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Accommodation Rule of Double Faults for Seven Inertial Sensors

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Abstract: This paper considers a fault accommodation problem for inertial navigation systems (INS) which have 7 inertial sensors such as gyroscopes and accelerometers. When it is decided that double faults occur in the inertial sensors due to the fault detection and isolation (FDI), it is necessary to decide whether the faulty sensors should be excluded or not. An accommodation rule of double faults for 7 sensors is obtained based on the error covariance of triad-solution of redundant inertial sensors, which is related to the navigation accuracy of INS. Monte Carlo simulation is performed for coplanar configuration and the obtained accommodation rule is drawn in the decision space of two-dimensional Cartesian coordinate system.

Keywords: fault detection and isolation, fault accommodation, inertial sensors, parity equation

1. INTRODUCTION

The reliability of the systems can be enhanced by fault detection and isolation (FDI) method and fault accommodation. FDI methods have been studied from 1960's in various areas of engineering problems. As reported in literature such as survey papers [Betta et. al., 2000, Frank, 1990] and books [Chow et. al, 1984, Gertler, 1998], various methods of FDI have been studied and applied to diverse applications.

To obtain reliability and to enhance navigation accuracy, INS may use redundant sensors. A lot of studies on FDI for the redundant sensors have been performed so far. There are many papers for FDI such as look-up table and squared error(SE) method[Gilmore et. al, 1972], generalized likelihood test(GLT) method[Daly et. al., 1979] and optimal parity test(OPT) method[Jin et.al., 1999] for hardware redundancy.

[Yang et.al., 2006, 2007] suggested an accommodation threshold for single fault and accommodation rules for double faults based on the error covariance of an estimated variable, which is related to the navigation accuracy of INS. The accommodation threshold and rules give decision rules to determine whether a faulty sensor should be excluded or not.

[Yang et.al., 2007] suggested accommodation rules for double faults when 6 sensors are used. When 6 sensors are used, double faults can be detected, but the faults cannot be isolated in some cases. This paper suggests accommodation rules for double faults when 7 inertial sensors are used, where any double faults can be detected and isolated

2. FAULT DETECTION, ISOLATION, AND ACCOMMODATION (FDIA)

Consider a typical measurement equation for redundant inertial sensors.

$$m(t) = Hx(t) + f(t) + \varepsilon(t) \tag{1}$$

where

$m(t) = [m_1 \quad m_2 \quad \cdots \quad m_n]^T \in R^n$: inertial sensor measurement.

$H = [h_1 \quad \cdots \quad h_n]^T$: n×3 measurement matrix with rank(H^T)=3.

$x(t) \in R^3$: triad-solution(acceleration or angular rate).

$f(t) = [f_1 \quad f_2 \quad \cdots \quad f_n]^T \in R^n$: fault vector.

$\varepsilon(t) \sim N(0_n, \sigma I_n)$: a measurement noise vector, normal distribution(white noise).

$N(x, y)$: Gaussian probability density function with mean x and standard deviation y.

A parity vector $p(t)$ is obtained using a matrix V as follows:

$$p(t) = Vm(t) = Vf(t) + V\varepsilon(t) \tag{2}$$

where the matrix V satisfies

$$VH = 0 \ (V \in R^{(n-3)\times n}) \quad \text{and} \quad VV^T = I, \ V = [v_1 \quad v_2 \quad \cdots \quad v_n] \tag{3}$$

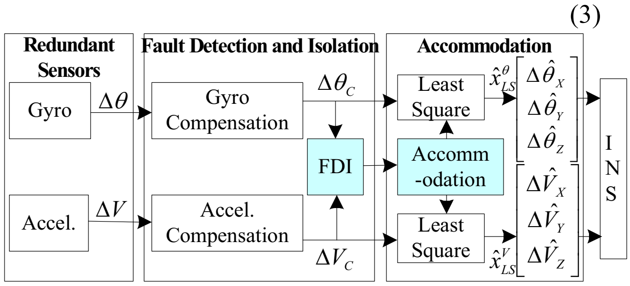

Figure 1: FDI and Accommodation for INS with redundant sensors.

Terminologies

Figure 1 shows the block diagram of FDIA (fault detection, isolation and accommodation) procedure in inertial navigation systems. From the sensor measurement, a parity equation is generated, and FDIA is performed. Triad solutions are calculated by the least square method and entered into the navigation equations. The navigation accuracy depends on the estimation error of the triad solutions, i.e., acceleration or angular rate. Triad solution $\hat{x} = [\ \hat{x}_x\ \ \hat{x}_y\ \ \hat{x}_z\ ]^T$ in Fig. 1, which is acceleration or angular rate, can be obtained by least square method as follows:

$$\hat{x}(t) = (H^T H)^{-1} H^T m(t) \tag{4}$$

In this paper only fault accommodation part is considered.

Two assumptions are made as follows.

[Assumption 1]: Any three input axes of the sensors are not on the same plane.

[Assumption 2]: All sensors have same noise characteristics, i.e., same standard deviation σ of white Gaussian noise.

3. ACCOMMODATION RULE FOR DOUBLE FAULTS

3.1 Navigation performance analysis

For equation (1), suppose that double faults f_i and f_j occur, which means that $f(t) = [0\ \cdots\ f_i\ \ 0\ \cdots\ f_j\ \ 0\ \cdots]^T$.

To analyze the navigation performance, the error covariance of triad solution $\hat{x}(t)$ needs to be calculated. The covariance matrices are defined as follows. Matrix C_{+i+j} denotes the error covariance of $\hat{x}(t)$ including i-th and j-th sensor outputs, and C_{-i-j} the error covariance of $\hat{x}(t)$ excluding i-th and j-th sensors, and so on for C_{-i+j} and C_{+i-j}.

Covariance matrix C_{+i+j}

The error for $\hat{x}(t)$ can be calculated as follows

$$\hat{x}_{+i+j} - x = (H^T H)^{-1}\{ f_i h_i + f_j h_j + H^T \varepsilon \} \tag{5}$$

where $\hat{x}_{+i+j} = [\ \hat{x}_{++x}\ \ \hat{x}_{++y}\ \ \hat{x}_{++z}\ ]^T$.

Then the estimation error of x can be described as the error covariance matrix C_{+i+j} in (6)

$$C_{+i+j} = E\,[\ (\hat{x}_{+i+j} - x)\ (\hat{x}_{+i+j} - x)^T\ ] \tag{6}$$

$$= \sigma^2\ (H^T H)^{-1} + (H^T H)^{-1}[h_i\ \ h_j]\cdot\begin{bmatrix} f_i^2 & f_i f_j \\ f_i f_j & f_j^2 \end{bmatrix}\begin{bmatrix} h_i^T \\ h_j^T \end{bmatrix}(H^T H)^{-1}$$

Covariance matrix C_{-i-j}

The error for $\hat{x}$ can be calculated as follows

$$\hat{x}_{-i-j} - x = (H^T W_{ij} H)^{-1}\ H^T W_{ij}\,\varepsilon \tag{7}$$

where $\hat{x}_{-i-j} = [\ \hat{x}_{--x}\ \ \hat{x}_{--y}\ \ \hat{x}_{--z}\ ]^T$ and W_{ij} is a diagonal matrix with diagonal elements of 1 except (i,i) component and (j,j) component which components are 0.

Then the estimation error of $\hat{x}$ can be described as the error covariance matrix C_{-i-j} in (8).

$$C_{-i-j} = E\,[\ (\hat{x}_{-i-j} - x)\ (\hat{x}_{-i-j} - x)^T\ ] \tag{8}$$

$$= \sigma^2\ (H^T H)^{-1} + \frac{\sigma^2}{D_{ij}}(H^T H)^{-1}[h_i\ \ h_j]\cdot\begin{bmatrix} \| v_j \|_2^2 & -v_j^T v_i \\ -v_i^T v_j & \| v_i \|_2^2 \end{bmatrix}\begin{bmatrix} h_i^T \\ h_j^T \end{bmatrix}(H^T H)^{-1}$$

where

$D_{ij} = \| v_i \|_2^{\,2} \| v_j \|_2^{\,2} - < v_i\ ,\ v_j >^2 = \| v_i \|_2^{\,2} \| v_j \|_2^{\,2} \sin^2\theta_{ij}$ and θ_{ij} is the angle between two vectors v_i and v_j, which are column vectors of matrix V defined in (3).

Covariance matrix C_{-i+j}

The error for $\hat{x}$ can be calculated as follows

$$\hat{x}_{-i+j} - x = (H^T W_i\ H)^{-1}\ H^T W_i\ (V_{Fj} f_j + \varepsilon) \tag{9}$$

where $\hat{x}_{-i+j} = [\ \hat{x}_{-+x}\ \ \hat{x}_{-+y}\ \ \hat{x}_{-+z}\ ]^T$ and $V_{Fj} = [\ 0\ \cdots\ 0\ 1\ 0\ \cdots\ 0\ ]^T \in R^{n\times 1}$ with j-th component of 1, which results in $H^T W_i V_{Fj} = h_j$.

Then the estimation error of $\hat{x}$ can be described as the error covariance matrix C_{-i+j} in (10)

$$\begin{aligned} C_{-i+j} &= f_j^{\,2}(H^T W_i\ H)^{-1} h_j h_j^T (H^T W_i\ H)^{-1} + \sigma^2 (H^T W_i\ H)^{-1} \\ &= f_j^{\,2}(H^T W_i\ H)^{-1} h_j h_j^T (H^T W_i\ H)^{-1} + \sigma^2 (H^T H)^{-1} \\ &\quad + \frac{\sigma^2}{\| v_i \|_2^2}(H^T H)^{-1} h_i h_i^T (H^T H)^{-1} \end{aligned} \tag{10}$$

3.2 Accommodation rule for double faults

In this section three Lemmas are used from the results of [Yang et.al., 2007], which provide accommodation rules for double faults for 7 sensors.

Lemma 1 [Yang et.al., 2007] Consider the measurement equation (1) and the triad solution (4), and suppose that i-th and j-th sensors have faults. For the two estimation error covariance matrices (6) and (8), the following two inequalities are equivalent:

i) $tr(C_{+i+j}) < tr(C_{-i-j})$

where tr(•) denotes the trace of a matrix.

ii) $$f_i^2 \| (H^T H)^{-1} h_i \|_2^2 + f_j^2 \| (H^T H)^{-1} h_j \|_2^2 + 2 f_i f_j < (H^T H)^{-1} h_i , (H^T H)^{-1} h_j > \langle\, \zeta_1 \tag{11}$$

where < , > denotes an inner product and

$$\zeta_1 = \sigma^2 \frac{\| (H^T H)^{-1} h_i \|_2^2 \| v_j \|_2^2 + \| (H^T H)^{-1} h_j \|_2^2 \| v_i \|_2^2 - \gamma}{D_{ij}},$$

$\gamma = 2 < (H^T H)^{-1} h_i , (H^T H)^{-1} h_j > < v_i\ ,\ v_j\ >$. ■

Remark 1 : Lemma 1 means that if faults f_i and f_j occur, and the magnitudes of the two faults satisfy (11) located inside an ellipse, then the corresponding faulty sensors should not be excluded to obtain less estimation error by using them.

Lemma 2[Yang et.al., 2007] Consider the measurement equation (1) and the triad solution (4), and suppose that i-th and j-th sensors have faults. For the two estimation error covariance matrices (8) and (10), the following two inequalities are equivalent:

i) $\mathrm{tr}(C_{-i+j}) < \mathrm{tr}(C_{-i-j})$

ii) $f_j^2 < \zeta_2$ (12)

where $\zeta_2 = \dfrac{\mathrm{tr}(A)}{\mathrm{tr}(B)}$ and

$$A = \sigma^2 (H^T H)^{-1} \left\{ \frac{1}{D_{ij}} [h_i \; h_j] \begin{bmatrix} \| v_j \|_2^2 & -v_j^T v_i \\ -v_i^T v_j & \| v_i \|_2^2 \end{bmatrix} \begin{bmatrix} h_i^T \\ h_j^T \end{bmatrix} - \frac{1}{\| v_i \|_2^2} h_i h_i^T \right\} (H^T H)^{-1}$$

$$B = (H^T W_i H)^{-1} h_j h_j^T (H^T W_i H)^{-1} .$$ ■

Remark 2 : Lemma 2 means that even though faults f_i and f_j are located outside the ellipse in (11) and $|f_j| < |f_i|$, if (12) is satisfied, then the j-th sensor should not be excluded since less estimation error can be obtained by using j-th sensor.

Lemma 3[Yang et.al., 2007] Consider the measurement equation (1) and the triad solution (4), and suppose that i-th and j-th sensors have faults. For the two estimation error covariance matrices (6) and (10), the following two inequalities are equivalent:

i) $\mathrm{tr}(C_{-i+j}) < \mathrm{tr}(C_{+i+j})$

ii) $$f_i^2 + f_j^2 \frac{\{\| (H^T H)^{-1} h_j \|_2^2 - \| (H^T W_i H)^{-1} h_j \|_2^2\}}{\| (H^T H)^{-1} h_i \|_2^2} + \frac{2 f_i f_j < (H^T H)^{-1} h_i, (H^T H)^{-1} h_j >}{\| (H^T H)^{-1} h_i \|_2^2} > \frac{\sigma^2}{\| v_i \|_2^2} \quad (13)$$ ■

Remark 3 : Lemma 3 means that even though faults f_i and f_j satisfy (11), located inside the ellipse, and $|f_j| < |f_i|$, if (13) is satisfied, then i-th sensor should be excluded since less estimation error can be obtained by excluding i-th sensor.

According to the results of Lemma 1 through Lemma 3, double faults can be categorized into four groups.

Category Ⅰ: When double faults satisfy the following three inequalities

i) $f_i^2 \| (H^T H)^{-1} h_i \|_2^2 + f_j^2 \| (H^T H)^{-1} h_j \|_2^2 + 2 f_i f_j < (H^T H)^{-1} h_i, (H^T H)^{-1} h_j > < \zeta_1$

ii) $$f_i^2 + f_j^2 \frac{\{\| (H^T H)^{-1} h_j \|_2^2 - \| (H^T W_i H)^{-1} h_j \|_2^2\}}{\| (H^T H)^{-1} h_i \|_2^2} + \frac{2 f_i f_j < (H^T H)^{-1} h_i, (H^T H)^{-1} h_j >}{\| (H^T H)^{-1} h_i \|_2^2} < \frac{\sigma^2}{\| v_i \|_2^2}$$

iii) $|f_j| < |f_i|$

the two faulty sensors should not be excluded.

Category Ⅱ: When double faults satisfy the following three inequalities

i) $f_i^2 \| (H^T H)^{-1} h_i \|_2^2 + f_j^2 \| (H^T H)^{-1} h_j \|_2^2 + 2 f_i f_j < (H^T H)^{-1} h_i, (H^T H)^{-1} h_j > < \zeta_1$

ii) $$f_i^2 + f_j^2 \frac{\{\| (H^T H)^{-1} h_j \|_2^2 - \| (H^T W_i H)^{-1} h_j \|_2^2\}}{\| (H^T H)^{-1} h_i \|_2^2} + \frac{2 f_i f_j < (H^T H)^{-1} h_i, (H^T H)^{-1} h_j >}{\| (H^T H)^{-1} h_i \|_2^2} \geq \frac{\sigma^2}{\| v_i \|_2^2}$$

iii) $|f_j| < |f_i|$

the i-th sensor should be excluded, but not for the j-th sensor.

Category Ⅲ: When double faults satisfy the following three inequalities

i)

$f_i^2 \| (H^T H)^{-1} h_i \|_2^2 + f_j^2 \| (H^T H)^{-1} h_j \|_2^2 + 2 f_i f_j < (H^T H)^{-1} h_i, (H^T H)^{-1} h_j >$
$\geq \zeta_1$

ii) $f_j^2 < \zeta_2$

iii) $|f_j| < |f_i|$

the i-th sensor should be excluded, but not for the j-th sensor.

Category Ⅳ: When double faults satisfy the following three inequalities

i)

$f_i^2 \| (H^T H)^{-1} h_i \|_2^2 + f_j^2 \| (H^T H)^{-1} h_j \|_2^2 + 2 f_i f_j < (H^T H)^{-1} h_i, (H^T H)^{-1} h_j >$
$\geq \zeta_1$

ii) $f_j^2 \geq \zeta_2$

iii) $|f_j| < |f_i|$

the two faulty sensors should be excluded.

Remark 4 : For the 4 categories above, we consider only the half of the first quadrant in two dimensional space. i.e., $0 \leq \theta \leq \pi/4$.

4 Accommodation rule for double faults with 7 sensors in coplanar configuration

In order to show the decision rule for a real configuration for redundant inertial sensors, we use the coplanar configuration as Figure 2, which uses 7 identical sensors. In this case the measurement matrix

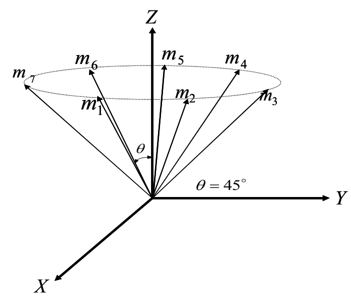

Figure 2: Coplanar configuration with 7 identical sensors.

and parity matrix have the following relations.

$H^T H = \frac{7}{3} I_3$, $\| h_i \|_2 = 1$, $\| v_i \|_2 = 0.7756$ (i = 1 , 2 , ⋯ , 6)

Table 1 through 3 can be plotted in a two-dimensional plane as in Figure 3 through 5.

Table 1 Four categories of double faults of 1st and 2nd sensors with coplanar configuration

($0 \le \theta \le \pi/4$ region only, ○: use, x : exclusion)

Category	Conditions	i-th faulty sensor	j-th faulty sensor
I	$f_1^2 + 1.4980 f_1 f_2 + f_2^2 < 7.2652\sigma^2$, $f_1^2 - 1.1570 f_2^2 + 1.4980 f_1 f_2 < 1.750\sigma^2$, $\lvert f_2 \rvert < \lvert f_1 \rvert$	○	○
II	$f_1^2 + 1.4980 f_1 f_2 + f_2^2 < 7.2652\sigma^2$, $f_1^2 - 1.1570 f_2^2 + 1.4980 f_1 f_2 \ge 1.750\sigma^2$, $\lvert f_2 \rvert < \lvert f_1 \rvert$	x	○
III	$f_1^2 + 1.4980 f_1 f_2 + f_2^2 \ge 7.2652\sigma^2$, $\lvert f_2 \rvert < 1.5990\sigma$, $\lvert f_2 \rvert < \lvert f_1 \rvert$	x	○
IV	$f_1^2 + 1.4980 f_1 f_2 + f_2^2 \ge 7.2652\sigma^2$, $\lvert f_2 \rvert \ge 1.5990\sigma$, $\lvert f_2 \rvert < \lvert f_1 \rvert$	x	x

Table 2 Four categories of double faults of 1st and 3rd sensors with coplanar configuration

($0 \le \theta \le \pi/4$ region only, ○: use, x : exclusion)

Category	Conditions	i-th faulty sensor	j-th faulty sensor
I	$f_1^2 + 0.370 f_1 f_3 + f_3^2 < 3.6603\sigma^2$, $f_1^2 - 0.0706 f_3^2 + 0.370 f_1 f_3 < 1.750\sigma^2$, $\lvert f_3 \rvert < \lvert f_1 \rvert$	○	○
II	$f_1^2 + 0.370 f_1 f_3 + f_3^2 < 3.6603\sigma^2$, $f_1^2 - 0.0706 f_3^2 + 0.370 f_1 f_3 \ge 1.750\sigma^2$, $\lvert f_3 \rvert < \lvert f_1 \rvert$	x	○
III	$f_1^2 + 0.370 f_1 f_3 + f_3^2 \ge 3.6603\sigma^2$, $\lvert f_3 \rvert < 1.3358\sigma$, $\lvert f_3 \rvert < \lvert f_1 \rvert$	x	○
IV	$f_1^2 + 0.370 f_1 f_3 + f_3^2 \ge 3.6603\sigma^2$, $\lvert f_3 \rvert \ge 1.3358\sigma$, $\lvert f_3 \rvert < \lvert f_1 \rvert$	x	x

Table 3 Four categories of double faults of 1st and 4th sensor with coplanar configuration

($0 \le \theta \le \pi/4$ region only, ○: use, x : exclusion)

Category	Conditions	i-th faulty sensor	j-th faulty sensor
I	$f_1^2 - 0.5346 f_1 f_4 + f_4^2 < 3.8420\sigma^2$, $f_1^2 - 0.1474 f_4^2 - 0.5346 f_1 f_4 < 1.750\sigma^2$, $\lvert f_4 \rvert < \lvert f_1 \rvert$	○	○
II	$f_1^2 - 0.5346 f_1 f_4 + f_4^2 < 3.8420\sigma^2$, $f_1^2 - 0.1474 f_4^2 - 0.5346 f_1 f_4 \ge 1.750\sigma^2$, $\lvert f_4 \rvert < \lvert f_1 \rvert$	x	○
III	$f_1^2 - 0.5346 f_1 f_4 + f_4^2 \ge 3.8420\sigma^2$, $\lvert f_4 \rvert < 1.3503\sigma$, $\lvert f_4 \rvert < \lvert f_1 \rvert$	x	○
IV	$f_1^2 - 0.5346 f_1 f_4 + f_4^2 \ge 3.8420\sigma^2$, $\lvert f_4 \rvert \ge 1.3503\sigma$, $\lvert f_4 \rvert < \lvert f_1 \rvert$	x	x

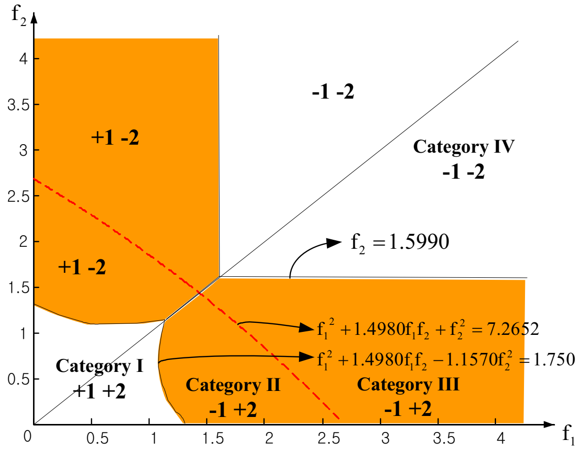

Figure 3: Decision rule for exclusion of 1st and 2nd sensors of faulty sensors for the coplanar configuration

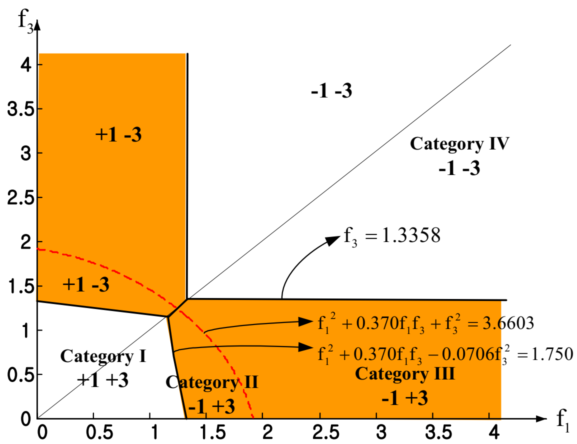

Figure 4: Decision rule for exclusion of 1st and 3rd sensors of faulty sensors for the coplanar configuration

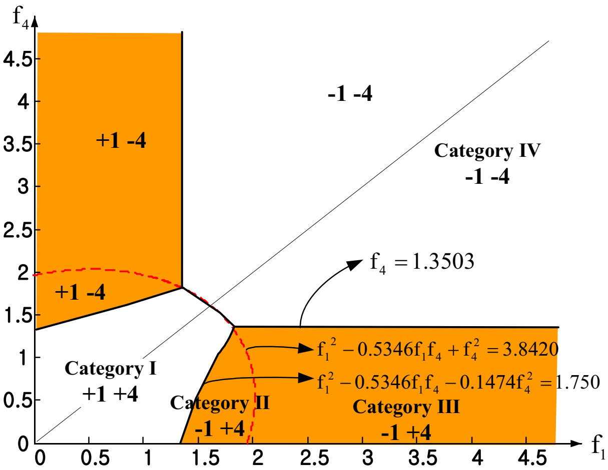

Figure 5: Decision rule for exclusion of 1st and 4th sensors of faulty sensors for the coplanar configuration

5. SIMULATIONS

In this section, Monte Carlo simulations are performed 10,000 times for each fault to confirm the accommodation rules for double fault case correct. Seven identical sensors are used with coplanar configuration as Figure 2.

The measurement matrices H and V satisfying $VH = 0$ and $VV^T = I$ can be obtained as follows:

$$H = \begin{bmatrix} 0.8165 & 0 & 0.5774 \\ 0.5091 & 0.6384 & 0.5774 \\ -0.1817 & 0.7960 & 0.5774 \\ -0.7356 & 0.3543 & 0.5774 \\ -0.7356 & -0.3543 & 0.5774 \\ -0.1817 & -0.7960 & 0.5774 \\ 0.5091 & -0.6384 & 0.5774 \end{bmatrix} \quad (14)$$

where $\|v_1\| = \|v_2\| = \cdots = \|v_6\| = 1/\sqrt{2}$.

5.1 Double faults of 1st and 2nd sensors

We assume that the first and second sensors have fault like $f(t) = [f_1 \;\; f_2 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T$, and the faults f_1 and f_2 are constants and satisfy the straight line as Figure 6, and the measurement noise is white Gaussian with mean 0 and variance $\sigma = 1$.

Figure 7 shows the results of accommodation rule for double faults according to the fault size in Figure 6. When fault f_1 and f_2 belong to the region of Category I, the trace of C_{+1+2} is the minimum among three traces. When fault f_1 and f_2 belong to that of Category II and III, the trace of C_{-1+2} is the minimum, and to the Category IV, the trace of C_{-1-2} is the minimum.

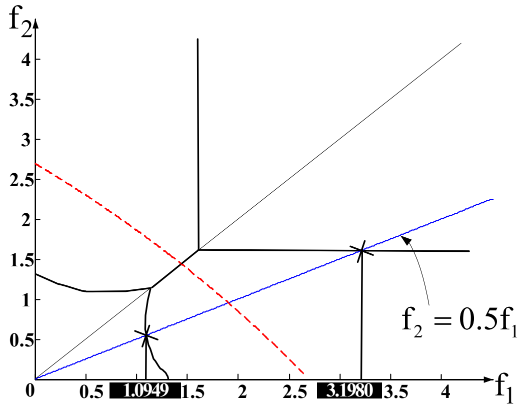

Figure 6: Decision rule for exclusion of faulty 1st and 2nd sensor and the relation of two fault magnitudes for simulation.

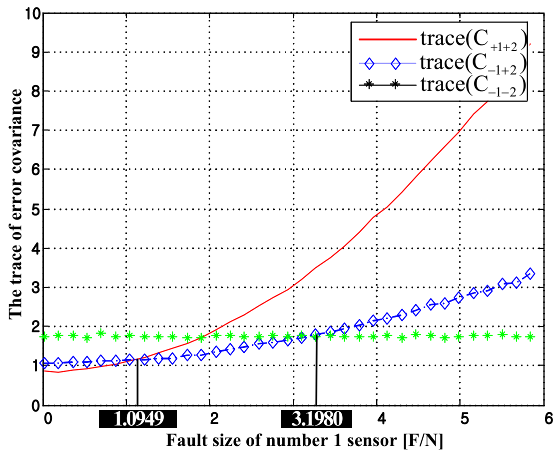

Figure 7: $\mathrm{trace}(C_{+1+2}(t))$, $\mathrm{trace}(C_{-1+2}(t))$ and $\mathrm{trace}(C_{-1-2}(t))$ with respect to fault magnitude

5.2 Double faults of 1st and 3rd sensors

Figure 9 shows the results of accommodation rule for double faults according to the fault size in Figure 8. The results are same as in section 5.1

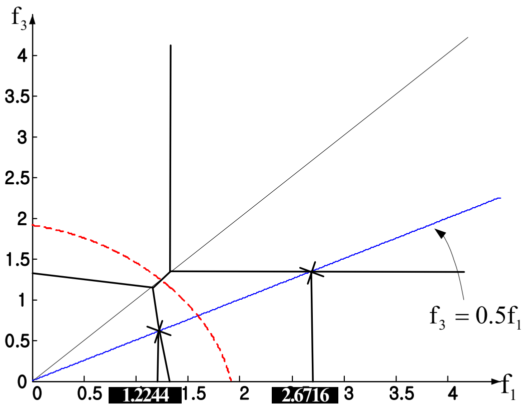

Figure 8: Decision rule for exclusion of faulty 1st and 3rd sensor and the relation of two fault magnitudes for simulation.

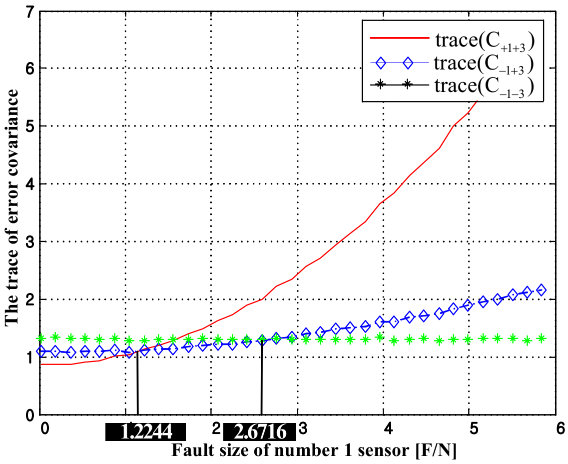

Figure 9: trace($C_{+1+3}(t)$), trace($C_{-1+3}(t)$) and trace($C_{-1-3}(t)$) with respect to fault magnitude

5.3 Double faults of 1st and 4th sensors

Figure 11 shows the results of accommodation rule for double faults according to the fault size in Figure 10. The results are same as in section 5.1

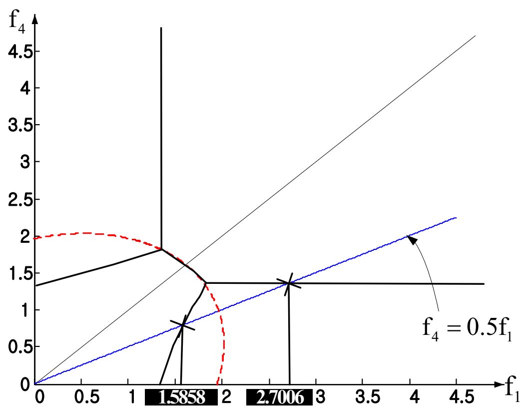

Figure 10: Decision rule for exclusion of faulty 1st and 4th sensor and the relation of two fault magnitudes for simulation.

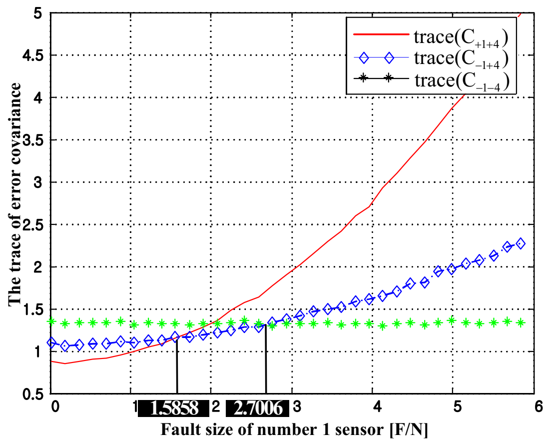

Figure 11: trace($C_{+1+4}(t)$), trace($C_{-1+4}(t)$) and trace($C_{-1-4}(t)$) with respect to fault magnitude

6. CONCLUSIONS

For inertial navigation systems which use seven sensors with coplanar configuration, double faults can be isolated in any cases. This paper suggests accommodation rules for double faults when seven inertial sensors are used. Since identical sensors are used, we consider only 3 cases and suggest accommodation rules for the 3 cases. Figures 3 through 5 show different decision rules and the results are confirmed by the Monte Carlo simulations with Fig, 7, 9, and 11, which shows the results for the fault combinations of Fig, 6, 8, and 10, respectively.

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