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Robust Output Feedback Control for a Class of Uncertain Switching Fuzzy Systems*

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Abstract: The problem of robust output-feedback control for a class of uncertain switching fuzzy control systems is investigated. Asymptotic stability of the observer, based on the error equation, is obtained by using the switching technique and the common Lyapunov function method. Switching laws are also designed via using single Lyapunov function method for observer equation such that the closed-loop system is asymptotically stable. The sufficient condition for the asymptotic stability of the uncertain switching fuzzy control system is transformed into standard solvable LMIs. An illustrative example along with the respective simulation results is give to demonstrate the effectiveness of the proposed design synthesis.

1. INTRODUCTION

Recently, the research activities on the synergy of fuzzy system based controls, as an important intelligent control approach, combined with some of math-analytical control theories has attracted great attention. In particular, the class of Takagi-Sugeno (T-S) fuzzy models has been found to be most effective as system model in various fuzzy system based methods. Based on T-S fuzzy model representations and the feedback control strategy, stability and robust analysis and design as well as handling parameter uncertainties for fuzzy systems have acquired considerable number of fruitful results(see e.g., Berenji, 1992; Dimirovski, et al., 2006; Lee, et al., 2001; Lo and Lin 2004; Ma, et al., 1998; Tanaka, et al., 1996; Tong and Zhou, 2001; Wang, et al., 1995; Wang, et al., 1996;).

Also recently, the switching systems, which are an important class of hybrid systems and have wide background and technological applications, have also been one of the main research focuses. In turn, considerable number of fruitful results in analysis and design of switching systems have been derived too, (see e.g., Branicky, 1998; Liberzon and Morse 1999; Wang, et al., 2003; Zhao and Spong, 2001; Zhao and Nie, 2003; Zhao and Dimirovski, 2004). Moreover, considerable ongoing research is oriented towards the synergetic use of fuzzy system and switched system concepts for the purpose of designing efficient and robust control systems (see e.g., Hiroshi, et al., 2003; Hiroshi, et al., 2006; Tanaka, et al., 2000; Tanaka, et al., 2001; Yang, et al., 2006a; Yang, et al., 2006b).

Conceptually, a switching fuzzy system is a type of switching systems in which all of the respective subsystems are fuzzy systems. Many nonlinear systems with switching features can be modeled as switching fuzzy systems. However, the results for switching fuzzy systems in the literature seem to be rather limited, (see e.g., Hiroshi, et al., 2003; Hiroshi, et al., 2006; Tanaka, et al., 2000; Tanaka, et al., 2001; Yang, et al., 2006a; Yang, et al., 2006b) and references therein. Zhao and Dimirovski (2004) have given a profound study of the problems of stability and controller switching for switching fuzzy systems. Hiroshi, et al. (2001) and Tanaka, et al. (2000) have developed fuzzy system based controls employing a fuzzy system with two level of fuzzy rules. Hiroshi, et al. (2006) designed a switching fuzzy controller via the augmented system design method. Following these results Yang, et al. (2006a) and Yang, et al. (2006b) have developed a novel approach to representation modeling, stability analysis, and synthesis design for switched fuzzy systems.

The present study, while exploiting previous results in Tong and Zhou, (2001), is focused on the robust control problem for fuzzy systems due to its considerable importance for practical applications. In Tong and Zhou, (2001), a fuzzy state feedback controller and robust observer are designed on the background of fuzzy modeling of the uncertain nonlinear process to be controlled. On the grounds of the methodology of linear matrix inequalities (LMIs), fuzzy state feedback gain matrix and fuzzy observer gain matrix algorithms are given. Since the states of system are not always measurable,

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the observer error $e(t)$ is also unknown, and therefore $e(t)$ can not be used to design the switching law.

The present study considers uncertain switching fuzzy systems and finds an alternative solution that overcomes the deficiency of results in Tong and Zhou, (2001). For this purpose, we reconstruct the system states by means of observer design and study stability as to ensure its stable asymptotic behaviour. The observer error is made to converge to zero under an arbitrary switching law, which has been achieved by using the common Lyapunov function method. The switching law is designed via single Lyapunov function method for the observer such that the overall closed-loop control system is guaranteed to be asymptotically stable. The remainder is written up as follows. Section II presents the representation modeling of the plant and the derivation of the designed fuzzy output-feedback controller. In Section III, the stability analysis and the design synthesis of the switching law are carried out; here the main new result is given. Section IV presents an illustrative example along with the respective numeric and simulation results. Conclusion and references are given thereafter.

2. FUZZY OUTPUT-FEEDBACK CONTEOLLER DESIGN

Consider the switching fuzzy system with uncertainty. The T-S fuzzy model of every subsystem is described as follows:

R_σ^i : if $z_1(t)$ is $M_{\sigma 1}^i \cdots$ and $z_p(t)$ is $M_{\sigma p}^i$, then

$$\begin{aligned} \dot{x}(t) &= (A_{\sigma i} + \Delta A_{\sigma i})x(t) + (B_{\sigma i} + \Delta B_{\sigma i})u_\sigma(t) \\ y(t) &= C_{\sigma i}x(t), \qquad i = 1,2,\cdots N_\sigma \end{aligned} \tag{1}$$

In here the symbols represent the following: $M_{\sigma j}^i$ represent fuzzy subsets; $z(t) = [z_1(t), z_2(t), \cdots, z_p(t)]^T$ is the premise vector; $\sigma \in M = \{1,2,\cdots l\}$ is a piecewise constant function representing the switching signal; $x(t) \in R^n$ is the system state vector; $u_\sigma(t) \in R^m$ is the system input vector; $y(t) \in R^\vartheta$ is the system output vector; $A_{\sigma i} \in R^{n\times n}$, $B_{\sigma i} \in R^{n\times m}$, $C_{\sigma i} \in R^{\vartheta \times n}$ are the system matrices; and $\Delta A_{\sigma i}$, and $\Delta B_{\sigma i}$ are time varying matrices of appropriate dimension, which represent uncertainties of the system.

Since the system states often are not directly measurable, an observer for each subsystem of (1) has to be designed and implemented in order to reconstruct them via the measurements of the system input and output. For System (1), design the rules of switching subsystem state observer

O_σ^i : if $z_1(t)$ is $M_{\sigma 1}^i \cdots$ and $z_p(t)$ is $M_{\sigma p}^i$, then

$$\begin{aligned} \dot{\hat{x}}(t) &= A_{\sigma i}\hat{x}(t) + B_{\sigma i}u_\sigma(t) + L_{\sigma i}(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C_{\sigma i}\hat{x}(t), \qquad i = 1,2,\cdots N_\sigma \end{aligned} \tag{2}$$

where, $\hat{x}(t) \in R^n$ and $\hat{y}(t) \in R^\vartheta$ are the state and the output vectors, respectively, of the fuzzy observer, and $L_{\sigma i}$ represents observer gain matrix for the i-th fuzzy rule of the σ-th switching subsystem.

Since the system states often are not directly measurable, we consider the switching signal $\sigma = \sigma(\hat{x}(t))$, which depends on observer states. Suppose $\widetilde{\Omega}_1, \widetilde{\Omega}_2, \cdots \widetilde{\Omega}_l$ is a partition of R^n, i.e. $\bigcup_{i=1}^{l} \widetilde{\Omega}_i = R^n \setminus \{0\}$, and $\widetilde{\Omega}_i \bigcap \widetilde{\Omega}_j = \Phi, i \neq j$. The switching signal is $\sigma = \sigma(\hat{x}(t)) = r$, which depends on $\widetilde{\Omega}_1, \widetilde{\Omega}_2, \cdots \widetilde{\Omega}_l$. when $\hat{x}(t) \in \widetilde{\Omega}_r$, the switching signal $\sigma(\hat{x}(t))$ subjects to

$$v_r(\hat{x}(t)) = \begin{cases} 1 & \hat{x}(t) \in \widetilde{\Omega}_r \\ 0 & \hat{x}(t) \notin \widetilde{\Omega}_r \end{cases}, \; r \in M$$

That is, if and only if $\sigma = \sigma(\hat{x}(t)) = r$, $v_r(\hat{x}(t)) = 1$. $\widetilde{\Omega}_1, \widetilde{\Omega}_2, \cdots \widetilde{\Omega}_l$ and the switching law σ will be designed later.

It can be shown the overall switching fuzzy observer is represented by

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{r=1}^{l}\sum_{i=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))[A_{ri}\hat{x}(t) + B_{ri}u_r(t)] \\ &+ \sum_{r=1}^{l}\sum_{i=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))L_{ri}(y(t) - \hat{y}(t)) \end{aligned} \tag{3}$$

$$\hat{y}(t) = \sum_{r=1}^{l}\sum_{i=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))C_{ri}\hat{x}(t) \tag{4}$$

where, $\mu_{ri}(z(t)) = \dfrac{\prod_{j=1}^{p} M_{rj}^i(z_j(t))}{\sum_{i=1}^{N_r}\prod_{j=1}^{p} M_{rj}^i(z_j(t))}$, $0 \le \mu_{ri}(z(t)) \le 1$, $\sum_{l=1}^{N_r}\mu_{ri}(z(t)) = 1$, and $M_{rj}^i(z_j(t))$ represents the membership function of $z_j(t)$ belonging to fuzzy set M_{rj}^i.

Substituting $y(t)$ and $\hat{y}(t)$ into Eq. (3) yields

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{r=1}^{l}\sum_{i=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))A_{ri}[\hat{x}(t) + B_{ri}u_r(t)] \\ &+ \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t))L_{ri}C_{rj}(x(t) - \hat{x}(t)) \end{aligned} \tag{5}$$

Now the task is to design the fuzzy feedback controls that are represented as follows:

K_σ^i : if $z_1(t)$ is $M_{\sigma 1}^i \cdots$ and $z_p(t)$ is $M_{\sigma p}^i$, then

$$u_\sigma(t) = -K_{\sigma i}\hat{x}(t), \qquad i = 1,2,\cdots N_r \tag{6}$$

The overall switching fuzzy controller is then represented by means of

$$u_r(t) = -\sum_{r=1}^{l}\sum_{i=1}^{N_r} v_r(\hat{x}(t))\mu_{ri}(z(t))K_{ri}\hat{x}(t) \tag{7}$$

Upon introducing the observer error $e(t) = x(t) - \hat{x}(t)$, we can obtain:

$$\dot{x}(t)=\sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}\left\{\begin{array}{l}v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t))(A_{ri}+\Delta A_{ri}\\-(B_{ri}+\Delta B_{ri})K_{rj})x(t)\end{array}\right\} \\ +\sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t))(B_{ri}+\Delta B_{ri})K_{rj}e(t) \tag{8}$$

$$\dot{\hat{x}}(t)=\sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t))(A_{ri}-B_{ri}K_{rj})\hat{x}(t) \\ +\sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t))L_{ri}C_{rj}e(t) \tag{9}$$

$$\dot{e}(t)=\sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}\left\{\begin{array}{l}v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t))(A_{ri}+\\\Delta A_{ri}-L_{ri}C_{rj})e(t)\end{array}\right\} \\ +\sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}v_r(\hat{x}(t))\mu_{ri}(z(t))\mu_{rj}(z(t))(\Delta A_{ri}-\Delta B_{ri}K_{rj})\hat{x}(t) \tag{10}$$

3. STABILITY ANALYSIS AND SWITCHING LAW DESIGN

In this section, we give sufficient conditions for global asymptotic stability of the uncertain switching fuzzy control system (1). For observer equation (10), we choose a common Lyapunov function, such that the observer error $e(t)$ is asymptotically stable under arbitrary switching law. With single Lyapunov function method, a switching rule is designed by using observed state $\hat{x}(t)$ such that the output-feedback control system is asymptotically stable. In what follows the next assumption and lemma are needed.

Assumption 1. The parameter uncertainty matrices are norm bounded

$$[\Delta A_{ri}\quad \Delta B_{ri}]=D_{ri}F_{ri}(t)[E_{1ri}\quad E_{2ri}]$$

where, D_{ri}, E_{1ri} and E_{2ri} are constant matrices of appropriate dimension, $F_{ri}(t)$ is a unknown time varying matrix, satisfying $F_{ri}^T(t)F_{ri}(t)\le I$, $i=1,2,\cdots N_r$.

Lemma 1. Given constant matrices X and Y, for arbitrary $\varepsilon>0$, the following inequality

$$X^TY+Y^TX\le\varepsilon X^TX+\frac{1}{\varepsilon}Y^TY$$

holds.

Lemma 2: Given constant matrices D and E and a symmetric constant matrix Y of appropriate dimension, the following inequality holds:

$$Y+DFE+E^TF^TD^T<0$$

where F satisfies $F^TF\le R$, if and only if for some $\varepsilon>0$

$$Y+\varepsilon DD^T+\varepsilon^{-1}E^TE<0$$

Theorem 1. Suppose there exist positive definite matrix P_1 and P_2, controller gain K_{rj} and observer gain L_{ri}, and parameter $\lambda_r\in[0,1]$, $\sum_{r=0}^{l}\lambda_r=1$, such that

$$(A_{ri}-L_{ri}C_{rj})^TP_2+P_2(A_{ri}-L_{ri}C_{rj})+I+E_{1ri}^TE_{1ri} \\ +2P_2D_{ri}D_{ri}^TP_2<0 \tag{11}$$

$$\sum_{r=1}^{l}\lambda_r\Gamma_{rij}<0 \tag{12}$$

where

$$\Gamma_{rij}=(A_{ri}-B_{ri}K_{rj})^TP_1+P_1(A_{ri}-B_{ri}K_{rj})+P_1L_{ri}C_{rj}C_{rj}^TL_{ri}^TP_1+\Pi_{rij}$$

$$\Pi_{rij}=(E_{1ri}-E_{2ri}K_{rj})^T(E_{1ri}-E_{2ri}K_{rj})$$

Then there is a switching law $\sigma=\sigma(t)\in M=\{1,2,\cdots,l\}$, such that the uncertain switching fuzzy system (1) is global in asymptotically stable.

Proof. Consider the Lyapunov function

$$V(t)=\hat{x}^T(t)P_1\hat{x}(t)+e^T(t)P_2e(t) \tag{13}$$

where P_1 and P_2 are two positive definite matrices. For any $e(t)\ne0$, it follows form (11) that

$$e^T(t)[(A_{ri}-L_{ri}C_{rj})^TP_2+P_2(A_{ri}-L_{ri}C_{rj})+I \\ +E_{1ri}^TE_{1ri}+2P_2D_{ri}D_{ri}^TP_2]e(t)<0 \tag{14}$$

which means that under arbitrary switching law, observer error satisfies $\lim_{t\to\infty}e(t)=0$.

For any $\hat{x}(t)\ne0$, inequality (12) yields

$$\sum_{r=1}^{l}\lambda_r\hat{x}^T(t)\Gamma_{rij}\hat{x}(t)<0$$

Therefore there exists at least a r, such that

$$\hat{x}^T(t)[(A_{ri}-B_{ri}K_{rj})^TP_1+P_1(A_{ri}-B_{ri}K_{rj}) \\ +P_1L_{ri}C_{rj}C_{rj}^TL_{ri}^TP_1+\Pi_{rij}]\hat{x}(t)<0 \tag{15}$$

For arbitrary $r\in M$, let

$$\Omega_r=\left\{\hat{x}(t)\in R^n\left|\begin{array}{l}\hat{x}^T(t)[(A_{ri}-B_{ri}K_{rj})^TP_1\\+P_1(A_{ri}-B_{ri}K_{rj})\\+P_1L_{ri}C_{rj}C_{rj}^TL_{ri}^TP_1+\Pi_{rij}]\hat{x}(t)<0\end{array}\right.,\forall\hat{x}(t)\ne0\right\}$$

then $\bigcup_r\Omega_r=R^n\setminus\{0\}$, construct set $\tilde{\Omega}_1=\Omega_1$, …, $\tilde{\Omega}_r=\Omega_r-\bigcup_{i=1}^{r-1}\tilde{\Omega}_i$, it is apparent that $\bigcup_{i=1}^{l}\tilde{\Omega}_i=R^n\setminus\{0\}$, and $\tilde{\Omega}_i\bigcap\tilde{\Omega}_j=\Phi, i\ne j$.

Next, we design a switching law as follows:

$$\sigma(\hat{x}(t))=r\text{, if }\hat{x}(t)\in\tilde{\Omega}_r,\ r\in M \tag{16}$$

Let $V_1(\hat{x}(t))=\hat{x}^T(t)P_1\hat{x}(t)$ and $V_2(e(t))=e^T(t)P_2e(t)$, and then we obtain:

(1)The time derivative of $V_1(\hat{x}(t))$ satisfies

$$\dot V_1(\hat x(t)) = \dot{\hat x}^T(t)P_1\hat x(t) + \hat x^T(t)P_1\dot{\hat x}(t)$$
$$= \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}\left\{\begin{array}{l} v_r(\hat x(t))\mu_{ri}(z(t))\mu_{rj}(z(t))\hat x^T(t)[(A_{ri} \\ -B_{ri}K_{rj})^T P_1 + P_1(A_{ri}-B_{ri}K_{rj})]\hat x(t)\end{array}\right\}$$
$$+\sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}\left\{\begin{array}{l} v_r(\hat x(t))\mu_{ri}(z(t))\mu_{rj}(z(t))[e(t)^T(L_{ri}C_{rj})^T P_1\hat x(t) \\ +\hat x^T(t)P_1(L_{ri}C_{rj})e(t)]\end{array}\right\} \tag{17}$$

According to *Lemma 1,* the second term on the right hand side of (17) is

$$\begin{array}{l} e^T(t)(L_{ri}C_{rj})^T P_1\hat x(t) + \hat x^T(t)P_1(L_{ri}C_{rj})e(t) \\ \le \hat x^T(t)P_1L_{ri}C_{rj}C_{rj}^TL_{ri}^TP_1\hat x(t) + e^T(t)e(t)\end{array} \tag{18}$$

Substituting (18) into (17) gives

$$\dot V_1(\hat x(t)) \le \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}\left\{\begin{array}{l} v_r(\hat x(t))\mu_{ri}(z(t))\mu_{rj}(z(t)) \\ \hat x^T(t)[(A_{ri}-B_{ri}K_{rj})^T P_1 + P_1(A_{ri}-B_{ri}K_{rj}) \\ +P_1L_{ri}C_{rj}C_{rj}^TL_{ri}^TP_1]\hat x(t)\end{array}\right\}$$
$$+\sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} v_r(\hat x(t))\mu_{ri}(z(t))\mu_{rj}(z(t))e^T(t)e(t) \tag{19}$$

(2)The time derivative of $V_2(e(t))$ is

$$\dot V_2(e(t)) = \dot e^T(t)P_2e(t) + e^T(t)P_2\dot e(t)$$
$$= \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}\left\{\begin{array}{l} v_r(\hat x(t))\mu_{ri}(z(t))\mu_{rj}(z(t))e^T(t)[(A_{ri}+\Delta A_{ri}-L_{ri}C_{rj})^T P_2 \\ +P_2(A_{ri}+\Delta A_{ri}-L_{ri}C_{rj})]e(t)\end{array}\right\}$$
$$+\sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}\left\{\begin{array}{l} v_r(\hat x(t))\mu_{ri}(z(t))\mu_{rj}(z(t))[\hat x^T(t)(\Delta A_{ri}-\Delta B_{ri}K_{rj})^T P_2e(t) \\ +e^T(t)P_2(\Delta A_{ri}-\Delta B_{ri}K_{rj})\hat x(t)]\end{array}\right\} \tag{20}$$

According to *Lemma 2* and *Assumption 1,* the first term on the right hand side of (20) is

$$\begin{array}{l} (A_{ri}+\Delta A_{ri}-L_{ri}C_{rj})^T P_2 + P_2(A_{ri}+\Delta A_{ri}-L_{ri}C_{rj}) \\ = (A_{ri}-L_{ri}C_{rj})^T P_2 + P_2(A_{ri}-L_{ri}C_{rj}) \\ +P_2D_{ri}F_{ri}(t)E_{1ri} + E_{1ri}^TF_{ri}^T(t)D_{ri}^TP_2 \\ \le (A_{ri}-L_{ri}C_{rj})^T P_2 + P_2(A_{ri}-L_{ri}C_{rj}) + P_2D_{ri}D_{ri}^TP_2 + E_{1ri}^TE_{1ri}\end{array},$$
$$i,j = 1,2,\cdots N_r \tag{21}$$

According to *Lemma 1*and *Assumption 1,* the second term on the right hand side of (20) is

$$\begin{array}{l} \hat x^T(t)(\Delta A_{ri}-\Delta B_{ri}K_{rj})^T P_2e(t) + e^T(t)P_2(\Delta A_{ri}-\Delta B_{ri}K_{rj})\hat x(t) \\ \le e^T(t)P_2D_{ri}D_{ri}^TP_2e(t) + \hat x^T(t)\Pi_{rij}\hat x(t)\end{array} \tag{22}$$

where $\Pi_{rij} = (E_{1ri}-E_{2ri}K_{rj})^T(E_{1ri}-E_{2ri}K_{rj})$

Substituting (21) and (22) into (20), results in

$$\dot V_2(e(t)) \le \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}\left\{\begin{array}{l} v_r(\hat x(t))\mu_{ri}(z(t))\mu_{rj}(z(t))e^T(t)[(A_{ri} \\ -L_{ri}C_{rj})^T P_2 + P_2(A_{ri}-L_{ri}C_{rj}) \\ +E_{1ri}^TE_{1ri} + 2P_2D_{ri}D_{ri}^TP_2]e(t)\end{array}\right\} \tag{23}$$
$$+\sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r} v_r(\hat x(t))\mu_{ri}(z(t))\mu_{rj}(z(t))\hat x^T(t)\Pi_{rij}\hat x(t)$$

In view of (19), (23) and (13), we have

$$\dot V(t) \le \sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}\left\{\begin{array}{l} v_r(\hat x(t))\mu_{ri}(z(t))\mu_{rj}(z(t))\hat x^T(t) \\ [(A_{ri}-B_{ri}K_{rj})^T P_1 + P_1(A_{ri}-B_{ri}K_{rj}) \\ +P_1L_{ri}C_{rj}C_{rj}^TL_{ri}^TP_1 + \Pi_{rij}]\hat x(t)\end{array}\right\}$$
$$+\sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}\left\{\begin{array}{l} v_r(\hat x(t))\mu_{ri}(z(t))\mu_{rj}(z(t))e^T(t)[(A_{ri}-L_{ri}C_{rj})^T P_2 \\ +P_2(A_{ri}-L_{ri}C_{rj}) + I + E_{1ri}^TE_{1ri} + 2P_2D_{ri}D_{ri}^TP_2]e(t)\end{array}\right\} \tag{24}$$

From (14) and (15), we know that under the switching law (16), for arbitrary $\hat x(t) \ne 0$ and $e(t) \ne 0$, i.e., $x(t) \ne 0$, $\dot V(t) < 0$ holds. Therefore, the closed-loop system is asymptotically stable, and the observer error $e(t)$ asymptotically converges to zero.

From *Theorem 1*, after an appropriate analysis, the stability conditions for the uncertain switching fuzzy control system are transformed into the following matrix inequalities:

$$(A_{ri}-B_{ri}K_{rj})^T P_1 + P_1(A_{ri}-B_{ri}K_{rj}) + P_1L_{ri}C_{rj}C_{rj}^TL_{ri}^TP_1 + \Pi_{rij} < 0 \tag{25}$$

$$\begin{array}{l} (A_{ri}-L_{ri}C_{rj})^T P_2 + P_2(A_{ri}-L_{ri}C_{rj}) + I \\ +E_{1ri}^TE_{1ri} + 2P_2D_{ri}D_{ri}^TP_2 < 0\end{array} \tag{26}$$

Observe closely matrix inequalities (25) and (26). It should be noted, we may first compute inequality (26) to obtain the common Lyapunov function P_2 and the observer gain L_{ri} , and then to substitute L_{ri} into (25) to get the single Lyapunov function P_1 and the feedback gain K_{rj} . Furthermore, the inequalities (25) and (26) can be transformed into LMIs.

To this end, let it be defined

$$\begin{array}{l} T_{rij} = (A_{ri}-L_{ri}C_{rj})^T P_2 + P_2(A_{ri}-L_{ri}C_{rj}) + I + E_{1ri}^TE_{1ri} \\ = A_{ri}^TP_2 + P_2A_{ri} - C_{rj}^TN_{ri}^T - N_{ri}C_{rj} + I + E_{1ri}^TE_{1ri}\end{array}$$

where $N_{ri} = P_2L_{ri}$

Now, using Schur's complement, we obtain the following LMI

$$\begin{bmatrix} T_{rij} & P_2D_{ri} \\ (P_2D_{ri})^T & -0.5I\end{bmatrix} < 0 \tag{27}$$

Via substituting the result L_{ri} of the LMI (27) into inequality (25) and by letting $Q = P_1^{-1}, M_{rj} = K_{rj}Q$, upon multiplication of both sides of inequality (25) by $Q = P_1^{-1}$, we obtain the following LMI

$$\begin{bmatrix} \Phi_{rij} & (E_{1ri}Q - E_{2ri}M_{rj})^T \\ E_{1ri}Q - E_{2ri}M_{rj} & -I\end{bmatrix} < 0 \tag{28}$$

where $\Phi_{rij} = QA_{ri}^T + A_{ri}Q - M_{rj}^TB_{ri}^T - B_{ri}M_{rj} + L_{ri}C_{rj}C_{rj}^TL_{ri}^T$

Thus, the stability conditions of the uncertain switching fuzzy control system are transformed into the LMIs (27) and (28), which are tractable by LMI Toolbox of the Mathlab.

4. SIMULATION EXAMPLE

Consider the following switching fuzzy system:

$$\begin{aligned}
&R_1^1: \text{if } z_1(\text{t}) \text{ is } M_{11}^1, \text{ then}\\
&\quad \dot{x}(t)=(A_{11}+\Delta A_{11})x(t)+(B_{11}+\Delta B_{11})u_1(t)\\
&\quad y(t)=C_{11}x(t)\\
&R_1^2: \text{if } z_1(\text{t}) \text{ is } M_{11}^2, \text{ then}\\
&\quad \dot{x}(t)=(A_{12}+\Delta A_{12})x(t)+(B_{12}+\Delta B_{12})u_1(t)\\
&\quad y(t)=C_{12}x(t)\\
&R_2^1: \text{if } z_1(\text{t}) \text{ is } M_{21}^1, \text{ then}\\
&\quad \dot{x}(t)=(A_{21}+\Delta A_{21})x(t)+(B_{21}+\Delta B_{21})u_2(t)\\
&\quad y(t)=C_{21}x(t)\\
&R_2^2: \text{if } z_1(\text{t}) \text{ is } M_{21}^2, \text{ then}\\
&\quad \dot{x}(t)=(A_{22}+\Delta A_{22})x(t)+(B_{22}+\Delta B_{22})u_2(t)\\
&\quad y(t)=C_{22}x(t)
\end{aligned} \tag{29}$$

where,

$$A_{11}=\begin{bmatrix}-10 & -9.3\\ 0.01 & -1\end{bmatrix},\ B_{11}=\begin{bmatrix}1\\0\end{bmatrix},\ C_{11}=[0 \quad 1];$$

$$A_{12}=\begin{bmatrix}0 & 0.1\\ -32 & -4.5\end{bmatrix},\ B_{12}=\begin{bmatrix}1\\0.1\end{bmatrix},\ C_{12}=[0 \quad 1];$$

$$A_{21}=\begin{bmatrix}-10 & 10\\ 0.1 & -0.1\end{bmatrix},\ B_{21}=\begin{bmatrix}2\\1\end{bmatrix},\ C_{21}=[0 \quad 1];$$

$$A_{22}=\begin{bmatrix}0 & 0.8\\ -8 & -0.9\end{bmatrix},\ B_{22}=\begin{bmatrix}1.2\\1\end{bmatrix},\ C_{22}=[0 \quad 1];$$

$$D_{11}=D_{12}=\begin{bmatrix}-0.2 & 1\\ 1 & 0\end{bmatrix};\ D_{21}=D_{22}=\begin{bmatrix}0.01 & 1\\ 1 & 0\end{bmatrix};$$

$$E_{111}=E_{112}=\begin{bmatrix}1 & 0.2\\ 0 & 0\end{bmatrix};\ E_{121}=E_{122}=\begin{bmatrix}0.5 & 1\\ 0 & 0\end{bmatrix};$$

$$E_{211}=E_{212}=\begin{bmatrix}0\\0.6\end{bmatrix};\ E_{221}=E_{222}=\begin{bmatrix}0\\0.8\end{bmatrix};$$

$$F_{11}(t)=F_{12}(t)=F_{21}(t)=F_{22}(t)=\begin{bmatrix}\sin t & 0\\ 0 & \cos t\end{bmatrix}.$$

The membership functions chosen are as follows:

$$\mu_{M_{11}^1}(x_1)=\mu_{M_{21}^1}(x_1)=1-1/(1+e^{-4x_1}),$$

$$\mu_{M_{11}^2}(x_1)=\mu_{M_{21}^2}(x_1)=1/(1+e^{-4x_1}),$$

By carrying out computations for LMI (27), we obtain

$$P_2=\begin{bmatrix}0.2511 & 0.1240\\ 0.1240 & 0.3287\end{bmatrix};$$

$$L_{11}=[-23.6903 \quad 14.7707]^T;\ L_{21}=[8.2002 \quad 5.1755]^T;$$

$$L_{12}=[-53.7228 \quad 22.6392]^T;\ L_{22}=[-14.1609 \quad 13.0788]^T;$$

Upon choosing $\lambda_1=0.2, \lambda_2=0.8$, computing LMI (28) yields

$$P_1=\begin{bmatrix}0.0199 & 0.0223\\ 0.0223 & 0.0303\end{bmatrix};$$

$$K_{11}=[-374.9297 \quad -602.5408];$$

$$K_{12}=[-374.9297 \quad -602.5408];$$

$$K_{21}=[70.1912 \quad 112.8234];$$

$$K_{22}=[70.1912 \quad 112.8234];$$

let

$$\Omega_1=\left\{\hat{x}(t)\in R^n \left| \begin{aligned}&\hat{x}^T(t)[(A_{1i}-B_{1i}K_{1j})^T P_1\\ &+P_1(A_{1i}-B_{1i}K_{1j})\\ &+P_1L_{1i}C_{1j}C_{1j}^TL_{1i}^TP_1+\Pi_{1ij}]\hat{x}(t)<0\end{aligned}\right., \forall\hat{x}(t)\neq 0\right\}$$

$$\Omega_2=\left\{\hat{x}(t)\in R^n \left| \begin{aligned}&\hat{x}^T(t)[(A_{2i}-B_{2i}K_{2j})^T P_1\\ &+P_1(A_{2i}-B_{2i}K_{2j})\\ &+P_1L_{2i}C_{2j}C_{2j}^TL_{2i}^TP_1+\Pi_{2ij}]\hat{x}(t)<0\end{aligned}\right., \forall\hat{x}(t)\neq 0\right\}$$

Then

$\Omega_1 \bigcup \Omega_2 = R^2 \setminus \{0\}$. we design a switching law as follows:

$$\sigma(\hat{x}(t))=\begin{cases}1, \hat{x}(t)\in\Omega_1\\ 2, \hat{x}(t)\in\Omega_2\setminus\Omega_1.\end{cases}$$

In Matlab-Simulink carried computer simulation the initial condition $x(0)=[1,-2]^T$ has been adopted. This way the simulation results depicted in Figures 1 to 3 have been obtained. Figure 1 and Figure 2 show the system state and the observer state trajectories respectively, i.e. these represent the respective evolution time histories. The state observer error trajectories are depicted in Figure 3. These trajectories indicate that uncertain switching fuzzy system (29) is asymptotically stable via switching law (16), and also the observer error $e(t)$ asymptotically converges to zero at the same time.

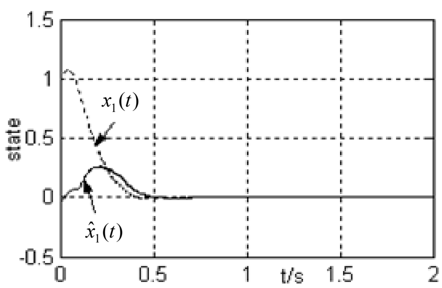

Fig. 1. The trajectory of state $x_1(t)$ and observer state $\hat{x}_1(t)$

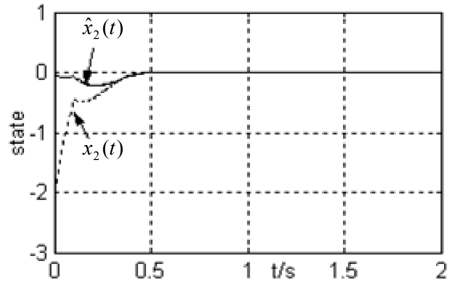

Fig. 2. The trajectory of state $x_2(t)$ and observer state $\hat{x}_2(t)$

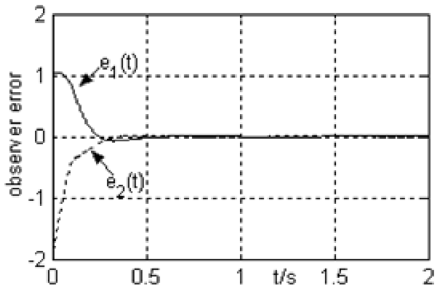

Fig. 3. The observer error trajectory

6. CONCLUSIONS

The problems of simultaneous output-feedback control and observer design for a class of uncertain switching fuzzy control systems has been investigated and a novel solution derived. Observers for subsystem states and a switching law are designed. A sufficient condition for asymptotic stability of the overall system in the closed-loop is derived. The simulation results demonstrate the feasibility of this design method, its effectiveness, and the achievable control performance of the designed system, which is superior to many results reported in the literature.

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