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Robust adaptive sliding mode control for a class of uncertain hybrid linear systems with Markovian jump parameters

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Abstract: The robust sliding mode control problem of a class of uncertain hybrid linear systems with Markovian jump parameters is considered. Under the assumption of unknown upper bounds matched system uncertainties, the sufficient conditions are proposed to guarantee exponentially stable in mean square of reduced-order sliding mode system. Under the conditions of unknown upper bounds of system uncertainties, adaptive robust sliding mode control law is proposed. The control methods guarantees that the trajectory of system arrives at the sliding surface in finite time interval and is kept here thereafter. It has been shown that the sliding mode control problem for the Markovian jump systems is solvable if a set of linear matrix inequalities(LMIs) have solution. Latsly a simulation is given to illustrate the effectiveness of the proposed approach.

1. INTRODUCTION

Many dynamical systems have variable structures subject to random abrupt changes, which may result from abrupt phenomena such as random failures and repairs of the components, changes in the interconnections of subsystems, sudden environment changes, modication of the operating point of a linearized model of a nonlinear systems, etc. Systems with this character may be modelled as hybrid ones, that is, to the continuous state variable, a discrete random variable called the mode, or regime, is appended. The mode describes the random jumps of the system parameters and the occurrence of discontinuities. A special class of hybrid systems is Markovian jump systems (MJSs). In MJSs, the random jumps in system parameters are represented by a Markov process taking values in a nite set. This class of systems may represent a large variety of processes including those in the aircraft flight control systems, manufacturing systems, communication systems, and economic systems, etc (Sowrder et al., 1983; Moerder et al., 1989; Boukas et al., 1995). Over the past decade, many important issues have been extensively studied for Markovian jump linear systems (MJLSs). More recently, H1 control for uncertain Markovian jump systems has been considered (Shi et al., 1997); stabilization of Billinear uncertain time-delay stochastic systems can be found (Wang et al., 2002); stability and output feedback stabilization of Markov jump systems can be found (Boukas et al., 2002); and Guaranteed cost control of a Markov jump linear uncertain system has been discussed (Boukas et al., 2003).

In another active research area, sliding mode control (SMC) is an effective method of robustness control (Choi, 1999; Niu et al., 2003; Hu et al., 2000). SMC has a good performance that is the system behavior is insensitive to the internal parameter variations and external disturbances. Thought many work has been conducted on Markonian jumping systems, very few results are available for sliding mode control problem of Markovian jump systems.Recently, the problem of design of sliding mode control for Markonian jumping systems without system uncertainties has appeared (shi et al., 2006). However, the problem of design of sliding mode control for Markonian jumping systems with system uncertainties is still of interest. This motivated us to study the robust sliding mode control problem of a class of uncertain hybrid linear systems with Markovian jump parameters.

In this note, the robust sliding mode control problem of a class of uncertain hybrid linear systems with Markovian jump parameters is considered. Unknown upper bounds matched uncertainties is introduced into Markovian jump systems. Concepts of exponentially stable in mean square for underlying system are proposed. The sliding surface and reaching motion controller for system will be designed. Above problems are solved in terms of linear matrix inequalities(LMIs).

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2. PROBLEM FORMULATION AND PRELIMINARIES

Consider a class of uncertain hybrid linear systems with Markovian jump parameter in the following form:

$$\dot{x}(t) = (A(\eta_t) + \Delta A(\eta_t))x(t) + B(\eta_t)(u(t) + G(\eta_t)w(t)).\quad(1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R$ is the control input vector, $w(t) \in R$ is the disturbance, η_t is a finite-state Markovian process having a state space $S = \{1, 2, ..., N\}$,generator(π_{ij}) with transition probability from mode i at time t to mode j at time $t + \sigma$, $i, j \in S$:

$$p_{ij} = P(\eta_{t+\sigma} = j|\eta_t = i) = \begin{cases} \pi_{ij}\sigma + o(\sigma), & if\ i \neq j, \\ 1 + \pi_{ii}\sigma + o(\sigma), & if\ i = j. \end{cases} \quad (2)$$

where $\pi_{ii} = -\sum_{j=1,j\neq i}^{N} \pi_{ij},\ \pi_{ij} \geq 0\ \forall i,j \in S, i \neq j,\ \sigma > 0$, and $lim_{\sigma\downarrow 0}\ o(\sigma)/\sigma = 0$.

For each possible value $\eta_t \in S$, $A(\eta_t) \in R^{n\times n}$, $B(\eta_t) \in R^{n\times 1}$ are known real constant matrices, $G(\eta_t)$ are known real constant scalar with appropriate dimensions, $\Delta A(\eta_t) \in R^{n\times n}$ is parameter uncertainty matrix.

For the sake of simplicity,we will denote the system matrix associated with mode $\eta_t = i$ by $M(\eta_t) = M(i)$.

For the convenience of proof,the definitions ,lemma and assumption are given as follows:

Definition 1: For stochastic Lyapunov function $V(x(t), \eta_t = k)$, the weak infinitesimal operator Ψ of the Markov process$(x(t), \eta_t), t \geq 0$ is given by

$$\Psi V(x(t), \eta_t) = \lim_{\sigma\to 0} 1/\sigma[E\{V(x(t+\sigma), \eta_{t+\sigma})|x(t), \eta_t\} - V(x(t), \eta_t)]$$

Definition 2: System (1) with $u(t) = 0, w(t) = 0, \Delta A(\eta_t) = 0$ is said to be mean exponentially stable(MES) if there exist constants $\varrho > 0, a > 0, b > 0$ such that $E\{\|x(t)^2\|\} \leq b\|x(0)\|^2 e^{-at}$ when $\|x(0)\| < \varrho$, where $x(0)(t = 0, \eta_t = \eta_0, \eta_0 \in S)$ is the initial condition.

Lemma 1: For any matrices $D \in R^{n\times nf}$, $E \in R^{nf\times n}$,and $F \in R^{nf\times nf}$ with $P > 0$, $\|F\| \leq 1$, and scalar $\varepsilon > 0$,we have:

$$DFE + E^T F^T D^T \leq \varepsilon^{-1} DD^T + \varepsilon E^T E$$

Assumption 1: $\|G(i)w(t)\| \leq g(i), i \in S$, where $g(i), i \in S$ are positive scalars.

The objective of this paper is

1)The sliding motion is mean exponentially stable;

2)System(1) is mean exponentially stable with the reaching control law $u(t)$.

3. MATCHED UNCERTAINTIES

Assumption 3: System uncertainties satisfies the following matched condition:

$\Delta A(\eta_t) = B(\eta_t)\Delta\bar{A}(\eta_t)$. where $\|\bar{A}(\eta_t)\| \leq M_{\Delta A}$,and $M_{\Delta A}$ are unknown upper bounds positive scalars.

From assumption 3, system(1) can be rewritten

$$\dot{x}(t) = A(\eta_t)x(t) + B(\eta_t)(u(t) + G(\eta_t)w(t) + \Delta\bar{A}(\eta_t)x(t)).\quad(3)$$

In order to obtain a regular form of system(3),we can choose a nonsingular matrix $T(\eta_t)$ such that

$T(\eta_t)B(\eta_t) = \begin{bmatrix} 0_{(n-1)\times 1} \\ B_2(\eta_t)_{1\times 1} \end{bmatrix}$ where $B_2(\eta_t) \neq 0$.For convenience[13],let us partition

$$T(\eta_t) = \begin{bmatrix} U_2{}^T(\eta_t) \\ U_1{}^T(\eta_t) \end{bmatrix}.$$

where $U_1(\eta_t) \in R^{n\times 1}$ and $U_2(\eta_t) \in R^{n\times(n-1)}$ are two subblocks of a unitary matrix resulting from the singular value decomposition of $B(\eta_t)$, that is

$$B(\eta_t) = [\, U_1(\eta_t)\ U_2(\eta_t)\,] \begin{bmatrix} \Sigma(\eta_t) \\ 0_{(n-1)\times 1} \end{bmatrix} \Gamma^T(\eta_t).$$

where $\Sigma(\eta_t) \in R$, and $\Sigma(\eta_t) > 0$, $\Gamma(\eta_t) = 1$.By the state transformation $z = T(\eta_t)x$, system(3) has the regular form

$$\dot{z}(t) = \bar{A}(\eta_t)z(t) + \begin{bmatrix} 0 \\ B_2(\eta_t) \end{bmatrix} \begin{pmatrix} u(t) + G(\eta_t)w(t) \\ +\Delta\bar{A}(\eta_t)T^{-1}(\eta_t)z(t) \end{pmatrix} \quad (4)$$

where $\bar{A}(\eta_t) = T(\eta_t)A(\eta_t)T^{-1}(\eta_t)$.

System(4) can be rewritten as:

$$\dot{z}_1(t) = \bar{A}_{11}(\eta_t)z_1(t) + \bar{A}_{12}(\eta_t)z_2(t), \quad (5)$$

$$\dot{z}_2(t) = \bar{A}_{21}(\eta_t)z_1(t) + \bar{A}_{22}(\eta_t)z_2(t) + B_2(\eta_t) \begin{pmatrix} u(t) + G(\eta_t)w(t) \\ +\Delta\bar{A}(\eta_t)T^{-1}(\eta_t)z(t) \end{pmatrix} \quad (6)$$

where $z_1 \in R^{(n-1)\times 1}$,$z_2 \in R$.

The sliding surface can be chosen as follows:

$$s(t) = C(\eta_t)z(t) = [\, C_1(\eta_t)\ C_2(\eta_t)\,]\, z(t) \quad (7)$$

where $C_1(\eta_t) \in R^{1\times(n-1)}$,$C_2(\eta_t) \in R$,$C_2(\eta_t) \neq 0$ for any $\eta_t \in S$. so on the sliding surface we have

$$s(t) = C_1(\eta_t)z_1(t) + C_2(\eta_t)z_2(t) = 0 \text{ and}$$

$$z_2(t) = -C_2(\eta_t)^{-1}C_1(\eta_t)z_1(t).$$

Let $K(\eta_t) = C_2(\eta_t)^{-1}C_1(\eta_t)$, and substitute $z_2(t) = -K(\eta_t)z_1(t)$ to (5) gives the sliding motion

$$\dot{z}_1(t) = [\bar{A}_{11}(\eta_t) - \bar{A}_{12}(\eta_t)K(\eta_t)]z_1(t). \quad (8)$$

Let $\tilde{A}_1(\eta_t) = [\bar{A}_{11}(\eta_t) - \bar{A}_{12}(\eta_t)K(\eta_t)]$,

then the sliding motion(8) can be rewritten

$$\dot{z}_1(t) = \tilde{A}_1(\eta_t)z_1(t). \quad (9)$$

Theorem 1:The reduced order system(9)is mean exponentially stable if there exist symmetric positive-definite matrix $P(i) \in R^{(n-1)\times(n-1)}$, and general matrix $K(i) \in R^{1\times(n-1)}$, $i \in S$,such that the following inequalities hold for all $i \in S$

$$\begin{bmatrix} \Pi_{11}(i) & \Pi_{12}(i) \\ \Pi_{12}^T(i) & \Pi_{22}(i) \end{bmatrix} < 0 \quad (10)$$

where

$X(i) = P(i)^{-1}$, $W(i) = K(i)P(i)^{-1}$,

$$\Pi_{11}(i) = A_{11}(i)X(i) - A_{12}(i)W(i) + X(i)A_{11}^T(i) - W^T(i)A_{12}^T(i) + \pi_{ii}X(i),$$

$$\Pi_{12}(i) = [\, \pi_{i1}^{1/2}X(i) \ \pi_{i2}^{1/2}X(i) \ \dots \ \pi_{iN}^{1/2}X(i) \,]$$

$$\Pi_{22}(i) = -diag\{\, X(1) \ X(2) \ \dots \ X(N) \,\}.$$

Proof:Take the stochastic Lyapunov function as

$$V(z_1(t), \eta_t, t) = z_1^T(t)P(\eta_t)z_1(t). \quad (11)$$

Let the mode at time t be i, then

$$\begin{aligned}\Psi V(z_1(t), i, t) &= z_1^T(t)P(i)\dot{z}_1(t) + \dot{z}_1^T(t)P(i)z_1(t) + \sum_{j=1}^N \pi_{ij}V(j) \\ &= z_1^T(t)P(i)\tilde{A}_1(i)z_1(t) + z_1^T(t)\tilde{A}_1^T(i)P(i)z_1(t) + z_1^T(t)\sum_{j=1}^N \pi_{ij}P(j)z_1(t) \\ &= z_1^T(t)[P(i)\tilde{A}_1(i) + \tilde{A}_1^T(i)P(i) + \sum_{j=1}^N \pi_{ij}P(j)]z_1(t).\end{aligned}$$

Let $\Theta(i) = P(i)\tilde{A}_1(i) + \tilde{A}_1^T(i)P(i) + \sum_{j=1}^N \pi_{ij}P(j)$.

Then $\Psi V < 0$ when $\Theta(i) < 0$.

so we have

$\frac{\Psi V}{V} = \frac{z_1(t)^T\Theta(i)z_1(t)}{z_1^T(t)P(i)z_1(t)} \le -\gamma$,

where $\gamma = min_{i\in S}\left[\frac{\lambda_{min}(-\Theta(i))}{\lambda_{max}P(i)}\right] > 0$.

which implies

$$E\left[\int_0^t \Psi V(z_1(\xi), \eta_\xi, \xi)d\xi\right] \le -\gamma E\left[\int_0^t V(z_1(\xi), \eta_\xi, \xi)d\xi\right].$$

Using Dynkin's formula and Gronwall-Bellman lemma,

$$E\left[V(z_1(t), \eta_t, t)\right] \le e^{-\gamma t}E\left[z_1^T(0)P(0)z_1(0)\right]$$

Moreover,it is easy check that

$$\begin{aligned}E[\|z_1(t)\|^2] &\le \frac{1}{min_{i\in S}\{\lambda_{min}P(i)\}}E[z_1^T(i)P(i)z_1(i)] \\ &\le \frac{1}{min_{i\in S}\{\lambda_{min}P(i)\}}E[V(z_1(t), i, t)] \\ &\le \frac{1}{min_{i\in S}\{\lambda_{min}P(i)\}}e^{-\gamma t}E[z_1^T(0)P(0)z_1(0)] \\ &\le \frac{max_{i\in S}\{\lambda_{max}P(0)\}}{min_{i\in S}\{\lambda_{min}P(i)\}}e^{-\gamma t}\|z_1(0)\|^2.\end{aligned}$$

From the definition 2,the reduced order system(9)is mean exponentially stable if there exist symmetric positive-definite matrix $P(i) \in R^{(n-1)\times(n-1)}$, $i \in S$,such that $\Theta(i) < 0$ for all $i \in S$.

$\Theta(i) < 0$ is equivalent to

$$P(i)\tilde{A}_1(i) + \tilde{A}_1^T(i)P(i) + \sum_{j=1}^N \pi_{ij}P(j) < 0.$$

Pre- and post-multiplying above inequalities by $P(i)^{-1}$ on both sides,we have

$$\begin{pmatrix} \tilde{A}_1(i)P(i)^{-1} + P(i)^{-1}\tilde{A}_1^T(i) + \pi_{ii}P(i)^{-1} \\ +P(i)^{-1}\sum_{j=1, j\ne i}^N \pi_{ij}P(j)P(i)^{-1} \end{pmatrix} < 0.$$

Applying Schur complement formula and let

$X(i) = P(i)^{-1}$, $W(i) = K(i)P(i)^{-1}$,

$$\Pi_{11}(i) = A_{11}(i)X(i) - A_{12}(i)W(i) + X(i)A_{11}^T(i) - W^T(i)A_{12}^T(i) + \pi_{ii}X(i),$$

$$\Pi_{12}(i) = [\, \pi_{i1}^{1/2}X(i) \ \pi_{i2}^{1/2}X(i) \ \dots \ \pi_{iN}^{1/2}X(i) \,]$$

$$\Pi_{22}(i) = -diag\{\, X(1) \ X(2) \ \dots \ X(N) \,\}.$$

we can have inequalities (10).

Theory 2:Assume the condition in theory 1 holds,i.e.,inequalities (10) have solutions $X(i), W(i)$, and the linear sliding surface is given by (7), and there exist selected $\Xi(i)$,$i \in S$ such that the inequalities $-\Omega(i)\Xi(i) - \Xi(i)^T\Omega(i) + \sum_{j=1}^N \pi_{ij}\Omega(i) < 0$ have definite positive matrix solutions$\Omega(i)$.Then the following adaptive control makes the sliding surface mean exponentially stable and globally attractive in finite time:

$$\begin{aligned}u(t) = -\left(C_2(i)B_2(i)\right)^{-1}\{&[C_1(i) \ C_2(i)]\bar{A}z(t) \\ &+\Xi(i)s(t) \\ &+\|C_2(i)B_2(i)\|[\|\Omega(i)s(t)\|\epsilon(i) \\ &+g(i) + \hat{M}_{\Delta A}\|T^{-1}(i)\|\|z(t)\|] \\ &\times sign(\Omega(i)s(t))\}\end{aligned} \quad (12)$$

where $C_1(i)$, $C_2(i)$ are designed in theorem 1, $\epsilon(i)$ are given positive constants.

The adaptive gain is designed as

$$\dot{\hat{M}}_{\Delta A} = \|\Omega(i)s(t)\|\|C_2(i)B_2(i)\|\|T^{-1}(i)\|\|z(t)\| \quad (13)$$

Proof: Let $y(t) = \left[\, s(t)^T \ \tilde{M}_{\Delta A} \,\right]^T$, where $\tilde{M}_{\Delta A} = \hat{M}_{\Delta A} - M_{\Delta A}$ is adaptive gain error.

Take the stochastic Lyapunov function as

$$\begin{aligned}V(s(t), \eta_t, t) &= y(t)^T \begin{bmatrix} \Omega(i) & 0 \\ 0 & 1 \end{bmatrix} y(t) \\ &= s^T(t)\Omega(\eta_t)s(t) + \tilde{M}_{\Delta A}^2.\end{aligned} \quad (14)$$

Let the mode at time t be i, then

$$\Psi V(s(t), i, t) = s^T(t)\Omega(i)\dot{s}(t) + \dot{s}^T(t)\Omega(i)s(t) + \sum_{j=1}^N \pi_{ij}V(j) + 2\tilde{M}_{\Delta A}\dot{\tilde{M}}_{\Delta A}. \quad (15)$$

Substitute (4),(7),(12)to(15),we have

$$\begin{aligned}
\Psi V(s(t),i,t) = & s^T(t)\{-\Omega(i)\Xi(i)-\Xi(i)^T\Omega(i) \\
& +\sum_{j=1}^{N}\pi_{ij}\Omega(i)\} \\
& +s^T(t)\Omega(i)\{C_2(i)B_2(i)[G(i)w(t) \\
& +\Delta\bar{A}(i)T^{-1}(i)z(t)] \\
& -\|C_2(i)B_2(i)\|[\|\Omega(i)s(t)\|\epsilon(i) \\
& +g(i)+\hat{M}_{\Delta A}\|T^{-1}(i)\|\|z(t)\|] \\
& \times sign(\Omega(i)s(t))\} \\
& +\{C_2(i)B_2(i)[G(\eta_t)w(t) \\
& +\Delta\bar{A}(\eta_t)T^{-1}(i)z(t)] \\
& -\|C_2(i)B_2(i)\|[\|\Omega(i)s(t)\|\epsilon(i) \\
& +g(i)+\hat{M}_{\Delta A}\|T^{-1}(i)\|\|z(t)\|] \\
& \times sign(\Omega(i)s(t))\}^T\Omega(i)s(t) \\
& +2\tilde{M}_{\Delta A}\dot{\hat{M}}_{\Delta A}.
\end{aligned}$$

when $-\Omega(i)\Xi(i)-\Xi(i)^T\Omega(i)+\sum_{j=1}^{N}\pi_{ij}\Omega(i)<0$,we have

$$\begin{aligned}
\Psi V(s(t),i,t) \le & s^T(t)\Omega(i)\{\|C_2(i)B_2(i)\|[\|G(i)w(t)\| \\
& -g(i)sign(\Omega(i)s(t))] \\
& +\|C_2(i)B_2(i)\|[M_{\Delta A}\|T^{-1}(i)\|\|z(t)\| \\
& -\hat{M}_{\Delta A}\|T^{-1}(i)\|\|z(t)\|sign(\Omega(i)s(t))] \\
& -\|C_2(i)B_2(i)\|[\|\Omega(i)s(t)\|\epsilon(i)sign(\Omega(i)s(t))\} \\
& +\{\|C_2(i)B_2(i)\|[\|G(i)w(t)\| \\
& -g(i)sign(\Omega(i)s(t))] \\
& +\|C_2(i)B_2(i)\|[M_{\Delta A}\|T^{-1}(i)\|\|z(t)\| \\
& -\hat{M}_{\Delta A}\|T^{-1}(i)\|\|z(t)\|sign(\Omega(i)s(t))] \\
& -\|C_2(i)B_2(i)\|[\|\Omega(i)s(t)\|\epsilon(i)sign(\Omega(i)s(t))\}^T \\
& \times\Omega(i)s(t)+2(\hat{M}_{\Delta A}-M_{\Delta A})\dot{\hat{M}}_{\Delta A}.
\end{aligned}$$

Substitute(13)to above inequalities,we have

$\Psi V(s(t),i,t) \le -2\|C_2(i)B_2(i)\|\|\Omega(i)s(t)\|^2\epsilon(i)$.

Because

$$\begin{aligned}
\|\Omega(i)s(t)\|^2 &= [\Omega(i)s(t)]^T[\Omega(i)s(t)] = s(t)^T\Omega(i)^2s(t) \\
&\ge \lambda_{min}(\Omega(i)^2)s(t)^Ts(t),
\end{aligned}$$

$$\begin{aligned}
\Psi V(s(t),i,t) &\le -2\epsilon(i)\|C_2(i)B_2(i)\|\lambda_{min}(\Omega(i)^2)s(t)^Ts(t). \\
&\le -y(t)^T\begin{bmatrix}\beta & 0\\ 0 & 0\end{bmatrix}y(t).
\end{aligned}$$

where $\beta = 2\epsilon(i)\|C_2(i)B_2(i)\|\lambda_{min}(\Omega(i)^2) > 0$.

$$\frac{\Psi V(s(t),i,t)}{V(s(t),i,t)} \le -\frac{y^T(t)\begin{bmatrix}\beta & 0\\ 0 & 0\end{bmatrix}y(t)}{y^T(t)\begin{bmatrix}\Omega(i) & 0\\ 0 & 1\end{bmatrix}y(t)} \le -\gamma.$$

where $\gamma = \frac{\beta}{max_{i\in S}\{\lambda_{max}(\Omega(i),\ 1\}} > 0$.

From the proof of theorem 1,we can know the sliding surface is mean exponentially stable and globally attractive in finite time.

4. EXAMPLE

In this section ,for purpose of illustrating the usefulness of the theory developed in this note,we present a simulation example.

Let us consider the following system with generator for Markov process governing the mode switching being:

$$\Im = \begin{bmatrix}\pi_{11} & \pi_{12}\\ \pi_{21} & \pi_{22}\end{bmatrix} = \begin{bmatrix}-4 & 4\\ 3 & -3\end{bmatrix}.$$

For the two operating modes,the associated data are

mode 1

$A(1) = \begin{bmatrix}-3 & -2\\ 1 & 0\end{bmatrix}$,

$\Delta A(1) = \begin{bmatrix}0 & 0\\ 0.2sin(t) & 0.2sin(t)\end{bmatrix}$

$B(1) = \begin{bmatrix}0\\ 1\end{bmatrix}$, $G(1) = 1$, $w(t) = sin(t)$.

mode 2

$A(2) = \begin{bmatrix}-1 & 0\\ 2 & -2\end{bmatrix}$,

$\Delta A(2) = \begin{bmatrix}0.2sin(t) & 0.2sin(t)\\ 0 & 0\end{bmatrix}$

$B(2) = \begin{bmatrix}1\\ 0\end{bmatrix}$, $G(2) = 1$, $w(t) = sin(t)$.

Taking $T(1) = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $T(2) = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, using Theorem 1 and LMI method,we have

$P(1) = 1.6077$, $K(1) = -0.2635$,

$P(2) = 1.6081$, $K(2) = -0.2636$.

Then

$C_1(1) = -0.1186$, $C_2(1) = 0.45$,

$C_1(2) = -0.0923$, $C_2(2) = 0.35$.

Taking $\Xi(1) = 0.82$, $\Xi(2) = 0.56$, by theorem 2 and LMI method,we have $\Omega(1) = 2.2287$, $\Omega(2) = 2.4022$.

Taking $\varepsilon(1) = \varepsilon(2) = 1$, $g(1) = g(2) = 1$, the initial value of $\hat{M}_{\Delta A}$ and $\hat{M}_{\Delta Ad}$ is 0.5, $x(0) = [\,1\ \ 1\,]^T$, $\eta_0 = 1$, we have the following simulation results.

From Figs.1 and 2,we can see that trajectories of the system state and sliding motion converge to the origin quickly and will be in the stead state.

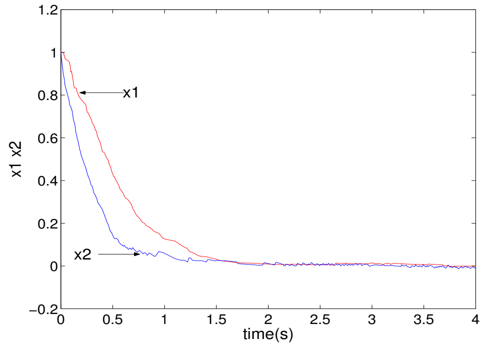

Fig. 1. Responses of system state variables (x1,x2)

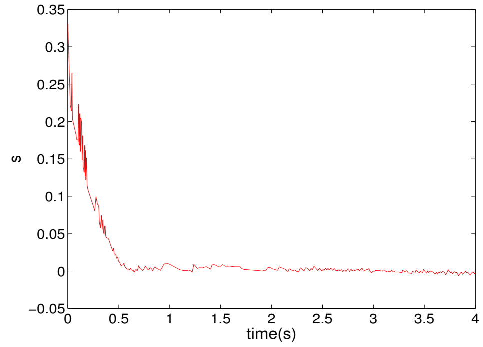

Fig. 2. Responses of system sliding motion s

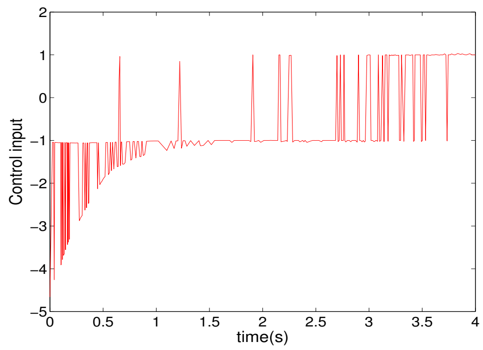

Fig. 3. Responses of system control input u

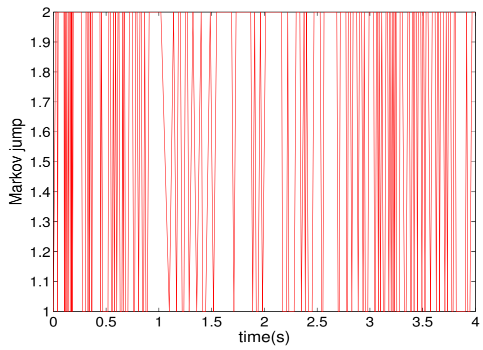

Fig. 4. Markovian jumping parametrs

5. CONCLUSION

In this note,the robust sliding mode control problem of a class of uncertain hybrid linear systems with Markovian jump parameter is considered.By nonsingular state transformation,system has been transformed the regular form.In term of LMIs,the sufficient conditions are proposed to guarantee the mean exponentially stability of reduced-order sliding mode system for unknown upper bounds matched uncertain system uncertainties and known upper bounds mismatched system uncertainties.Then, corresponding reaching motion controllers are designed such that the resulting closed-loop system can be driven onto sliding surface in a finite time.The simulation resulting has shown the sliding mode control in this note is robust stable to the internal parameter uncertainties and external disturbances.

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