

Stable Schooling for Multiple Underactuated AUVs

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Abstract: This paper presents a stable schooling scheme for multiple autonomous underwater vehicles (AUVs) where the number of independent actuators for each vehicle are less than the degrees of freedom (DOF). In most of the formation schemes presented so far, the multiple dynamic agents usually have been modeled as particle systems whose motions can be expressed as simple double-integrator. Therefore, these formation algorithms could not directly apply to the most of actual systems, typically to the case of underwater vehicle systems whose dynamics are highly nonlinear. Moreover, the AUVs considered in this paper are underactuated that each vehicle's 6 DOF motion is steered only by one thruster, one rudder, and one stern plane. For this kind of multiple underwater vehicles, proposed schooling scheme can guarantee the stability of the formation and further guarantee the velocity and heading matching of the group while keeping obstacle avoidances. Numerical simulations are carried out to illustrate the effectiveness of the proposed schooling scheme.

1. INTRODUCTION

Recently, the multiple AUVs are being widely applied in the applications of various underwater inspections (Guo *et al.*, 2004; Edwards *et al.*, 2004; Fiorelli *et al.*, 2006; Sousa and Pereira, 2002). Compare with traditional inspections using single ROV (Remotely Operated Vehicle), AUV, or some other underwater equipment, using multiple AUVs has great benefits in efficiency improvement and cost saving. Since the dynamics of the underwater vehicles are highly nonlinear and further according to the vehicles mechanical restricting characteristics such as underactuation in flying AUVs, it is usually difficult to derive satisfactory tracking control scheme for each vehicle. For this reason, formation control for these multiple underwater vehicles now emerges as a tough challenge in the control community.

In this paper, we consider the schooling of multiple torpedo-type AUVs where only three control inputs (surge force, yaw and pitch moments) are available for each vehicle's 6DOF motion. In fact, most of the present commercial AUVs, such as the REMUS AUVs (Pestero, 2001) and the HUGIN AUVs (Marthiniussen *et al.*, 2004), all have this kind of mechanical structure. By now, various formation algorithms have been presented for multiple agent systems (Reynolds, 1987; Leonard and Fiorelli, 2001; Fiorelli *et al.*, 2006; Saber and Murray, 2002, 2003; Saber, 2006; Do, 2007). However, in these works, all of the dynamical agents are modeled as certain particle system expressed as simple double-integrator. Besides these works, some nonlinear models are also used for underwater vehicles (Dubar and Murray, 2002) and wheeled robots (Fax and Murray, 2004). In both of Dubar and Murray (2002) and Fax and Murray (2004), the nonlinear dynamics are fully actuated. Therefore, all of these results cannot be directly applicable to the schooling of multiple underactuated vehicles.

On the other hand, for each of these underactuated vehicles, since there is a non-integrable constraint in the acceleration dynamics (Reyhanoglu, 1997), the underwater vehicles do not satisfy the Brockett's necessary condition (Brockett *et al.*, 1983) and therefore cannot be asymptotically stabilizable to a desired equilibrium point using traditional time-invariant continuous feedback laws (Reyhanoglu, 1997; Bacciotti and Rosier, 2005). Moreover, the vehicle's dynamics is not transformable into a driftless chained form (Murray and Sastry, 1993), therefore the tracking algorithm presented in (Jiang and Nijmeijer, 1999) cannot be directly applicable to the tracking of these underactuated vehicles. To overcome this kind of difficulty, in this paper we apply a certain polar coordinates transformation to convert the vehicle's model to a form which can be easily handled.

The common method of formation algorithms currently is to apply certain potential function to conduct the group behavior (Leonard and Fiorelli, 2001; Fiorelli *et al.*, 2006; Saber and Murray, 2002, 2003; Saber, 2006; Do, 2007). The potential function method has been initially used in the robotics for mobile robot motion planning (Latombe, 1991; Rimon and Koditschek, 1992), and, recently, has been applied in the formation of multiple agent systems. In this paper, similar to Saber (2006), the potential function consists of three different components: one is for the interactions between vehicles, the second is for the group navigation, and the last one is for the obstacle avoidance. Proposed formation scheme guarantees asymptotic stability of the vehicles schooling and the velocity and heading matching of the group while keeping obstacles avoidance.

2. AUV'S DYNAMICS MODEL

In this paper, we consider a group of n torpedo type AUVs where only the surge force and the yaw moment are available

for the vehicle's 3D¹ horizontal motion (Prestero, 2001; Lee *et al.*, 2003). For this kind of underactuated underwater vehicles, their horizontal model can be expressed as following (Fossen, 2002)

$$\begin{aligned} \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\psi}_i \end{bmatrix} &= \begin{bmatrix} \cos \psi_i & -\sin \psi_i & 0 \\ \sin \psi_i & \cos \psi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ r_i \end{bmatrix}, \\ \begin{bmatrix} \dot{u}_i \\ \dot{v}_i \\ \dot{r}_i \end{bmatrix} &= \begin{bmatrix} f_{ui}(u_i, v_i, r_i) \\ f_{vi}(u_i, v_i, r_i) \\ f_{ri}(u_i, v_i, r_i) \end{bmatrix} + \begin{bmatrix} b_{ui} & 0 \\ 0 & 0 \\ 0 & b_{ri} \end{bmatrix} \begin{bmatrix} \tau_{ui} \\ \tau_{ri} \end{bmatrix}, \end{aligned} \quad (1)$$

where $i = 1, \dots, n$, (x_i, y_i) is the coordinate of i th vehicle and ψ_i is yaw angle both in the earth-fixed frame, and u_i, v_i and r_i denote the velocities in the surge, sway and yaw directions in the vehicle's body-fixed frame. $f_{ui}(\cdot)$, $f_{vi}(\cdot)$ and $f_{ri}(\cdot)$ denote the vehicle's nonlinear dynamics including hydrodynamic damping, inertia and added mass terms each in the surge, sway and yaw directions. The surge force τ_{ui} and the yaw moment τ_{ri} are two available control inputs with constant control gains b_{ui} and b_{ri} .

Remark 1. In general, the underwater vehicles are designed to have non-symmetry in y-z plane. This means that the vehicle's backward dynamics is not exactly equal to its forward dynamics. Moreover, because of high nonlinearity, the exact mathematical model of the vehicle dynamics is hard to be obtained and usually a simplified model is applicable in most of practical applications. And this kind of simplification highly depends on the vehicle's forward speed. This indicates that, for given nonlinear dynamics $f_{ui}(\cdot)$, $f_{vi}(\cdot)$ and $f_{ri}(\cdot)$, they are validated under certain restricting conditions such as $u_i \geq u_{\min} > 0$ (Fossen, 2002). For this reason, in the rest of this paper, we only consider the case where $u_i > 0$ for all time.

As aforementioned, for the underactuated vehicles, since the sway force is unavailable, most difficulty in the tracking is how to properly handle the vehicle's sway dynamics. To deal with this problem, this paper introduces a polar coordinates transformation which is defined in the vehicle's body-fixed frame as following

$$u_{li} = \sqrt{u_i^2 + v_i^2}, \quad \psi_{li} = \psi_i + \psi_{ai}, \quad (2)$$

where $\psi_{ai} = \arctan(v_i/u_i)$ is the polar angle or so-called sideslip angle (Fossen, 2002). Since $u_i > 0$, ψ_{ai} is smooth in the domain $(-\pi/2, \pi/2)$. Differentiating the first equation of (2) and further according to $u_i = u_{li} \cos \psi_{ai}$ and $v_i = u_{li} \sin \psi_{ai}$, we have

$$\dot{u}_{li} = \dot{u}_i \cos \psi_{ai} + \dot{v}_i \sin \psi_{ai}. \quad (3)$$

Using polar coordinates u_{li} and ψ_{li} , the vehicle's kinematics can be rewritten as

$$\dot{x}_i = u_{li} \cos \psi_{li}, \quad \dot{y}_i = u_{li} \sin \psi_{li}. \quad (4)$$

Consequently, the vehicle model (1) can be rewritten as

¹ For the convenience of discussion, in this paper we only consider the vehicle's 3 DOF motion in the horizontal plane instead of its 6 DOF motion.

$$\begin{aligned} \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\psi}_{li} \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \\ \dot{\psi}_{ai} \end{bmatrix} + \begin{bmatrix} \cos \psi_{li} & 0 \\ \sin \psi_{li} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{li} \\ r_i \end{bmatrix}, \\ \begin{bmatrix} \dot{u}_{li} \\ \dot{r}_i \end{bmatrix} &= \begin{bmatrix} f_{ui} \cos \psi_{ai} + f_{vi} \sin \psi_{ai} \\ f_{ri} \end{bmatrix} + \begin{bmatrix} b_{ui} \cos \psi_{ai} & 0 \\ 0 & b_{ri} \end{bmatrix} \begin{bmatrix} \tau_{ui} \\ \tau_{ri} \end{bmatrix}. \end{aligned} \quad (5)$$

It is easy to see that (5) is in the form where the vehicle's sway dynamics can be kinematically handled by the surge force τ_u .

3. VEHICLES SCHOOLING RULES

3.1 Geometry of Schooling

In this paper, we consider a group of n vehicles, where each vehicle's dynamics can be expressed as (1). For these multiple vehicles, in this paper, we model its geometry of the schooling using α -Lattice (Saber, 2006), which satisfies the following sets of constraints

$$\|q_j - q_i\| = d_\alpha, \quad \forall j \in N(q_i), \quad i = 1, \dots, n, \quad (6)$$

where $q_i = (x_i, y_i) \in \mathfrak{R}^2$, $\|\cdot\|$ denotes Euclidean norm, d_α is the desired distance between vehicles in the schooling, and $N(q_i)$ is the neighbor of i th vehicle defined as following

$$N(q_i) = \{q_j : \|q_j - q_i\| < r_\alpha, \quad j = 1, \dots, n\}, \quad (7)$$

where $r_\alpha > 0$ is a design parameter.

Remark 2. For the convenience of discussion, in this paper, we assume all vehicles in the group have the same dynamics as (1) with the same neighboring radius r_α . Further, the desired distances between the vehicles in the same neighbors are all equal to d_α .

In this paper, the potential function for the configuration of the schooling is defined as following similar to Saber (2006)

$$V_\alpha = \frac{1}{2} \sum_i \sum_{j \in N(q_i)} \theta_\alpha(\|q_j - q_i\|_\sigma), \quad (8)$$

where $\|\cdot\|_\sigma$ denotes σ -norm defined as

$$\|\cdot\|_\sigma = (\sqrt{1 + \varepsilon \|\cdot\|^2} - 1) / \varepsilon, \quad (9)$$

with $\varepsilon > 0$ design parameter. In (8), the attractive/repulsive potential $\theta_\alpha(\zeta)$ is defined as following

$$\theta_\alpha(\zeta) = \int_{d_\alpha}^{\zeta} \varphi_\alpha(s) ds, \quad (10)$$

$$\varphi_\alpha(s) = \frac{1}{2} \rho_h(s/r_{\alpha\sigma}) \left[(a_\alpha + b_\alpha) \frac{s + c_\alpha - d_{\alpha\sigma}}{\sqrt{1 + (s + c_\alpha - d_{\alpha\sigma})^2}} + (a_\alpha - b_\alpha) \right], \quad (11)$$

where $b_\alpha \geq a_\alpha > 0$, and $c_\alpha = |a_\alpha - b_\alpha| / \sqrt{4a_\alpha b_\alpha}$ are design parameters, $r_{\alpha\sigma} = \|r_\alpha\|_\sigma$, $d_{\alpha\sigma} = \|d_\alpha\|_\sigma$, and the bump function $\rho_h(\cdot)$ is defined as

$$\rho_h(\zeta) = \begin{cases} 1, & \zeta \in [0, h) \\ 0.5 \{1 + \cos[\pi(\zeta - h)/(1 - h)]\}, & \zeta \in [h, 1] \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

where $h \in (0, 1)$ is a design parameter.

3.2 Group Movement

In so far the literature, the group movements usually have been led by certain leaders. Here the leaders can be the specific actual agents (Guo *et al.*, 2004; Edwards *et al.*, 2004), or are the certain virtual ones (Fiorelli *et al.*, 2006; Leonard and Fiorelli, 2001; Saber, 2006). By adding certain restricting conditions between these leaders and the corresponding followers, the group movements can be designed through designing the reference paths for these leaders. In this paper, we apply the virtual leader-follower concept, and the corresponding potential function is given as following

$$V_\beta = \frac{1}{2} \sum_k \sum_{j \in \Omega_k} \theta_\beta(\|q_j - q_{v,k}\|_\sigma), \quad k = 1, \dots, m, \quad (13)$$

where $m \leq n$ is the number of the virtual leaders needed to design the group movement, Ω_k denotes a subgroup of the vehicles following the k th virtual leader $q_{v,k}$, and have $\Omega_1 \cup \dots \cup \Omega_m = \{1, \dots, n\}$ and $\Omega_i \cap \Omega_j = \emptyset, \forall i \neq j \in \{1, \dots, m\}$. Moreover, in (13), the attractive/ repulsive potential $\theta_\beta(\xi)$ has the same structure as $\theta_\alpha(\xi)$ defined in (10) and (11) with the subscript letter α replaced by β .

Remark 3. Also, for the convenience of discussion, we assume that the desired distances between the virtual leaders and the corresponding vehicles (in the same subgroup) are all equal to d_β , and the neighboring radiuses of Ω_k with $k = 1, \dots, m$ are all equal to r_β . Moreover, all of m virtual leaders are assumed to have the same velocity of u_{iv} and heading ψ_{iv} .

3.3 Obstacles Avoidance

In this paper, the obstacles are modeled as in Fig. 1. Inside of the circle centered at i th vehicle q_i with r_γ as radius, each obstacle block is modeled as the point, which is the closest one from the center point q_i . In Fig. 1, B1 and B2, which are two parts of the same obstacle block B, are considered to be independent from each other and modeled as different points $q_{i,2}$ and $q_{i,3}$.

The potential function for the obstacles avoidance is chosen

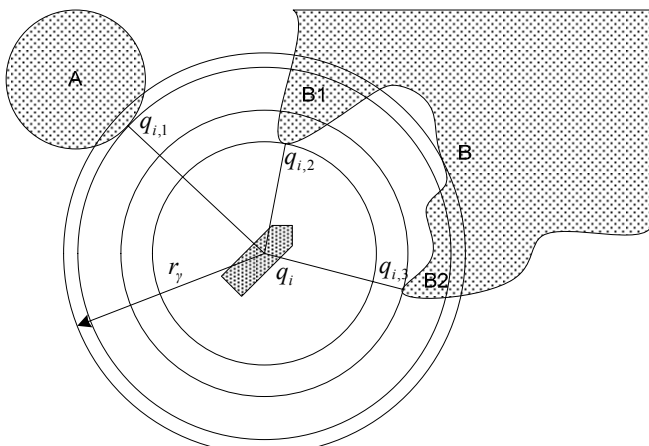


Fig. 1. Obstacle modeling.

as following

$$V_\gamma = \frac{1}{2} \sum_i \sum_{p \in \Omega_{oi}} \theta_\gamma(\|q_{i,p} - q_i\|_\sigma), \quad i = 1, \dots, n, \quad (14)$$

where Ω_{oi} is the subgroup of obstacle points $q_{i,p}$ included in the circle centered at the i th vehicle q_i with r_γ as radius as. And similar to the previous subsection, the attractive/ repulsive potential $\theta_\gamma(\xi)$ is chosen to has the same structure as $\theta_\alpha(\xi)$ defined in (10) and (11) with the subscript letter α replaced by γ .

Assumption 1. $\dot{q}_{i,p} = 0$. This means that all of the obstacles considered in this paper are position fixed.

Assumption 2. All of the virtual leaders are designed to move away from the obstacles such that $\|q_{v,k} - q_p\| \rightarrow \infty$ as $t \rightarrow \infty$ for all $k \in \{1, \dots, m\}$ and $p \in \Omega_o$ with $\Omega_o = \Omega_{o1} \cup \dots \cup \Omega_{on}$. Moreover, the virtual leaders are also designed to satisfy following inequalities

$$\sum_{j \in \Omega_k} (x_{kj} \cos \psi_v + y_{kj} \sin \psi_v) \leq 0, \quad \forall k = 1, \dots, m. \quad (15)$$

Remark 4. If we locate the virtual leader $q_{v,k}$ at the central point of the geometry of the subgroup Ω_k , then we have

$$x_{v,k} = \frac{1}{m_k} \sum_{j \in \Omega_k} x_j, \quad y_{v,k} = \frac{1}{m_k} \sum_{j \in \Omega_k} y_j, \quad (16)$$

where m_k is the number of the vehicles in the neighbour Ω_k . From (16), it is easy to see that $\sum_{j \in \Omega_k} x_{kj} = \sum_{j \in \Omega_k} y_{kj} = 0$, and therefore (15) satisfies.

4. STABLE FORMATION CONTROL DESIGN

The purpose of this paper is to propose a stable schooling scheme for multiple underactuated AUVs and to guarantee the velocity and heading matching of the schooling while keeping obstacles avoidance. This kind of control objective coincides with the Reynolds three heuristic rules (Reynolds, 1987).

For the two inputs second-order nonlinear system (5), we solve the schooling problem using general backstepping method (Krstic *et al.*, 1995).

Step 1. In this step, we consider the following Lyapunov function candidate

$$V_1 = \gamma_\alpha V_\alpha + \gamma_\beta V_\beta + \gamma_\gamma V_\gamma + \frac{1}{2} \left[\sum_i \gamma_u (u_{li} - u_v)^2 + \sum_i (\psi_{li} - \psi_v)^2 \right], \quad (17)$$

where $\gamma_\alpha, \gamma_\beta, \gamma_\gamma, \gamma_u > 0$ are certain weighting factors.

Differentiating (17) and substituting (8)~(14) into it, we have

$$\begin{aligned} \dot{V}_1 = & \gamma_\alpha \sum_i \left\{ \sum_{j \neq i} \varphi_\alpha(\|q_i - q_j\|^2) [x_{ij}[(\dot{x}_i - \dot{x}_v) - (\dot{x}_j - \dot{x}_v)]] \right. \\ & \left. + y_{ij}[(\dot{y}_i - \dot{y}_v) - (\dot{y}_j - \dot{y}_v)] \right\} \\ & + \gamma_\beta \sum_k \sum_{j \in \Omega_k} \varphi_\beta(\|q_j - q_{v,k}\|^2) [x_{jk}(\dot{x}_j - \dot{x}_{v,k}) + y_{jk}(\dot{y}_j - \dot{y}_{v,k})] \end{aligned}$$

$$\begin{aligned}
& + \gamma_\gamma \sum_i \sum_{p \in \Omega_u} \varphi_\gamma (\|q_i - q_{i,p}\|^2) \left\{ [x_{ip} (\dot{x}_i - \dot{x}_v + \dot{x}_v) \right. \\
& \left. + y_{jk} (\dot{y}_j - \dot{y}_v + \dot{y}_v)] \right\} + \sum_i \gamma_u u_{liv} (\dot{u}_{li} - \dot{u}_v) + \sum_i \psi_{liv} (\dot{\psi}_{li} - \dot{\psi}_v) \\
= & \gamma_\alpha \sum_i \sum_{j \neq i} \varphi_\alpha (\|q_i - q_j\|^2) \left\{ [x_{ij} (\cos \psi_{li} + \cos \psi_{lj}) \right. \\
& \left. + y_{ij} (\sin \psi_{li} + \sin \psi_{lj})] \cdot u_{liv} \right. \\
& \left. + 2u_v \left(x_{ij} \sin \frac{\psi_{li} + \psi_{lv}}{2} - y_{ij} \cos \frac{\psi_{li} + \psi_{lv}}{2} \right) \cdot \sin \frac{\psi_{liv}}{2} \right\} \\
& + \gamma_\beta \sum_k \sum_{j \in \Omega_i} \varphi_\beta (\|q_j - q_{v,k}\|^2) \left\{ [x_{jk} \cos \psi_{lj} + y_{jk} \sin \psi_{lj}] \cdot u_{liv} \right. \\
& \left. + 2u_v \left(y_{jk} \sin \frac{\psi_{lj} + \psi_v}{2} - x_{jk} \cos \frac{\psi_{lj} + \psi_v}{2} \right) \cdot \sin \frac{\psi_{liv}}{2} \right\} \\
& + \gamma_\gamma \sum_i \sum_{p \in \Omega_u} \varphi_\gamma (\|q_i - q_{i,p}\|^2) \left\{ [x_{ip} \cos \psi_{li} + y_{ip} \sin \psi_{li}] \cdot u_{liv} \right. \\
& \left. + 2u_v \left(y_{ip} \sin \frac{\psi_{li} + \psi_v}{2} - x_{ip} \cos \frac{\psi_{li} + \psi_v}{2} \right) \cdot \sin \frac{\psi_{liv}}{2} \right\} \\
& + \gamma_\gamma u_v \sum_i \sum_{p \in \Omega_u} \varphi_\gamma (\|q_i - q_{i,p}\|^2) (x_{ip} \cos \psi_v + y_{ip} \sin \psi_v) \\
& + \sum_i \gamma_u u_{liv} (\dot{u}_{li} - \dot{u}_v) + \sum_i \psi_{liv} (\dot{\psi}_{li} - \dot{\psi}_v) \\
= & \sum_i \left(\Lambda_{ui} \cdot u_{liv} + \Lambda_{\psi i} \cdot \sin \frac{\psi_{liv}}{2} \right) + \Lambda_o \\
& + \sum_i \gamma_u u_{liv} (\dot{u}_{li} - \dot{u}_v) + \sum_i \psi_{liv} (\dot{\psi}_{li} - \dot{\psi}_v), \tag{18}
\end{aligned}$$

where $u_{liv} = u_{li} - u_v$, $\psi_{liv} = \psi_{li} - \psi_v$ and

$$\Lambda_o = \gamma_\gamma u_v \sum_i \sum_{p \in \Omega_u} \varphi_\gamma (\|q_i - q_{i,p}\|^2) (x_{ip} \cos \psi_v + y_{ip} \sin \psi_v).$$

In the above expansion, we use the relation of $\sum_k \sum_{j \in \Omega_i} (\cdot) = \sum_i (\cdot)$. This is induced from, as aforementioned, $\Omega_1 \cup \dots \cup \Omega_m = \{1, \dots, n\}$ and $\Omega_i \cap \Omega_j = 0, \forall i, j \in \{1, \dots, m\}$.

Remark 5. In this paper, we choose the parameters d_γ and r_γ as $d_\gamma = r_\gamma$. Therefore, according to (10)~(12), we get that $\varphi_\gamma (\|q_i - q_{i,p}\|^2) < 0$ if $\|q_i - q_{i,p}\| < r_\gamma$, and $\varphi_\gamma (\|q_i - q_{i,p}\|^2) = 0$ if $\|q_i - q_{i,p}\| \geq r_\gamma$.

Further, substituting (5) into (18), we have

$$\begin{aligned}
\dot{V}_1 = & \sum_i \left(\Lambda_{ui} u_{liv} + \Lambda_{\psi i} \sin \frac{\psi_{liv}}{2} \right) + \Lambda_o \\
& + \sum_i \gamma_u u_{liv} (f_{ui} \cos \psi_{ai} + f_{vi} \sin \psi_{ai} + b_{ui} \cos \psi_{ai} \cdot \tau_{ui} - \dot{u}_v) \\
& + \sum_i \psi_{liv} (\alpha_{ri} - e_{ri} + \dot{\psi}_{ai} - \dot{\psi}_v), \tag{19}
\end{aligned}$$

where $e_{ri} = \alpha_{ri} - r_i$, and α_{ri} is the *stabilizing function* (Krstic et al., 1995) for the virtual input r_i .

According to (19), in the *Step 1*, we choose the control laws as following

$$\tau_{ui} = b_{ui}^{-1} \sec \psi_{ai} \left[\dot{u}_v - f_{ui} \cos \psi_{ai} - f_{vi} \sin \psi_{ai} - \gamma_u^{-1} (k_{ui} u_{liv} + \Lambda_{ui}) \right], \tag{20}$$

$$\alpha_{ri} = -k_{\psi i} \psi_{liv} - \dot{\psi}_{ai} + \dot{\psi}_v - \Lambda_{\psi i} \frac{\sin(\psi_{liv}/2)}{\psi_{liv}}, \tag{21}$$

where $k_{ui}, k_{\psi i} > 0$ are design parameters. Substituting (20) and (21) into (18), we get

$$\dot{V}_1 = \sum_i \left(-k_{ui} u_{liv}^2 - k_{\psi i} \psi_{liv}^2 - e_{ri} \psi_{liv} \right) + \Lambda_o. \tag{22}$$

Step 2. In this step, we will derive the control law for τ_{ri} . Rewrite the final equation of (5) as following

$$\dot{e}_{ri} = \dot{\alpha}_{ri} - f_{ri} - b_{ri} \tau_{ri}. \tag{23}$$

According to (19) and (23), we choose the control law for τ_{ri} as following

$$\tau_{ri} = b_{ri}^{-1} (k_{ri} e_{ri} + \dot{\alpha}_{ri} - f_{ri} - \psi_{liv}). \tag{24}$$

Theorem 1. Consider the schooling of multiple underactuated underwater vehicles whose dynamics can be expressed as (1). If we choose the formation control laws as (20) and (24), then all of the vehicles in the group are guaranteed to asymptotically move with the same velocity and heading while keeping obstacles avoidance.

Proof. Consider the following Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2} \sum_i e_{ri}^2. \tag{25}$$

Differentiating (25) and substituting (23), (24) into it, we get

$$\dot{V}_2 = \sum_i \left(-k_{ui} u_{liv}^2 - k_{\psi i} \psi_{liv}^2 - k_{ri} e_{ri}^2 \right) + \Lambda_o. \tag{26}$$

On the other hand, since $\|q_{v,k} - q_p\|_{t \rightarrow \infty} \rightarrow \infty$ for all $k \in \{1, \dots, m\}$ and $p \in \Omega_o$ (*Assumption 2*), after a certain period of ($t \geq t_0 > 0$), the following inequality can always be satisfied

$$q_{kp} \dot{q}_{kp} = u_v (x_{kp} \cos \psi_v + y_{kp} \sin \psi_v) > 0. \tag{27}$$

Since $u_v > 0$, (27) can be rewritten as following form

$$\sum_k \sum_{j \in \Omega_i} \sum_{p \in \Omega_o} (x_{kp} \cos \psi_v + y_{kp} \sin \psi_v) > 0. \tag{28}$$

Further rewriting (15) as following

$$\sum_k \sum_{j \in \Omega_i} N(p) (x_{kj} \cos \psi_v + y_{kj} \sin \psi_v) \leq 0, \tag{29}$$

where $N(p)$ is the number of p in Ω_{oi} . Subtracting (29) from (28), we have

$$\sum_i \sum_{p \in \Omega_o} (x_{ip} \cos \psi_v + y_{ip} \sin \psi_v) > 0. \tag{30}$$

Moreover, since $\varphi_\gamma (\cdot) \leq 0$ with $\gamma_\gamma, u_v > 0$, we can get that Λ_o is always non-positive. Consequently, according to (26), we have $\dot{V}_2 \leq 0$, and $\dot{V}_2 = 0$ iff $u_{liv} = \psi_{liv} = e_{ri} = 0$. \square

Remark 6. In practice, *Assumption 2*, particularly (15) may impose certain restricting conditions on the selection of the reference paths for virtual leaders, and the strictness of the restrictions might depend on the obstacles' geometry and their arrangements. How to choose the reference paths for the virtual leaders is out of this paper and will be considered in our future works.

5. NUMERICAL SIMULATIONS

In this section, we carry out certain numerical simulations to demonstrate the effectiveness of proposed schooling scheme. In the simulations, the group consists of three vehicles, each of which is modelled as 6DOF nonlinear dynamics of ISiMI AUV (Lee *et al.*, 2003), which has the same mechanical structure as REMUS AUV (Prestero, 2001). For this kind of AUV, we use the saturation conditions as $|\tau_{ui}| \leq 50N$ and $|\tau_{ri}| \leq 30\text{deg}$ in the simulation. And the initial conditions are: $x_1(0) = y_1(0) = 1m$, $\psi_1(0) = 0.1\text{rad}$, $u_1(0) = v_1(0) = r_1(0) = 0$, $x_2(0) = 5m$, $y_2(0) = 1m$, $\psi_2(0) = 0.3\text{rad}$, $u_2(0) = v_2(0) = r_2(0) = 0$, $x_3(0) = 1m$, $y_3(0) = 5m$, $\psi_3(0) = 2\text{rad}$, $u_3(0) = v_3(0) = r_3(0) = 0$. The control gain parameters are taken as $k_{ui} = k_{vi} = 1.0$, $k_{ri} = 0.5$, $i = 1, 2, 3$. In the simulation, we design one virtual leader whose reference path is taken as: for the first 30s, $u_v = 1.5m/s$, $\psi_v = 90\text{deg}$, then $u_v = 1.5m/s$, $\psi_v = 0\text{deg}$.

5.1 Selection of Potential Functions

For potential function V_α , the parameters are taken as $a_\alpha = b_\alpha = 50$, $r_\alpha = 25m$, $d_\alpha = 12\sqrt{3}m$, $h = 0.9$, $\varepsilon = 0.2$. For V_β , $a_\beta = b_\beta = 50$, $r_\beta = 50m$, $d_\beta = 12m$, $h = 0.9$, $\varepsilon = 0.2$, finally for V_γ , we take $a_\gamma = 200$, $b_\gamma = 950$, $r_\gamma = d_\gamma = 6m$, $h = 0.6$, $\varepsilon = 0.15$. From these parameters, it is easy to verify the geometry of the desired schooling of three vehicles is an equilateral triangle with the virtual leader at its center. Selected potential functions are depicted in Fig. 2.

5.2 Selection of Weighting Factors

In (17), the weighting factors γ_α , γ_β and γ_γ are taking two different roles in the schooling. One is scaling function and the other one is that these parameters reflect certain priorities between the potential functions V_α , V_β and V_γ in different situations. For example, in the case where vehicles approach to certain obstacles, the priority of V_γ will be higher than that of other two potentials. In the simulation, these parameters are chosen as: if $q_{ij} \leq r_\gamma$, $\gamma_\alpha = \gamma_\beta = 0.0001$, $\gamma_\gamma = 0.002$, $\gamma_u = 0.25$, else $\gamma_\alpha = \gamma_\beta = 0.003$, $\gamma_\gamma = 0.002$, $\gamma_u = 0.3$.

5.3 Simulation Results

Simulation results are presented in Fig. 3~6. Fig. 3 shows the vehicles' schooling motions with obstacles avoidance. In the simulation, we take $r_\beta = 50m$ which is much more larger than $r_\alpha(25m)$. This is for the purpose of strengthening the

attractive characteristic of V_β . From Fig. 4, we can see that the schooling of the vehicles converge to an equilateral triangle with the virtual leader at the centre as being desired. Fig. 5 shows the velocity and heading matching, from which we can see that the velocity and heading tracking in the polar frame are all asymptotically stable. Finally, the histories of proposed control inputs are plotted in Fig. 6.

6. CONCLUSIONS

This paper presents a stable schooling scheme for multiple underactuated AUVs, where only the surge force and yaw moment are available for the vehicle's 3D horizontal motion. Firstly, using certain polar coordinates transformation, we transform the vehicle's dynamics to a form where the sway dynamics can be handled by surge force. Upon this converted nonlinear model, a stable schooling scheme is proposed. Constructed potential function consists of three parts: one for the interactions between vehicles, the second one is for the group movement, and the final part is for the obstacle avoidances. The proposed scheme guarantees the stability of the schooling and further guarantees the velocity and heading matching of the group. However, the schooling scheme only guarantees a certain local minimum that $\dot{q}_{ij} \rightarrow 0$ instead of $q_{ij} \rightarrow d_\alpha$. It is of interest in our future works to upgrade the result of this paper to that can guarantee the global minimum of the schooling.

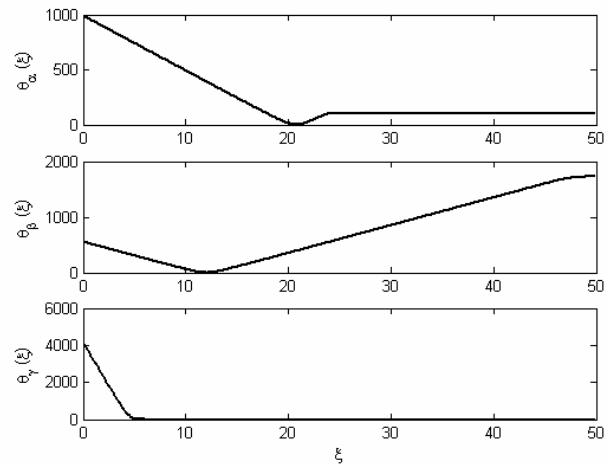


Fig. 2. Potential functions.

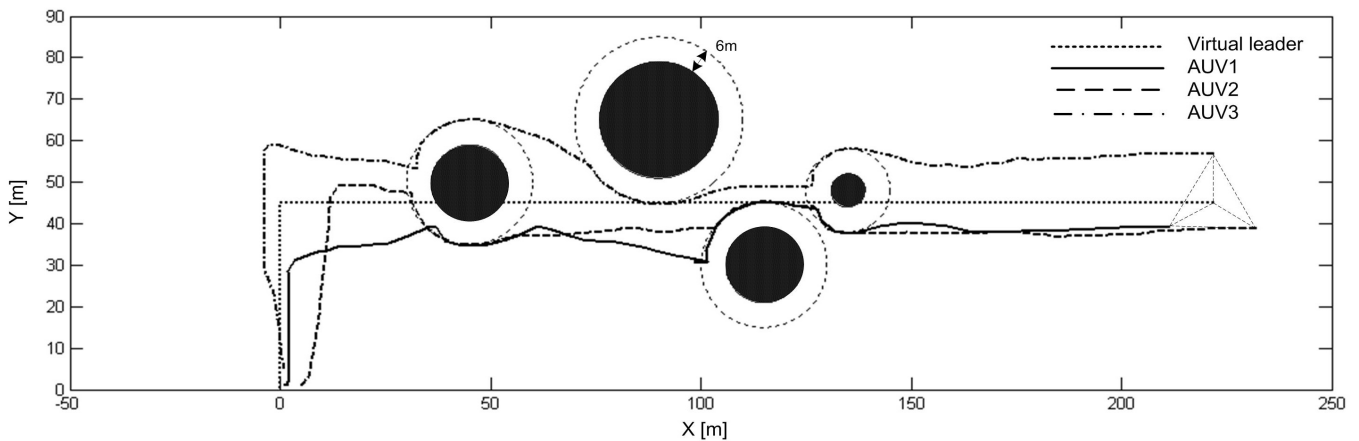


Fig. 3. Trajectories of the vehicles in the schooling with obstacle avoidances.

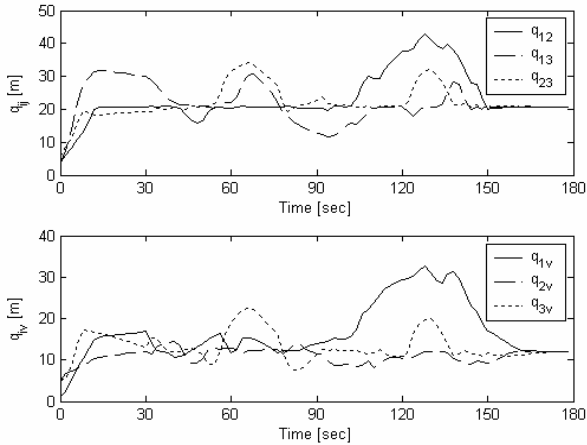


Fig. 4. Geometry of the schooling.

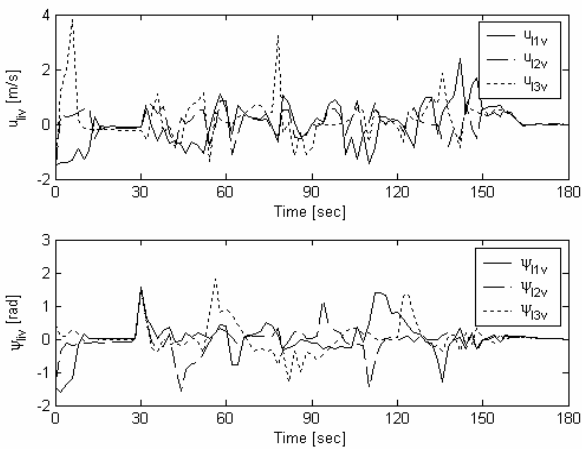


Fig. 5. Velocity and heading matching.

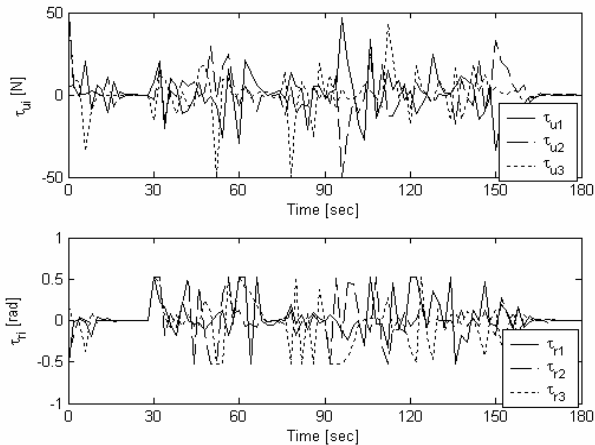


Fig. 6. Corresponding control inputs.

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REFERENCES

Bacciotti, A. and Rosier, L. (2005). *Liapunov Functions and*

Stability in Control Theory. Springer-Verlag, Berlin, Heidelberg.
 Brockett, R.W., Millman, R.S., and Sussmann, H.J. (1983). *Differential Geometric Control Theory*. Boston, MA: Birkhauser.
 Do, K.D. (2006). Formation control of mobile agents using local potential functions. *Proc. of the 2006 American Control Conference*, Minneapolis, Minnesota, USA (pp. 2148-2153).
 Dunbar, W.B. and Murray, R.M. (2002). Model predictive control of coordinated multi-vehicles formations. *Proc. of the 41st IEEE Conference on Decision and Control*, Las Vegas, Nevada, USA (pp. 4631-4636).
 Edwards, D.B, Bean, T.A., Odell, D.L., and Anderson, M.J. (2004). A leader-follower algorithm for multiple AUV formations. *Autonomous Underwater Vehicles, 2004 IEEE/OES Volume*, 17-18, 40-46.
 Fax, J.A and Murray, R.M. (2004). Information flow and cooperative control of vehicle formations. *IEEE Trans. On Automatic Control*, 49, 1465-1476.
 Fossen, T.I. (2002). *Marine Control Systems*. Trondheim, Norway: Marine Cybernetics.
 Fiorelli, E., Leonard, N.E., Bhatta, P., Paley, D.A., Bachmayer, R., and Fratantoni, D.M. (2006). Multi-AUV control and adaptive sampling in Monterey Bay. *IEEE Journal of Oceanic Engineering*, 31, 935-948.
 Guo, J., Wei, Y.Y, Chiu, F.C., and Cheng, S.W. (2004). A maximum entropy method for multi-AUV grouping. *Proc. Of IEEE/MTS Oceans 2004*, Kobe, Japan (pp. 532-536).
 Jiang, Z.P. and Nijmeijer, H. (1999). A recursive technique for tracking control of nonholonomic systems in chained form. *IEEE Trans. on Automatic Control*, 44, 265-279.
 Lee, P.M. et al. (2003). *Development of an Advanced Deep-Sea Unmanned Underwater Vehicle (II)*. Technical Report, UCM0043A-2442, KORDI, Daejeon, Korea.
 Leonard, N.E. and Fiorelli, E. (2001). Virtual leaders, artificial potentials and coordinated control of groups. *Proc. of the 40th IEEE Conference on Decision and Control*, Orlando, Florida, USA (pp. 2968-2973).
 Murray, R.M. and Sastry, S.S. (1993). Nonholonomic motion planning: Steering using sinusoids. *IEEE Trans. on Automatic Control*, 38, 700-716.
 Prestor, T. (2001). *Verification of a six-degree of freedom simulation model for the REMUS autonomous underwater vehicles*. Masters Thesis, Dept. of Ocean Eng. And Mech. Eng., MIT.
 Reyhanoglu, M. (1997). Exponential stabilization of an underactuated autonomous surface vessel. *Automatica*, 38, 301-309.
 Reynolds, C.W. (1987). Flocks, herds, and schools: A distributed behavioral model. *Computer Graphic. (ACM SIGGRAPH'87 Conference Proceeding)*, 21, 25-34.
 Saber, R.O. and Murray, R.M. (2002). Distributed cooperative control of multiple vehicle formations using structural potential functions. *Proc. of 15th IFAC World Congress*, Barcelona, Spain.
 Saber, R.O. and Murray, R.M. (2003). Flocking with obstacle avoidance: cooperation with limited communication in mobile networks. *Proc. of the 42nd IEEE Conference on Decision and Control*, Maui, Hawaii, USA (pp. 2022-2028).
 Saber, R.O. (2006). Flocking for multi-agent dynamic systems: algorithms and theory. *IEEE Trans. On Automatic Control*, 51, 401-420.
 Smith, T.R., Hanbmann, H., and Leonard, N.E. (2001). Orientation control of multiple underwater vehicles with with symmetry-breaking potentials. *Proc. of the 40th IEEE Conference on Decision and Control*, Orlando, Florida, USA (pp. 4598-4603).
 Sousa, J.B. and Pereira, F.L. (2002). Coordinated control strategies for networked vehicles: an application to autonomous underwater vehicles. *Proc. of International Workshop on Generalized Solutions in Control Problems*, Peraslavl-Zalessky, Russia.