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Persistency of excitation in subspace predictive control

R. Hallouzi * **M. Verhaegen** *

* *Delft Center for Systems and Control, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands, (e-mail: {R.Hallouzi, M.Verhaegen}@dcsc.tudelft.nl)*

Abstract: This paper presents a method that ensures persistency of excitation for subspace predictive control. This control method is characterized by the combination of a predictive control law with a subspace predictor. The subspace predictor is continuously being adapted to the controlled system by using input-output data from this system. For this purpose the input-output data should be persistently exciting. In this paper a method is proposed to ensure persistency of excitation by adding a term to the cost function used by the predictive control law. This term is designed such that only the least excited directions of the input space are additionally excited. An advantage of the method is that the optimization problem that needs to be solved for the predictive controller can still be solved by using quadratic programming. The proposed excitation method is evaluated in simulation on a detailed nonlinear model of a transport aircraft. The simulation results clearly show the usefulness of the proposed method.

1. INTRODUCTION

Subspace predictive control (SPC) is a control method that is characterized by the combination of a predictive control law and a subspace predictor. The concept of SPC was first described by Favoreel and De Moor [1999], after which it was used by several other researchers [Woodley et al., 2001, Kadali et al., 2003] . The subspace predictor is derived according to subspace identification theory using input-output data of the controlled system. This theory has gained much interest in the last two decades [Van Overschee and De Moor, 1996, Verhaegen and Dewilde, 1992] because of its efficient way of identifying state-space models for high order, multiple input, multiple output, linear time-invariant systems. The predictive control law that is used for SPC as described by Favoreel and De Moor [1999], is a standard generalized predictive control law [Maciejowski, 2002]. The main advantage of SPC is that it is capable of adapting to the controlled system because the subspace predictor is continuously updated by new input-output data. This advantage makes SPC very useful for fault-tolerant control (FTC) [Hallouzi and Verhaegen, 2007], for which it is required that the controller can adapt to post-fault conditions.

An important requirement for obtaining a subspace predictor that can accurately predict the system outputs, is that the available input-output data contains sufficient information on the system. This requirement can be met by ensuring that the input signals are persistently exciting the system. In this paper, the notion of persistency of excitation (PE) is directly linked to non-singularity of a data matrix that contains input signals stacked in a particular order. This matrix will be explained later in the paper. The requirement for PE can conflict with the control objective, e.g. in the steady-state case. In such a case the control objective can be met by using constant input signals, which are not persistently exciting the system. If the input-output data in this case is used to update the subspace predictor, this can lead to a drastic degradation of the performance of the predictor. Therefore, in order to ensure PE, even in the steady-state case, the system should be excited more than strictly necessary for control. Instead of using a randomly generated excitation signal, such additional excitation can be derived by minimizing some control-oriented measure of model mismatch, as is done for example by Bombois et al. [2006] and Forssell and Ljung [2000] for prediction error identification.

In the framework of simultaneous predictive control and system identification, methods for excitation have been proposed by formulating additional constraints to the optimization problem related to the predictive control law [Aggelogiannaki and Sarimveis, 2006, Shouche et al., 1998]. The drawback of these methods is that the additional constraints are non-convex. The result of this is that a non-convex optimization problem should be solved each time step to compute the control input. Standard constrained generalized predictive control problems require solving a quadratic program, which is a convex optimization problem [Maciejowski, 2002]. Since solving non-convex problems is significantly more involved than solving convex problems, the additional non-convex constraints are not desirable.

The contribution of this paper is that an excitation method is proposed that allows the required optimization problem to remain convex and quadratic. This excitation method only considers the input directions that are least excited. These directions are then additionally excited. In this aspect the proposed excitation method is related to the methods described by Bombois et al. [2006], Forssell and Ljung [2000]. The excitation method is integrated with

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SPC by adding an extra term to the cost function of the predictive control problem. The additional term allows the optimization problem to remain quadratic. This problem can therefore still be solved by using quadratic programming, which makes it possible to use the SPC algorithm with excitation for on-line applications. The SPC algorithm with excitation is evaluated in simulation on a detailed nonlinear model of a transport aircraft.

The organization of this paper is as follows. In Section 2, SPC theory is described briefly. Next, in Section 3 the proposed method for ensuring PE is explained. Section 4 contains the simulation results. Finally, concluding remarks are provided in Section 5.

2. SUBSPACE PREDICTIVE CONTROL

The SPC algorithm proposed by Favoreel and De Moor [1999] elegantly combines two theories, namely subspace identification and generalized predictive control. This combination results in a controller that does not require a model of the system in advance. Instead, based on input-output data, a subspace predictor can be identified that is used by the predictive controller. Because SPC does not require an explicit model of the system, it can be referred to as a "model-free" control method [Favoreel et al., 1999]. However, this name is arguable because the subspace predictor can be considered to be a kind of model [Woodley et al., 2001]. In this section, first the subspace predictor is derived. Next, this predictor is integrated with a predictive controller.

2.1 Subspace predictor

The general problem considered in linear subspace identification is to find system matrices A, B, C, and D, given measurements of the inputs $u_k \in \mathbb{R}^m$ and outputs $y_k \in \mathbb{R}^l$ of a linear time-invariant state-space system described by

$$x_{k+1} = Ax_k + Bu_k, \tag{1}$$
$$y_k = Cx_k + Du_k, \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state of the system. Matrix input-output relations are commonly used in subspace identification. These relations can be obtained by a recursive substitution of (1)-(2) and they are given by

$$Y_p = \Gamma_j X_p + H_j U_p, \tag{3}$$
$$Y_f = \Gamma_j X_f + H_j U_f. \tag{4}$$

Let the measurements of the inputs and outputs u_k and y_k be given for $k \in \{0, 1, \ldots, 2M+j-2\}$, then the Hankel matrices for the output are constructed as

$$Y_p = \begin{bmatrix} y_0 & y_1 & \cdots & y_{j-1} \\ y_1 & y_2 & \cdots & y_j \\ \vdots & \vdots & \ddots & \vdots \\ y_{M-1} & y_M & \cdots & y_{M+j-2} \end{bmatrix},$$
$$Y_f = \begin{bmatrix} y_M & y_{M+1} & \cdots & y_{M+j-1} \\ y_{M+1} & y_{M+2} & \cdots & y_{M+j} \\ \vdots & \vdots & \ddots & \vdots \\ y_{2M-1} & y_{2M} & \cdots & y_{2M+j-2} \end{bmatrix}, \tag{5}$$

and the Hankel matrices for the input (U_p and U_f) are constructed in a similar manner. The subscripts p and f denote "past" and "future", respectively. The matrices X_p and X_f are defined as

$$X_p = [x_0 \;\; x_1 \;\; \cdots \;\; x_{j-1}], \tag{6}$$
$$X_f = [x_M \;\; x_{M+1} \;\; \cdots \;\; x_{M+j-1}]. \tag{7}$$

The parameter M, which denotes the number of block rows of the Hankel matrices is typically chosen much smaller than the number of columns j. The observability matrix Γ_j and the block Toeplitz matrix H_j are given by

$$\Gamma_j = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{j-1} \end{bmatrix}, \quad H_j = \begin{bmatrix} D & 0 & \ldots & 0 \\ CB & D & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{j-2}B & CA^{j-3}B & \ldots & D \end{bmatrix}. \tag{8}$$

A relation between X_f, X_p, and U_p can be formulated as

$$X_f = A^j X_p + \Delta_j U_p, \tag{9}$$

where $\Delta_j = [A^{j-1}B \;\; A^{j-2}B \;\; \cdots \;\; B]$. The objective of the subspace predictor is to provide a prediction of future outputs (i.e. Y_f) given the past inputs and outputs (i.e. Y_p and U_p) and a future control input sequence (i.e. U_f). Such a prediction can be obtained by manipulation of the matrix relations (3), (4), and (9). This manipulation results in the following relation for $M, j \to \infty$

$$\begin{aligned} Y_f &= \Gamma_j \begin{bmatrix} A^j \Gamma_j^\dagger & \Delta_j - A^j \Gamma_j^\dagger \end{bmatrix} \begin{bmatrix} Y_p \\ U_p \end{bmatrix} + H_j U_f, \\ &= L_w W_p + L_u U_f, \end{aligned} \tag{10}$$

where $W_p = [Y_p^T \;\; U_p^T]^T$ and L_w and L_u are the predictor matrices. These predictor matrices can be computed approximately by solving the following least-squares problem

$$\min_{L_w, L_u} \left\| Y_f - [L_w \;\; L_u] \begin{bmatrix} W_p \\ U_f \end{bmatrix} \right\|_F^2. \tag{11}$$

This problem can be solved efficiently by computing the RQ-decomposition

$$\begin{bmatrix} W_p \\ U_f \\ Y_f \end{bmatrix} = \begin{bmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \end{bmatrix}, \tag{12}$$

and subsequently computing

$$L = [R_{31} \;\; R_{32}] \begin{bmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{bmatrix}^{-1} = [L_w \;\; L_u], \tag{13}$$

where $L_w \in \mathbb{R}^{Ml \times (m+l)M}$ and $L_u \in \mathbb{R}^{Ml \times Mm}$.

Each sample time new input-output data becomes available. This new data should be used to update the predictor matrices L_w and L_u. Computing a new RQ-decomposition, similar to (12), at each sample time would make the control algorithm computationally demanding and could prohibit its on-line implementation. Therefore, the update of the predictor matrices can be done more efficiently, e.g. by using Cholesky updates [Woodley et al., 2001] or by using Givens rotations [Hallouzi and Verhaegen, 2007].

2.2 Integration of subspace predictor with predictive control

Once the predictor matrices L_w and L_u are known, they are used in the following optimization problem to compute the control input:

$$\begin{aligned}\min_{u_f,\Delta u_f}\ & (\hat{y}_f - r_f)^T Q_a (\hat{y}_f - r_f) + u_f^T R_a u_f + \Delta u_f^T R_a^\Delta \Delta u_f,\\ \text{s.t. }& \Delta U_{\min} \le \Delta u_f \le \Delta U_{\max},\\ & U_{\min} \le u_f \le U_{\max},\\ & Y_{\min} \le \hat{y}_f \le Y_{\max},\end{aligned} \tag{14}$$

where $u_f = [u_{t+1}^T \ u_{t+2}^T \ \cdots \ u_{t+M}^T]^T$, r_f is a similarly defined vector containing the future reference signals, and t denotes the current time step. The matrices L_w and L_u are used to compute $\hat{y}_f$ as $\hat{y}_f = L_w w_p + L_u u_f$, where w_p denotes $[y_p^T \ u_p^T]^T$ with $y_p = [y_{t-M+1}^T \ y_{t-M+2}^T \ \cdots \ y_t^T]^T$ and $u_p = [u_{t-M+1}^T \ u_{t-M+2}^T \ \cdots \ u_t^T]^T$. Q_a, R_a, and R_a^Δ are weighting matrices that can be used for tuning the predictive controller. The vectors $U_{\min}, U_{\max}$, $Y_{\min}$, and $U_{\max}$ contain the limit values of the corresponding variables. In order to convert (14) to a problem with only Δu_f as an optimization variable, the relation

$$u_f = U_t + \begin{bmatrix} I_m & 0 & \dots & 0 \\ I_m & I_m & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_m & I_m & \dots & I_m \end{bmatrix} \Delta u_f = U_t + E \Delta u_f, \tag{15}$$

is used, where $U_t = [u_t^T \ u_t^T \ \cdots \ u_t^T]^T$. This relation leads to the following expression for $\hat{y}_f$:

$$\hat{y}_f = Y_t + L_w^{\text{inc}} \Delta w_p + L_u^{\text{inc}} \Delta u_f, \tag{16}$$

where $Y_t = [y_t^T \ y_t^T \ \cdots \ y_t^T]^T$ and L_w^{inc} can be obtained as follows:

$$L_w = \begin{bmatrix} L_w^{(1)} \\ L_w^{(2)} \\ \vdots \\ L_w^{(M)} \end{bmatrix}, \quad L_w^{\text{inc}} = \begin{bmatrix} L_w^{(1)} \\ L_w^{(1)} + L_w^{(2)} \\ \vdots \\ \sum_{i=1}^{M} L_w^{(i)} \end{bmatrix}, \tag{17}$$

where $L_w^{(i)} \in \mathbb{R}^{l \times (m+l)M}$. The term L_u^{inc} can be computed as follows

$$L_u^{\text{inc}} = L_u \begin{bmatrix} I_m & 0 & \dots & 0 \\ I_m & I_m & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_m & I_m & \dots & I_m \end{bmatrix}. \tag{18}$$

Next, the optimization problem can be formulated as a standard quadratic programming problem

$$\begin{aligned}\min_{\Delta u_f}\ & \frac{1}{2} \Delta u_f^T H \Delta u_f + c^T \Delta u_f,\\ \text{s.t. }& A_{\text{ineq}} \Delta u_f \le b_{\text{ineq}},\end{aligned} \tag{19}$$

with

$$H = 2\left((L_u^{\text{inc}})^T Q_a L_u^{\text{inc}} + E^T R_a E + R_a^\Delta\right), \tag{20}$$

$$\begin{aligned} c^T = 2\Big(& \left(Y_t + L_w^{\text{inc}} \Delta w_p - r_f\right)^T Q_a L_u^{\text{inc}} \\ & + U_t^T R_a E\Big), \end{aligned} \tag{21}$$

$$A_{\text{ineq}} = \left[-I \ \ -E^T \ \ -(L_u^{\text{inc}})^T \ \ I \ \ E^T \ \ (L_u^{\text{inc}})^T \right]^T, \tag{22}$$

$$\begin{aligned} b_{\text{ineq}} = \big[& -(b_1^{\min})^T \ \ -(b_2^{\min})^T \ \ -(b_3^{\min})^T \\ & (b_1^{\max})^T \ \ (b_2^{\max})^T \ \ (b_3^{\max})^T \big]^T, \end{aligned} \tag{23}$$

where $b_1^{\min} = \Delta U_{\min}$, $b_1^{\max} = \Delta U_{\max}$, $b_2^{\min} = U_{\min} - U_t$, $b_2^{\max} = U_{\max} - U_t$, $b_3^{\min} = Y_{\min} - (Y_t + L_w^{\text{inc}} \Delta w_p)$ and $b_3^{\max} = Y_{\max} - (Y_t + L_w^{\text{inc}} \Delta w_p)$. The interested reader is referred to the work by Hallouzi and Verhaegen [2007] for a more elaborate explanation of the derivations made in this section.

3. PERSISTENCY OF EXCITATION

The control law defined in (19) can result in a constant control signal (i.e. $\Delta u_f = 0$), e.g. in a steady-state case. The result of this is that the matrix to be inverted in (13) becomes singular. Therefore, a method is proposed that prevents this. This method is focused on the part of the R matrix computed in (12) that can be directly manipulated. Since U_f is the data matrix that is directly manipulated by the SPC algorithm, its corresponding part in the R-matrix, i.e. $[R_{21} \ \ R_{22}]$, can also be directly manipulated. Note that by directly manipulating $[R_{21} \ \ R_{22}]$, matrix R_{11} is also (indirectly) manipulated. Now that it is determined that $[R_{21} \ \ R_{22}]$ is the part of the R matrix that should be manipulated, it should be determined *how* it should be manipulated. A sensible approach to do this, is to additionally excite the least excited directions of U_f, i.e. those directions in the input space that actually need to be excited. The least excited directions can be determined by performing a singular value decomposition (SVD) on $[R_{21} \ \ R_{22}]$. Computing an SVD of $[R_{21} \ \ R_{22}]$ is equivalent to computing an SVD of U_f, but it can be done more efficiently. The SVD of $[R_{21} \ \ R_{22}]$ has the following form:

$$\begin{aligned} [R_{21} \ \ R_{22}] = & [c_1 \ \ c_2 \ \ \cdots \ \ c_{Mm-1} \ \ c_{Mm}] \cdot \\ & \begin{bmatrix} s_1 & 0 & 0 & \dots & 0 & 0 \dots 0 \\ 0 & s_2 & \ddots & \ddots & \vdots & 0 \dots 0 \\ 0 & \ddots & \ddots & \ddots & 0 & 0 \dots 0 \\ \vdots & \ddots & \ddots & s_{Mm-1} & 0 & 0 \dots 0 \\ 0 & \dots & 0 & 0 & s_{Mm} & 0 \dots 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_{Mm-1} \\ v_{Mm} \\ \vdots \\ v_j \end{bmatrix}^T, \end{aligned} \tag{24}$$

where $c_k \in \mathbb{R}^{Mm}$ and $v_k \in \mathbb{R}^{1 \times j}$. The least excited direction of $[R_{21} \ \ R_{22}]$ (and hence U_f) is c_{Mm} and the second least excited direction is c_{Mm-1}, etc.

3.1 Computing the SVD using inverse iterations

Computing the complete SVD of matrix $[R_{21} \ \ R_{22}]$ becomes a computationally burdensome task if the dimensions of this matrix are large. The need to compute such an SVD at each sample time of the control algorithm would make it unsuitable for on-line implementation. Moreover, the complete SVD is not required since only the least excited directions are desired. Therefore, only these least excited directions are computed efficiently by using the inverse iteration algorithm [Chan, 1984] that only computes that part of an SVD that corresponds to the smallest singular value. The inverse iteration algorithm is described as follows. Let c_{sv} and v_{sv} be vectors from the SVD of a matrix Φ that correspond to the smallest singular value σ_{sv}. In the case of (24), this would mean that $c_{\text{sv}} = c_{Mm}$, $v_{\text{sv}} = v_{Mm}^T$ and $\sigma_{\text{sv}} = s_{Mm}$. The inverse iteration algorithm starts with an initial guess for c_{sv} at $k = 0$, followed by an iteration that should be performed until convergence:

(1) $\Phi\tilde{v}_{\rm sv}(k+1) = c_{\rm sv}(k)$.
(2) $v_{\rm sv}(k+1) = \tilde{v}_{\rm sv}(k+1)/\|\tilde{v}_{\rm sv}(k+1)\|_2$.
(3) $\Phi^T\tilde{c}_{\rm sv}(k+1) = v_{\rm sv}(k+1)$.
(4) $c_{\rm sv}(k+1) = \tilde{c}_{\rm sv}(k+1)/\|\tilde{c}_{\rm sv}(k+1)\|_2$.

After convergence of the iteration, the smallest singular value can be computed as $\sigma_{\rm sv} = 1/\|\tilde{v}_{\rm sv}\|_2$. In order to run the inverse iteration algorithm the pseudo-inverse of matrix Φ, denoted by $\Phi^\dagger$, should be computed since it is required in steps 1 and 3. For example, step 1 of the iteration is solved by $\tilde{v}_{\rm sv}(k+1) = \Phi^\dagger c_{\rm sv}(k)$. It suffices to compute the pseudo-inverse only once, since $(\Phi^T)^\dagger = (\Phi^\dagger)^T$.

The described inverse iteration sequence computes only one direction that is least excited. It can be desirable to compute more than one direction. This can also be done very efficiently using inverse iterations. For this purpose the influence of the least excited direction is removed to compute the second least excited direction. Since not Φ itself, but $\Phi^\dagger$, is used in the inverse iteration algorithm, the influence of the least excited direction is removed directly from $\Phi^\dagger$ in the following way:

$$\tilde{\Phi}^\dagger = \Phi^\dagger - v_{\rm sv}c_{\rm sv}^T/\sigma_{\rm sv}. \tag{25}$$

Next, the second least excited direction can be computed by applying the inverse iteration algorithm using $\tilde{\Phi}^\dagger$. This second sequence of iterations would then result in c_{Mm-1}, v_{Mm-1}, and s_{Mm-1} from (24). The same procedure can be repeated several times to obtain more directions that are non-persistently excited.

3.2 Integration into cost function

Once one or more non-persistently excited directions are obtained, the cost function used in the optimization problem defined in (19) should be modified such that the system is additionally excited in the non-persistently excited directions. For this problem the following optimization is introduced:

$$\min_{u_f} \left\| \rho \begin{bmatrix} c_{Mm} \\ c_{Mm-1} \\ \vdots \\ c_{Mm-N_{\rm PE}+1} \end{bmatrix} - \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix} u_f \right\|_2^2, \tag{26}$$

where ρ denotes the excitation level and $N_{\rm PE}$ denotes the number of least excited excited directions that should be additionally excited. Since by definition of the SVD, the directions c_i are normalized vectors, the norm of vector $[(c_{Mm}^T \quad c_{Mm-1}^T \quad \cdots \quad c_{Mm-N_{\rm PE}+1}^T]^T$, which is $\sqrt{N_{\rm PE}}$, might be too small to persistently excite the system. Therefore, the parameter ρ is introduced. The higher ρ is chosen, the higher the excitation level. The objective of the optimization problem posed in (26) is to get u_f as close to the non-persistently excited directions as possible.

The optimization variable considered in (19) is Δu_f. Therefore the optimization variable in (26) should be converted from u_f to Δu_f. This can be done by substituting (15) into (26), which results in

$$\min_{\Delta u_f} \left\| \overbrace{\rho \begin{bmatrix} c_{Mm} \\ c_{Mm-1} \\ \vdots \\ c_{Mm-N_{\rm PE}+1} \end{bmatrix} - \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix} U_t}^{\mathcal{P}} - \overbrace{\begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix}}^{\mathcal{I}} E\,\Delta u_f \right\|_2^2. \tag{27}$$

Evaluation of the cost function in the above optimization problem results in

$$J_{\rm exc}(\Delta u_f) = (\Delta u_f)^T\mathcal{I}^T\mathcal{I}\Delta u_f - 2\mathcal{P}^T\mathcal{I}\Delta u_f. \tag{28}$$

This function can then be added to the cost function from (19). Before this is done, a multiplication factor τ is introduced that weights $J_{\rm exc}(\Delta u_f)$ before it is added. This factor τ allows $\tau J_{\rm exc}(\Delta u_f)$ to be in the same order of magnitude as the other terms in the cost function. It can be considered to be the same kind of tuning parameter as Q_a, R_a, and R_a^Δ, relative to which it should be chosen. After addition of $\tau J_{\rm exc}(\Delta u_f)$, the modified H and c matrices (defined in (20) and (21)) become

$$\begin{aligned} H_{\rm exc} = 2\Big(&(L_u^{\rm inc})^T Q_a L_u^{\rm inc} + E^T R_a E \\ &+ R_a^\Delta + \tau\mathcal{I}^T\mathcal{I}\Big), \end{aligned} \tag{29}$$

$$\begin{aligned} c_{\rm exc}^T = 2\Big(&\left(Y_t + L_w^{\rm inc}\Delta w_p - r_f\right)^T Q_a L_u^{\rm inc} \\ &+ U_t^T R_a E - \tau\mathcal{P}^T\mathcal{I}\Big), \end{aligned} \tag{30}$$

and $A_{\rm ineq}$ and $b_{\rm ineq}$ remain unaltered. It is apparent that the resulting optimization problem can still be solved by quadratic programming.

4. SIMULATION RESULTS

The proposed method for ensuring PE is evaluated on a detailed nonlinear model of a large transport aircraft. This model is used as a benchmark model in Action Group 16 (AG16) of the European GARTEUR project, which focuses on fault-tolerant control. This model has been originally developed for aircraft simulation and analysis by van der Linden [1998] and has since then been adapted and used by various researchers [Smaili et al., 2006].

In this paper a controller is developed for this model using SPC as described in Section 2. Although the model has a total of 30 different inputs, only 4 of them are used. These 4 inputs allow 3 elementary maneuvers to be performed, which are: descend/ascend, turn, and change speed. Furthermore, 7 output signals of the aircraft are used for the SPC algorithm. An overview of the inputs and outputs used by the SPC algorithm is given in Table 1. It

Table 1. Input and output signals used by the SPC algorithm

Outputs	Inputs
roll angle ϕ [deg]	Ailerons δ_a [deg]
pitch angle θ [deg]	Elevators δ_e [deg]
yaw angle ψ [deg]	Rudders δ_r [deg]
true airspeed $V_{\rm TAS}$ [m/s]	Engine Pressure Ratio (EPR) [-]
angle of attack α [deg]	
sideslip angle β [deg]	
altitude h [m]	

should be noted that the 4 input signals all drive more than one actuator. For example, δ_a drives all 4 aileron surfaces and EPR drives all 4 engines on the transport aircraft.

The simulation scenario consists of an initial phase of straight flight with $h = 980$ m, $V_{\text{TAS}} = 92.6$ m/s, and $\psi = 180$ deg. The parameters h and ψ are controlled by manipulating θ and ϕ, respectively. After the initial phase, a turn is made to $\psi = 300$ deg starting at $T = 70$ s. The last maneuver is a descent to $h = 100$ m, which is initiated at $T = 150$ s. This scenario is simulated for the cases with and without additional excitation. In the case of additional excitation, the three least excited directions are computed and additionally excited. Tuning of the parameters ρ and τ is performed empirically and is very much dependent on the parameters of the controlled system. The parameter ρ is chosen such that the vector containing the least excited directions and the vector containing the input signals (see (26)) have a norm that is in the same order of magnitude. The parameter τ is chosen such that the term responsible for the additional excitation is weighted properly relative to the other terms of the predictive control cost function.

The simulation results for the scenario performed with SPC without additional excitation are depicted in Figures 1 and 2. The simulation results obtained with SPC with additional excitation are depicted in Figures 3 and 4. An important observation that can be made concerns the behavior just after a maneuver change (i.e. just after $T = 70$ s and $T = 150$ s) for the two cases. It can be observed that for the case without additional excitation, the controller requires some time to adapt to the new situation. This is evident from the degraded control performance just after a maneuver due to uncontrolled excitation. In the case of SPC with additional excitation this phenomenon is not recognizable. The reason for this is that the controller can cope with more conditions, including the maneuvers, as a result of the controlled additional excitation. Another important observation that can be made is that in the steady-state cases the control performance of the controller with additional excitation is slightly worse than the controller without additional excitation. This is an inherent result of the additional excitation. Therefore, a trade-off should be made between controller performance and excitation level.

In order to evaluate how the proposed method affects the excitation of the system, the reciprocal condition number of $[R_{21} \;\; R_{22}]$ is analyzed. For the two cases, this condition number is depicted in Figure 5. A low reciprocal condition number indicates a (nearly) singular matrix and therefore a low excitation level. In Figure 5 it can be clearly seen that in the steady-state cases the condition number for the case without additional excitation becomes very small, while the condition number of the case with excitation remains at an acceptable level. Only in the time intervals in which the controller without excitation requires adaptation to a new condition, i.e. after a maneuver change, the condition number of the case without excitation becomes larger. Moreover, it can be seen in Figure 5 that just after the descent is initiated at $T = 150$ s, the condition number decreases for both cases. The reason for this is the EPR input. Since during a descent the aircraft picks up speed the engines should not be producing thrust in order to keep an appropriate airspeed. This means that the EPR

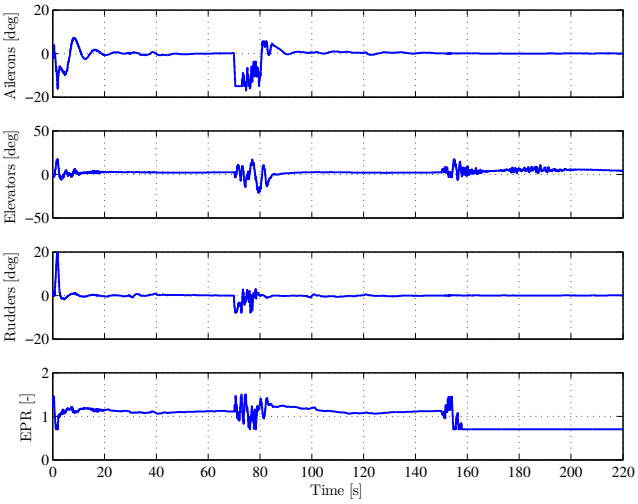

Fig. 1. SPC input signals for the scenario without additional excitation.

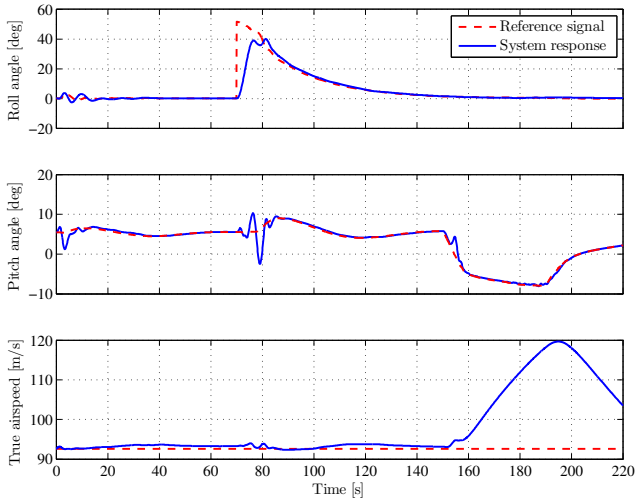

Fig. 2. Response of the controlled system variables for the scenario without additional excitation.

input is held at its minimum limit by the controller. So, in this case control performance has priority over excitation.

5. CONCLUSIONS

A method has been proposed to ensure persistency of excitation in subspace predictive control. This control method combines a predictive control law with a subspace predictor. The predictor that is identified from input-output data of the system requires that this data is persistently exciting. The notion of persistency of excitation is related to non-singularity of a Hankel matrix containing input signals. In order to ensure persistency of excitation an additional term is added to the cost function used by the subspace predictive controller. This additional term ensures that the system is excited in the least excited directions of the input space. A big advantage of the proposed method is that the optimization problem that

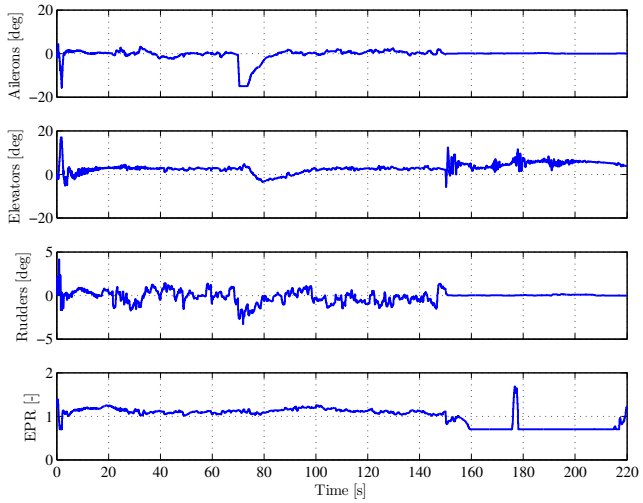

Fig. 3. SPC input signals for the scenario with additional excitation.

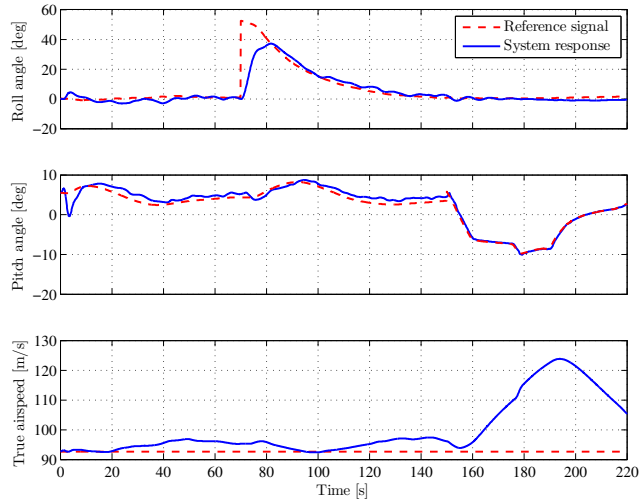

Fig. 4. Response of the controlled system variables for the scenario with additional excitation.

is required to be solved each time step can be solved by using quadratic programming. This is in contrast to methods proposed earlier that require a non-convex problem to be solved. The proposed method has been evaluated using a control problem of a detailed nonlinear model of a transport aircraft. It is shown that this system can be excited in closed loop such that the input data is persistently exciting. Although persistency of excitation comes at the cost of a slightly degraded control performance, it is shown that a satisfactory trade-off can be made between persistency of excitation and control performance.

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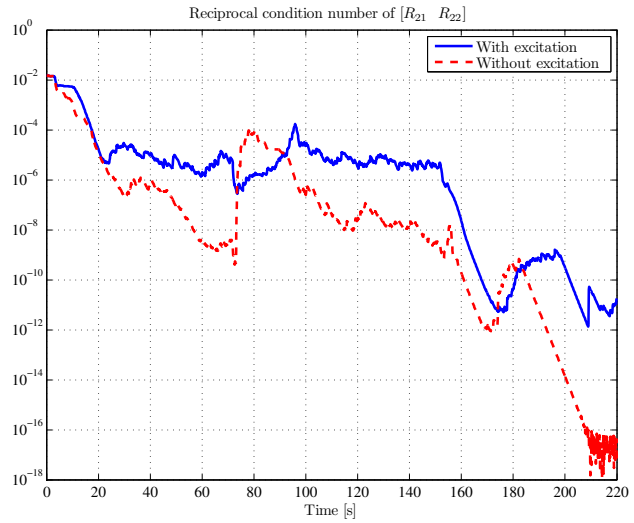

Fig. 5. Comparison of reciprocal condition number of $[R_{21}\ \ R_{22}]$ for the cases with and without additional excitation.

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