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DISCRETE-TIME OBSERVER FOR INDUCTION MACHINE IN PRESENCE MAGNETIC SATURATION

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Abstract: We are considering the problem of state estimation in induction motors. One key feature of this work is that the problem is dealt with based upon a model that accounts for the saturation effect of the magnetic circuit characteristic. Indeed, magnetic saturation cannot be ignored, especially when high power machines are considered. The above model is first based upon to analyse the machine observability. Then, a continuous-time high-gain observer is presented and discretized for omplementation purpose. Time-discretization is necessary because real-time implementation can only be done using digital equipments. The discretization task constitutes a crucial issue due to the nonlinear feature of the (continuous) observer. It is coped with using the Taylor-Lie method and the (discrete-time) observer thus obtained is validated experimentally using an asynchronous motor of 7.5 KW.

1. INTRODUCTION

When controlling induction machines one needs measurement of electromagnetic and mechanical variables (voltages, currents, flux, speed, position, etc). For some variables (e.g. stator voltages and currents), there exist reliable and not too expensive sensors that provide sufficiently accurate measures. This is not the case for other variables such as the rotor flux. Then, observers are resorted to get on-line estimates of the variables which are not accessible to measurement. The first observers (see e.g. Lubineau et al, 1999) were developed based on simplified assumptions namely, linear magnetic characteristics and constant (or slowly varying) rotor speed. Under these assumptions, the model of the induction motor becomes linear and, therefore, observability analysis and observer design may be dealt with using standard linear theory tools (e.g. pole placement design, Luenberger and Kalman observers). Interesting contributions came later proposing nonlinear observers developed without supposing a constant rotor speed (see e.g. De Leon et al, 2001). The proposed observers have been designed using different approaches such as high gain, sliding mode and dynamic state feedback. However, even in these contributions, the characteristics of the machine magnetic circuit are still supposed to be linear. In effect, this assumption is only valid when the machine operates close its nominal flux value. But, a constant-flux operation-mode cannot be optimal when large speed variations are needed. To achieve high observation performances, regardless the machine operation mode, the observer design should be based on a model that accounts for the nonlinear feature of the machine magnetic circuit. This has been done in few previous works, see e.g. (Krzeminski et al, 1993) and (Ouadi et al, 2005). However, the performances of the proposed observers have only been evaluated through simulation. Given the complexity of the involved observers, a real-time validation can only be done using digital equipments. To this end, a discrete-time version of the (continuous-time) observer of interest has to be developed. Time-discretization is a crucial issue when nonlinear dynamics are involved as this is the case for the nonlinear observer in (Ouadi et al, 2005). Furthermore, in the particular case of asynchronous induction motors, the limitation on the sampling period does not come from the involved digital equipment (used computer) but rather from the machine power supply inverter. Specifically, the sampling frequency is determined by the inverter commutation frequency.

In this paper, the focus is made on time-discretization and experimental validation of the nonlinear observer of (Ouadi et al, 2005). The latter has been developed using a model that accounts for the saturation effect in the machine magnetic characteristics (Ouadi et al, 2004). Observer discretization could be performed using standard methods such as those of Heun or Range-Kutta (Elfadili et al, 2006). But these have been discarded as they all share the disadvantage that the quality of approximation deteriorates rapidly when the sampling period increases. The discretization could also be coped with considering input-output models involving Volterra-series (Elfadili et al, 2006). But this too has been abandoned as it leads to complex models with a large number of parameters. More interesting techniques are those of Euler approximation, Carleman linearization and Taylor-Lie expansion (Monaco et al, 1993), (Elfadili et al, 2006), Svoronos et al, 1994). The first one is simple but requires high sampling rates. The second involves a compromise between model complexity and accuracy, but it is only appropriate for systems of weak dimension. The technique of Taylor-Lie leads to a better accuracy/complexity compromise and requires low computation time (Elfadili et al, 2006). It is therefore resorted, in the present work, to get a discrete-time version of the high-gain observer of Ouadi et al (2005). The performances of the discrete-time observer are experimentally evaluated using an asynchronous machine of 7.5KW.

The paper is organised as follows: the induction motor model is presented in Section 2; model observability is analysed in Section 3; the continuous high-gain observer is presented in Section 4 and its discrete-time version is developed in Section 5, the observer experimental validation is described in Section 6. A conclusion, a reference list end the paper.

2. INDUCTION MOTOR MODEL

In (Ouadi et al, 2004), a new model has been developed and experimentally validated for the considered induction motor. Its originality is that it accounts for the saturation effect in the machine magnetic characteristics (fig 1). It is defined by the following state-space representation:

$$\dot{x} = f(x,u) \overset{def}{=} \left[f_1(x,u) \quad \ldots \quad f_5(x,u) \right]^T$$
$$y = h(x) \overset{def}{=} \left[h_1(x) \quad h_2(x) \quad h_3(x) \right]^T \qquad (1)$$

With

$$x = [x_1, x_2, x_3, x_4, x_5]^T = [i_{s\alpha}, i_{s\beta}, \phi_{r\alpha}, \phi_{r\beta}, \Omega]^T$$
$$u = [u_{s\alpha}, u_{s\beta}]^T, \; y = [x_1, x_2, x_5]^T \qquad (2)$$

$$\begin{bmatrix} f_1(x,u) \\ f_2(x,u) \\ f_3(x,u) \\ f_4(x,u) \\ f_5(x,u) \end{bmatrix} = \begin{bmatrix} -a_2 i_{s\alpha} + \delta(t)\phi_{r\alpha} + a_3 p\Omega\phi_{r\beta} + a_3 u_{s\alpha} \\ -a_2 i_{s\beta} - a_3 p\Omega\phi_{r\alpha} + \delta(t)\phi_{r\beta} + a_3 u_{s\beta} \\ a_1 i_{s\alpha} - L_{seq}\delta(t)\phi_{r\alpha} - p\Omega\phi_{r\beta} \\ a_1 i_{s\beta} - L_{seq}\delta(t)\phi_{r\beta} + p\Omega\phi_{r\alpha} \\ \frac{p}{J}(\phi_{r\alpha} i_{s\beta} - \phi_{r\beta} i_{s\alpha}) - \frac{T_L}{J} \end{bmatrix} \qquad (3a)$$

$$\begin{bmatrix} h_1(x) \\ h_2(x) \\ h_3(x) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_5 \end{bmatrix} \qquad (3b)$$

Where:

. δ is a varying parameter that depends on the machine magnetic state, i.e.:

$$\delta = W(\Phi_r) \qquad (4)$$

where $W(.)$ is highly nonlinear function that has been given a polynomial approximation; specifically:

$$W(\Phi_r) = b_0 + b_1\Phi_r + \ldots + b_p\Phi_r^p \qquad (5)$$

The involved coefficients have been experimentally identified in (Ouadi et al, 2004) using Fig 1.

. Φ_r denotes the amplitude of the (instantaneous) rotor flux, denoted ϕ_r. Consequently, one has:

$$\Phi_r = \sqrt{\phi_{r\alpha}^2 + \phi_{r\beta}^2} \qquad (6)$$

where $\phi_{r\alpha}, \phi_{r\beta}$ denote the rotor flux $\alpha\beta$-components.

. ($i_{s\alpha}, i_{s\beta}$) and ($u_{s\alpha}$, $u_{s\beta}$) are the $\alpha\beta$-components of the stator current and stator voltage, respectively

. Ω represents the motor speed,

. R_s, R_r denote the stator and rotor resistances; T_L represents the load torque; p is the number of pole pairs; L_{seq} is the equivalent inductance (of both stator and rotor leakage) as this is seen from the stator,

. $a_1 = R_r$, $a_2 = \dfrac{R_s + R_r}{L_{seq}}$, $a_3 = \dfrac{1}{L_{seq}}$

The numerical values of the model parameters are those of (Ouadi et al, 2004) where the model is experimentally validated using an induction motor of 7.5 KW power.

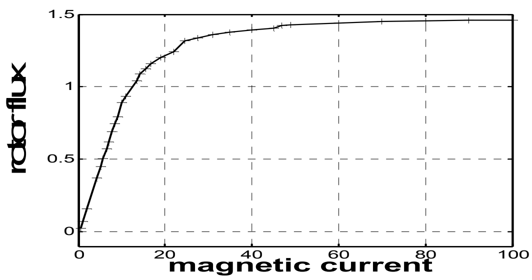

Fig 1. Magnetic characteristic experimentally obtained in (Ouadi et al, 2004) for a 7.5 induction motor. It represents the variation of the rotor flux norm Φ_r (Wb) in function of the magnetic current I_μ (A),

3. MODEL OBSERVABILITY ANALYSIS

In the rest of the paper, it is supposed that the stator currents and rotor speed are measurable, but the rotor flux is not. The observability of the induction machine will now be analysed, based on the model of Section 2, using Theorem 3.1, presented hereafter, due to (Chapra et al, 1988). In that theorem the following notations are used:

. $L_f g$: Lie derivative of a function $g : IR^n \to IR$ along the vector field $f : IR^n \to IR^n$ i.e.

$$L_f g(x) = \sum_{i=0}^{n} f_i(x) \frac{\partial g}{\partial x_i}(x) \qquad (7)$$

$$. L_f^m g = \underbrace{L_f(\ldots(L_f g))}_{m \, times} \quad \text{and} \quad L_f^0 g = g \qquad (8)$$

<u>Theorem 3.1</u> (The rank condition). The system (1) is said to be (locally) observable in the vicinity of a given point $x_0 \in IR^n$, if there exist a neighborhood U of x_0 and a set of q integers, denoted $\{l_1, l_2, \ldots, l_q\}$, such that:

$$\sum_{k=1}^{q} l_k = n \qquad (9)$$

and, for all $x \in U$, the matrix O is nonsingular, with:

$$O = \left[L_f^0\left(\frac{\partial h_1(x)}{\partial x}\right) \;\cdots\; L_f^{l_1-1}\left(\frac{\partial h_1(x)}{\partial x}\right) \;\Big|\; L_f^0\left(\frac{\partial h_2(x)}{\partial x}\right) \;\cdots\; L_f^{l_2-1}\left(\frac{\partial h_2(x)}{\partial x}\right) \;\Big|\; \cdots \;\Big|\; L_f^0\left(\frac{\partial h_q(x)}{\partial x}\right) \;\cdots\; L_f^{l_q-1}\left(\frac{\partial h_q(x)}{\partial x}\right)\right]^T \quad (10)$$

Let us apply the above theorem to the model (1)-(3) with the following parameters: $l_1 = 2,\ l_2 = 2,\ l_3 = 1$. Then, the observability matrix of Theorem 3.1 becomes:

$$O = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -a_2 & 0 & \delta + \frac{\partial \delta}{\partial x_3} x_3 & a_3 p x_5 + \frac{\partial \delta}{\partial x_4} x_3 & a_3 p x_4 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -a_2 & -a_3 p x_5 + \frac{\partial \delta}{\partial x_3} x_4 & \delta + \frac{\partial \delta}{\partial x_4} x_4 & -a_3 p x_3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

It follows from (6) that the derivatives of δ , involved in (11), can be expressed as follows:

$$\frac{\partial \delta}{\partial x_3} = \frac{d\delta}{d\Phi_r} \frac{x_3}{\sqrt{x_3^2 + x_4^2}}, \qquad \frac{\partial \delta}{\partial x_4} = \frac{d\delta}{d\Phi_r} \frac{x_4}{\sqrt{x_3^2 + x_4^2}} \quad (12)$$

Now we are in a position to establish the following observability result.

<u>Proposition 3.1</u>. *The induction motor represented by model (1)-(6), is (locally) observable in the rank sense* □

<u>Proof</u>. According to Theorem 3.1, the induction motor would be (locally) observable provided that the matrix O is nonsingular. The determinant of such a matrix is:

$$det(O) = \left(\delta + \frac{\partial \delta}{\partial x_3} x_3\right)\left(\delta + \frac{\partial \delta}{\partial x_4} x_4\right) - \left(a_3 p x_5 + \frac{\partial \delta}{\partial x_4} x_3\right)\left(\frac{\partial \delta}{\partial x_3} x_4 - a_3 p x_5\right) \quad (13)$$

Using (12), the determinant can be given the following compact form:

$$det(O) = \delta^2 + \Phi_r \delta \frac{d\delta}{d\Phi_r} + a_3^2 p^2 x_5^2 \quad (14)$$

It is clear that $det(O)$ will be different of zero if $\delta \frac{d\delta}{d\Phi_r} \geq 0$, i.e. if the function $y(\Phi_r) \overset{def}{=} \frac{1}{2}\delta^2$ is not decreasing. Using experimental data, the function $y(\Phi_r)$ has been plotted in Fig 2. From this figure it is clearly seen that the function $y(\Phi_r)$ is actually nondecreasing. Proposition 3.1 is thus proved □□□

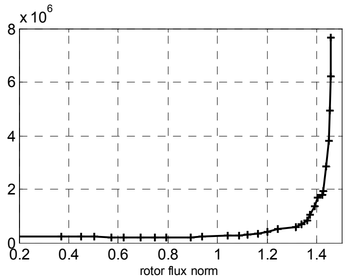

Fig. 2. Variation of $y(\Phi_r) = \frac{1}{2}\delta^2$ in function of Φ_r

4. CONTINUOUS-TIME HIGH-GAIN OBSERVER

In (Ouadi et al, 2005) a high-gain type observer has been developed based on the model (1)-(6) and supposing that the stator currents ($i_{s\alpha}, i_{s\beta}$) and the rotor speed (Ω) are accessible to measurement. The proposed observer has been shown to possess nice stability properties. However, it has not been possible to experimentally evaluate its performances mainly because no discrete-time version was available. The discretization task turned out to be a crucial issue, due to the highly nonlinear nature of the observer dynamics. Such an issue constitutes a major motivation of the present work. It is dealt with in Section 5. For convenience, let us make here a brief presentation of the continuous-time observer of (Ouadi et al, 2005).

Notice that, in the (α,β)-frame, the model (1)-(6) can be given the following more condensed state-affine representation:

$$\dot{x} = A(\omega)x + \varphi(u, y, x) \qquad y = Cx \quad (15)$$

with:

$$x = \left[I_s^T, \Psi_r^T\right]^T \quad I_s = [i_{s\alpha}, i_{s\beta}]^T, \ \Psi_r = [\phi_{r\alpha}, \phi_{r\beta}]^T$$
$$y = [i_{s\alpha} \quad i_{s\beta}]^T, \ u = [u_{s\alpha} \quad u_{s\beta}]^T \quad (16)$$

where $A(\omega)$ and $\varphi(u, y, x)$ are continuous functions defined by:

$$A(\omega) = \begin{bmatrix} 0 & 0 & \delta_L & a_3\omega \\ 0 & 0 & -a_3\omega & \delta_L \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0_2 & F(\omega) \\ 0_2 & 0_2 \end{bmatrix} \quad (17)$$

$$\varphi(y, u, x) = \begin{bmatrix} -a_2 I_s + \delta_v \Psi_r + \frac{1}{L_{seq}} u \\ a_1 I_s - k L_{seq} \delta \Psi_r - \omega \Im_2 \Psi_r \end{bmatrix} \quad (18)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \Im_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad (19)$$

In (17), the positive parameter δ_L denotes the constant value taken by the (varying) parameter $\delta(t)$ when the machine operates in the linear part of its magnetic characteristic. Then, we can write:

$$\delta(t) = \delta_L + \delta_v \tag{20}$$

where δ_v denotes the varying component of $\delta(t)$. That is, $\delta_v = 0$ whenever the motor operates in the linear part of its magnetic characteristic.

With the above notations, the proposed high-gain observer can simply be formulated as follows (see Ouadi et al, 2005):

$$\dot{\hat{x}} = A(\omega)\hat{x} + \varphi(u, y, \hat{x}) + M(\omega)\left(C\hat{x} - y\right) \tag{21}$$

$$\hat{\delta} = W(\hat{\Phi}_r) \tag{22}$$

With

$$\hat{\Phi}_r = \sqrt{\hat{\phi}_{r\alpha}^2 + \hat{\phi}_{r\beta}^2} \tag{23}$$

$$M(\omega) = \Gamma^{-1}(\omega)\Delta_\theta^{-1}K \tag{24}$$

$$\Gamma = \begin{bmatrix} I_{d2} & 0_2 \\ 0_2 & F(\omega) \end{bmatrix}, \Delta_\theta = \begin{bmatrix} \frac{1}{\theta} I_{d2} & 0_2 \\ 0_2 & \frac{1}{\theta^2} I_{d2} \end{bmatrix} \tag{25}$$

$$K = \begin{bmatrix} k_1 I_{d2} \\ k_2 I_{d2} \end{bmatrix} \tag{26}$$

Where:

- I_{d2} and 0_2 denote, respectively, the identity and null matrices of dimension (2×2)
- k_1 and k_2 are positive real constants chosen such that the matrix $\overline{A} - KC$ *is* Hurwitz with:

$$\overline{A} = \begin{bmatrix} 0_2 & I_{d2} \\ 0_2 & 0_2 \end{bmatrix}$$

In (Ouadi et al, 2005) it has been formally shown that the observer (21)-(26) is (locally) stable with a well defined attraction region. Its supremacy over to standard observers (those obtained from models that disregard the magnetic saturation effect) has there been demonstrated by simulation.

5. DISCRETIZATION OF THE OBSERVER

5.1 Discretization method

Compared to standard discretization methods, the Taylor-Lie technique leads to a better accuracy/complexity compromise and low calculation time (Elfadili et al, 2006). We first present the principle of this technique.

Consider a continuous-time model of the form:

$$\dot{x} = f(x, u) \tag{27}$$

where $x \in IR^n$, $u \in IR^m$ and $f : M \times IR^m \to IR^n$ for some open set $M \subset IR^n$. The control signal $u(t)$ is supposed to be generated through a ZOH, i.e.:

$$u(kT_e + \tau) = u(kT_e), \forall k \in IN, 0 \le \tau < T_e \tag{28}$$

Where $T_e > 0$ denotes the sampling period. To the above continuous model, we associate discrete-time models of the form:

$$x_d(k+1) = F^{T_e}(x_d(k), u(k)) \tag{29}$$

Where $x_d(k) \in M$ and $F^{T_e} : M \times IR^m \to IR^n$ is parameterized by the sampling period T_e. The discrete model is fully characterized by the function F^{T_e} which depends on the discretization method. An appropriate discretization method is one that yields a discrete model (29) able to generate an input-state behavior close enough to the continuous model behavior, for sufficiently small values of T_e. This requirement is better formulated in the following definition .

Definition 5.1 (Monaco and Normand-Cyrot, 1993). Consider the continuous-time model (27) and its discrete-time version (29), obtained with some sampling period T_e. The discrete model (29) is said to be an exact discretization of (27) if the following statement holds:

$$x_d(0) = x(0) \Rightarrow x_d(k) = x(kT_e) \quad (\forall k \in IN) \quad \square$$

Time-discretization of the system (27) by the Taylor-Lie method is described by the following proposition (Monaco and Normand-Cyrot, 1993):

Proposition 5.1. Let the function F^{T_e} in (29) be defined as follows:

$$\begin{aligned} F^{T_e}(x_d(k), u(k)) &= e^{T_e L_{f(x_d(k),u(k))}}(I_d)(x_d(k)) \\ &= e^{T_e L_{f(x_d(k),u(k))}}(x_d(k)) \end{aligned} \tag{32}$$

with :

$$e^{L_f} = \sum_{p \ge 0} \frac{1}{p!} L_f^p := 1 + L_f + \frac{1}{2!} L_f^2 + \ldots + \frac{1}{p!} L_f^p + \ldots \tag{33}$$

Where I and I_d denote respectively the identity function and operator.

Then, there exists a $T_{e0} > 0$ such that for any $T_e \le T_{e0}$, the model (29) constitutes an exact discretization of the continuous-time model (27) □

For practical applications, it is necessary to limit the development (33) to some order, say *N*. Then, expression (32) can be formulated as follows:

$$e^{T_e L_{f(x_d(k),u(k))}}(x_d(k)) :=$$

$$\left(1 + T_e L_f + \frac{T_e^2}{2!} L_f^2 + \ldots + \frac{T_e^N}{N!} L_f^N\right)(x_d(k)) + o(T_e^{N+1}) \tag{34}$$

where the quantity $o(T_e^{N+1})$ accounts for higher order terms and vanishes as rapidly as T_e^{N+1}, when T_e tends to zero. The usual practice consists in choosing T_e sufficiently small so that the above quantity can be neglected. The discrete-time model (29) can then be given the following compact form:

$$x_d(k+1) := F^{T_e}(x_d(k),u(k)) = \sum_{j=0}^{N} \frac{T_e^{\,j}}{j!} \overset{(j)}{x}(kT_e) \qquad (35)$$

where

$$\overset{(j)}{x}(kT_e) = L^j f_{(x_d(k),u(k))} \qquad (36)$$

Remark 5.1. In the particular case where $N = 1$, the Taylor method reduces to the Euler method.

5.2 Discretization of the high-gain observer (21)-(26)

Equation (21) suggests that the high gain observer can be rewritten as follows:

$$\dot{\hat{x}} = g(\hat{x}, v) \qquad (37)$$

with:

$$\hat{x} = [\hat{i}_{s\alpha}, \hat{i}_{s\beta}, \hat{\phi}_{r\alpha}, \hat{\phi}_{r\beta}]^T = [\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4]^T \qquad (38a)$$

$$v = [u_{s\alpha}, u_{s\beta}, i_{s\alpha}, i_{s\beta}, \Omega]^T \qquad (38b)$$

$$g(\hat{x}, v) = [g_1(\hat{x}, v), g_2(\hat{x}, v), g_3(\hat{x}, v), g_4(\hat{x}, v)]^T \qquad (39)$$

$$g_1(\hat{x}, v) = (-a_2 + k_1\theta)\hat{i}_{s\alpha} + \hat{\delta}\hat{\phi}_{r\alpha} + a_3\omega\hat{\phi}_{r\beta} + a_3 u_{s\alpha} - k_1\theta i_{s\alpha} \qquad (40)$$

$$g_2(\hat{x}, v) = (-a_2 + k_1\theta)\hat{i}_{s\beta} - a_3\omega\hat{\phi}_{r\alpha} + \hat{\delta}\hat{\phi}_{r\beta} + a_3 u_{s\beta} - k_1\theta i_{s\beta} \qquad (41)$$

$$g_3(\hat{x}, v) = (a_1 + \delta_L \frac{k_2\theta^2}{\delta_L^2 + (a_3\omega)^2})\hat{i}_{s\alpha} - (a_3\omega \frac{k_2\theta^2}{\delta_L^2 + (a_3\omega)^2})\hat{i}_{s\beta} - kL_{seq}\hat{\delta}\hat{\phi}_{r\alpha} - \omega\hat{\phi}_{r\beta} - \frac{k_2\theta^2}{\delta_L^2 + (a_3\omega)^2}(\delta_L i_{s\alpha} - a_3\omega i_{s\beta}) \qquad (42)$$

$$g_4(\hat{x}, v) = (a_3\omega \frac{k_2\theta^2}{\delta_L^2 + (a_3\omega)^2})\hat{i}_{s\alpha} + (a_1 + \delta_L \frac{k_2\theta^2}{\delta_L^2 + (a_3\omega)^2})\hat{i}_{s\beta} + \omega\hat{\phi}_{r\alpha} - kL_{seq}\hat{\delta}\hat{\phi}_{r\beta} - \frac{k_2\theta^2}{\delta_L^2 + (a_3\omega)^2}(a_3\omega i_{s\alpha} - \delta_L i_{s\beta}) \qquad (43)$$

Applying Proposition 5.1 to (37) with $T_e = 1ms$ N=4, one obtains the following discrete-time observer:

$$\hat{x}_{di}(k+1) = \hat{x}_{di}(k) + T_e L_f \hat{x}_{di}(k) + \frac{T_e^{\,2}}{2!} L_f^2 \hat{x}_{di}(k) + \frac{T_e^{\,3}}{3!} L_f^3 \hat{x}_{di}(k) + \frac{T_e^{\,4}}{4!} L_f^4 \hat{x}_{di}(k) \qquad (\mathrm{i} = 1, ..., 4) \quad (44)$$

where $\hat{x}_{di}(k)$ denotes the *i-th* component of the estimated state vector $\hat{x}_i$ (at time kT_e) and:

$$L_f \hat{x}_j = \sum_{i=1}^{4} f_i(\hat{x}) \frac{\partial \hat{x}_j}{\partial \hat{x}_i}(\hat{x}) \qquad (45)$$

$$L_f^r \hat{x}_j = \sum_{i=1}^{4} f_i(\hat{x}) \frac{\partial L_f^{r-1} \hat{x}_j}{\partial \hat{x}_i}(\hat{x}) \quad (\mathrm{j} = 1, ..., 4) \qquad (46)$$

Using the definitions (39)-(43) of the g_i's, the terms in the right side of (44) are computed according to the formulas (45)-(46). However, the involved calculations are too long to fit the allowed space.

6. EXPERIMENTAL VALIDATION OF THE DISCRETE-TIME OBSERVER

In this section, the performances of the discrete-time observer (44) are experimentally illustrated. First, the used experimental bench is described.

6.1 Experimental bench

The experimental bench is that of the Automatic Control lab (LAG) in Grenoble (France).it have the following main characteristics:

. the motor is a three-phase squirrel-cage asynchronous machine of 7.5 KW,

. the power converter is a three-phase inverter of 35 KVA,

. the load is a DC motor of 7.5 KW with the possibility to regulate either its speed or its couple

. all signals are measured and processed with a DSP card of the type TMS320C30.

6.2: Experimental evaluation of the high gain observer

The machine work in open-loop and is submitted to a specific stator voltage input (see fig 3). The applied input signal is given a profile that enforces the machine to operate successively in the linear part as well as in the nonlinear part of its magnetic characteristic. Specifically, the machine operates in the linear part ($\Phi_r \approx 0.4\,Wb$) over the interval $[0, 1.75\,s]$ and operates in the saturation region ($\Phi_r \approx 1.2\,Wb$) over $[1.75\,s, 4\,s]$ (see figs 4).

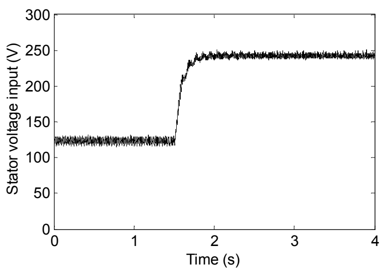

Fig 3: Stator voltage input

As mentioned In Section 4, the parameters of the design model (1)-(6) are given the numerical values of (Ouadi et al, 2004). Those of the (discrete-time) observer are given the following values which proved to be quite convenient: $k_1 = 0.1$, $k_2 = 1.5$, $\theta = 1500$. In all experiments, the initial values of the observed variables are different from the true values of the variables (Fig 5 and 7).

Figure 4 compares the estimated and the measured rotor flux norm. It is seen (from the lower curve) that the estimation error vanishes rapidly. This is more clearly illustrated by fig 5 which shows the machine and observer responses during the first 50 ms. Notice that, due to the used sensor, the flux norm measurement becomes noisier for high flux values.

Similar results are obtained for the stator current estimation (figs 6 and 7).

In summary, it follows from figures 4 to 7 that the observer (44) performs well in the linear region of the magnetic characteristic (time-interval $[0, 1.75\,s]$) as well as in the nonlinear region (time-interval $[1.75\,s, 4\,s]$). The state estimates converge to their true values after a transient period that lasts less than 5ms.

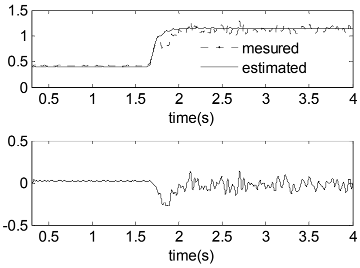

Fig 4: Rotor flux norm (Wb) estimation. Upper: real and observed flux norm; lower: estimation error

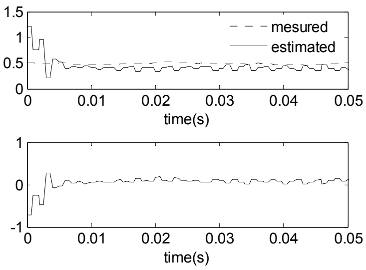

Fig 5: Zoom on the curves of Fig 4 over the interval $[0, 50\ ms]$.

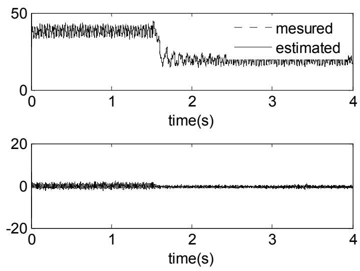

Fig 6: Stator current norm (A) estimation. Upper: measured and estimated current norm; lower: estimation error

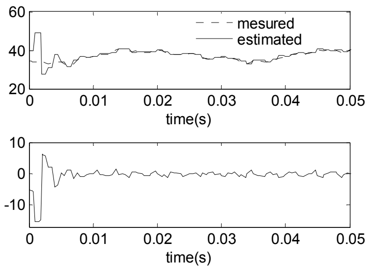

Fig 7: Zoom on the curves of Fig 6 over the time interval $[0, 50\ ms]$.

6. CONCLUSION

In this paper, we have considered the problem of obtaining discrete-time observers for induction motors. The originality lies in the fact that we seek an observer that accounts for the saturating feature of the machine magnetic characteristic. It is crucial to take into account such feature when seeking high control performances. Indeed, optimal operating conditions may correspond to large values of the flux and these are located in the saturating part of the magnetic characteristic. First, we have demonstrated that the machine model is observable in the rank condition sense. Then, the Taylor-Lie discretization method has been applied, up to the 4th order, to get a discrete-time observer based on the work of (Ouadi et al, 2005). The observer thus obtained is experimentally validated using a 7.5 KW machine. It is demonstrated that the observer estimates well the state variables, whatever the operation conditions (linear as well as saturating).

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