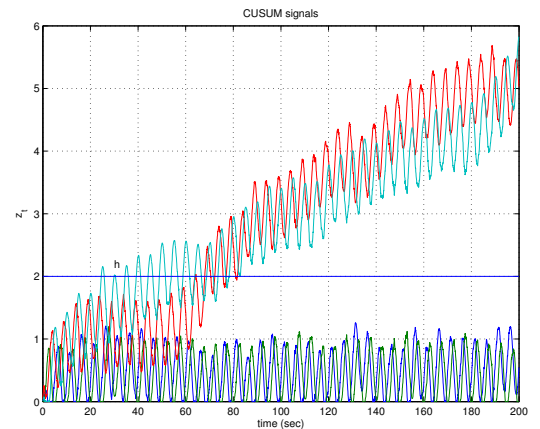


Fig. 7. The cusum signals for a fault in  $k$  at  $t=0 \text{ sec}$ .

The choice of  $\sigma_1$  was based on the assumed knowledge of (i.e. the possibility of estimating)  $\sigma_0^2$  and the effect of  $d_p$  in  $\epsilon$ .

The relevant signals are plotted in Fig. 5 and 6 for a change in  $k$  (-10 %) at  $t = 100 \text{ sec}$ . The cusum signals are plotted in Fig. 7-9 for an initial (at  $t = 0$ ) -10%, 50% and a 10 % change in each of the three parameters:  $k$ ,  $\zeta$  and  $\psi$ . From the plots in Fig. 7-9 it is clear that the shift in mean of  $m_c$  and  $m_s$  in (17)

$$\Delta \begin{bmatrix} m_c \\ m_s \end{bmatrix} = \begin{bmatrix} -0.0021 \\ -0.0023 \end{bmatrix} \quad \Delta \begin{bmatrix} m_c \\ m_s \end{bmatrix} = \begin{bmatrix} -0.0026 \\ 0.0013 \end{bmatrix}$$

$$\Delta \begin{bmatrix} m_c \\ m_s \end{bmatrix} = \begin{bmatrix} -0.0054 \\ -0.0070 \end{bmatrix}$$

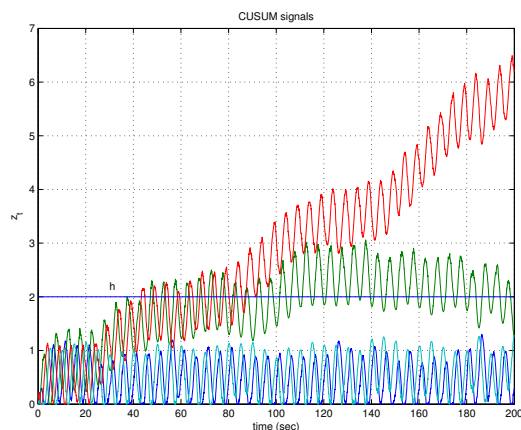


Fig. 8. The cusum signals for a fault in  $\zeta$  at  $t=0$  sec.

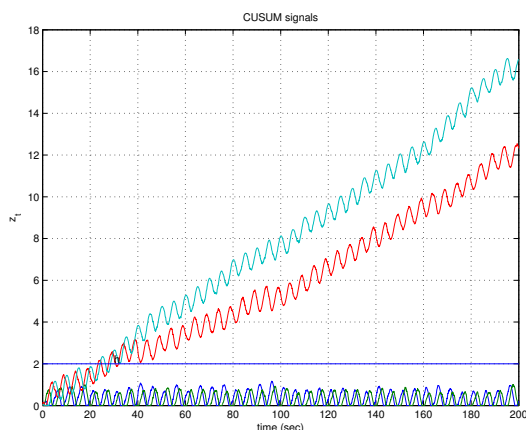


Fig. 9. The cusum signals for a fault in  $\psi$  at  $t=0$  sec.

for changes in  $k$ ,  $\zeta$  and  $\psi$  can be used for isolation of the fault. Note that the oscillations in Figs. 7 - 9 are caused by the time varying part in the  $c$  and  $s$  signals.

## 6. CONCLUSION

An active fault diagnosis approach has been investigated with respect to fault diagnosis based on different excitations of the system. Instead of using auxiliary inputs for the fault diagnosis, it is possible to base the diagnosis on the signatures from disturbances and/or reference inputs when these inputs has a well defined signature in the residuals. This does not require an exact knowledge of the external inputs. Changes in the signature can then be used to detect and isolate faults in the system.

Further, using the standard AFD approach, it is investigated under which conditions it is possible to include auxiliary inputs in the control-loop without affecting the controlled output. It has been shown that it is always possible to decouple the effect in some of the controlled outputs when the auxiliary input is a vector input.

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