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# Optimal LQI Synthesis for Speed Control of Synchronous Actuator under Load Inertia Variations

**S. Carrière * S. Caux * M. Fadel ***

* *Université de Toulouse; LAPLACE; CNRS, INPT, UPS; 2 rue Camichel, 31071 Toulouse Cedex France ( e-mail: sebastien.carriere@laplace.univ-tlse.fr).*

**Abstract:** Direct drives applications are more and more common and rise up the control weakness to parameters variation. New method have been created but are most of time complex to tune and need powerful processor. In this paper, state feedback controllers optimized by Linear Quadratic principle are shown up. These methods achieve cost requirements due to constant gain coefficients. Methods tune criterion to let the closed loop stable and unsensitive to load variation. The first one uses an iterative algorithm on few tuned parameter. The last one blends poles placement and Linear Quadratic principle to achieve a fast synthesis method. Experimental results have been obtained taking into account internal current control and inverter limitation.

## 1. INTRODUCTION

In lot of industrial domains, speed control of a load is still a problem mainly due to load parameter variations. Moreover it is commonly known that elasticity phenomenon must be taken into account in the transmission model between the actuator and its load. In different applications such as: crusher, drilling system, re-winder and paper machine, elasticity and inertia variation of the load are of paramount importance. For example, in machine tools operation, railway traction and rolling mill drives, the mechanical load varies considerably under certain operating conditions. Permanent Magnet Synchronous Machine (PMSM) is chosen in many industrial drive applications and the mechanical transfer elements (shafts, gears, friction, backlash, etc) introduce imperfections that must be considered in dynamic speed controllers. The control problem goals are in one hand to keep the closed loop system stable whatever the load parameters are and on the other hand to keep some performances as constant as possible (overshoot and response time). In the case studied a two masses model is used for the mechanical part under large load inertia variation.

In industry the controllers must be simple to implement. To be easily tuned on different systems, the synthesis has to take into account parameter variations. A classical polynomial PID approach regains interest for few years to obtain a self tuned PID controller Ang et al. [2005]. Using a PID with filters, anti windup and no zero structures keep the system stable but is not sufficient for the performances robustness point of view. In another way, using more information on the device and the state space formulation allow the designer to define robust state feedback controllers. The designer must use optimization principles and more performing pole placements. But, most of robust control theories provide high order controllers Chilali and Gahinet [1996]. To be implemented such controller must be reduced. But reduction of such controller is not so easy and provide the lost of the initial robustness. The mathematical approach must be structured depending on the parameter variation and closed loop desired behavior Kajiwara et al. [1999]. Moreover these methods use mathematical and optimization analyzes providing more problems to be understood in non academic research field.

The approach developed here, tries to obtain a compromise between complexity of the synthesis and performances obtained. In fact, using Linear Quadratic criterion for a state space feedback synthesis is well known to keep stable a controlled device. To take into account consideration of performances, the parameters used in the criterion must be accurately specified Anderson and Moore [1989]. Theory shows it is possible to tune the LQ optimization for a robust poles placement. The proposed approach links the state feedback to the LQ criterion and the poles placement taking into account actuators limitations and inertia variation.

Section 2 of this paper presents the two masses and elasticity model focusing on the inertia variation problems. The state space representation to be used in the state feedback speed controller is presented too. Section 3 presents the controllers synthesis with an integral term added. An optimal PID controller and different state feedback are presented for comparisons purposes. Note that state feedback approaches are used in speed control and not in a position control problem. The proposed approach for state feedback controller design is detailed for a simple four degree of freedom LQ criterion. To keep specified closed loop performances, the final LQ approach with poles placement is exposed. Validation results shown in section 4 are conduced with an actual synchronous actuator with bounded inertia variations on its driven load. The controller designed does not only let the closed loop system response stable, but also let constant the requested performances whatever the load inertia variation is.

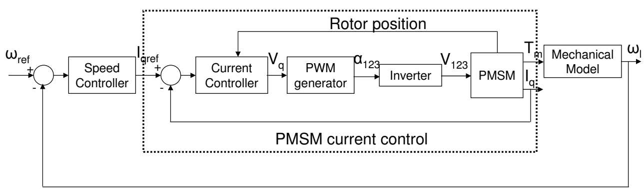



Fig. 1. PMSM driven in current for torque control

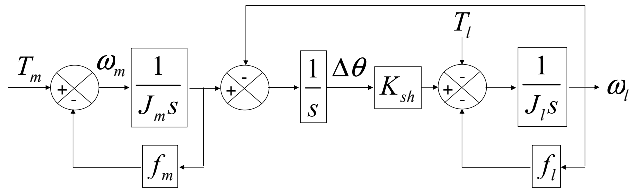


Fig. 2. Two mass model for speed mechanical load control

## 2. ACTUATOR AND LOAD MODELING

Permanent Magnet Synchronous Motor (PMSM) is the most used drive in machine tool servos and modern speed control applications due to its desirable features (compact structure, high air-gap flux density, high power density, high blocked torque). The influence of inverter in such control is commonly neglected if an accurate current control and torque control is effective to contribute to the accurate velocity on the load side.

To study the complete actuator elements, simulation-based synthesis are conduced with the structure presented on (Fig. 1) for a classic current control in d,q rotating frame.

The current and speed controllers are in cascade, so even if the internal closed loop is neglected, some assumptions must stay verified (speed dynamic lower than current one, torque reference lower than current limitation of the inverter and motor size). The following controllers tuning takes into account all this phenomena.

### *2.1 Two masses model*

Considering effective the torque control, the load speed control is only link to the mechanical behavior. In this paper, a two-mass system is considered for mechanical part. The PMSM and load are connected through a shaft. The mechanical simulation model is presented in (Fig. 2) corresponding to the experimental device (Fig. 3). Indices $l$ and $m$ represent load and motor parameters respectively. The joint and a long axis introduce a torque shaft $T_{sh}$ depending on the position difference and the elasticity $K_{sh}$. On speed control scheme, position can not be employed cause it leads to an uncontrollable model. Position difference is a finite variable so with known initial condition, it can be calculated by speeds difference integration. This points out the difference between position and speed control scheme.

Zero, one or two wheels can be mounted on the load axis to vary load inertia $J_l$. Controlling the brake can introduce a disturbance torque $T_l$. This experimental device allows

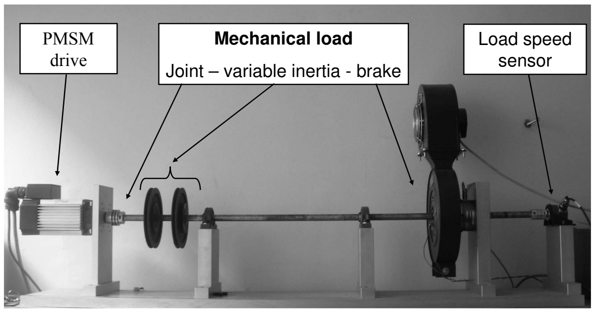



Fig. 3. Experimental setup with load inertia variation

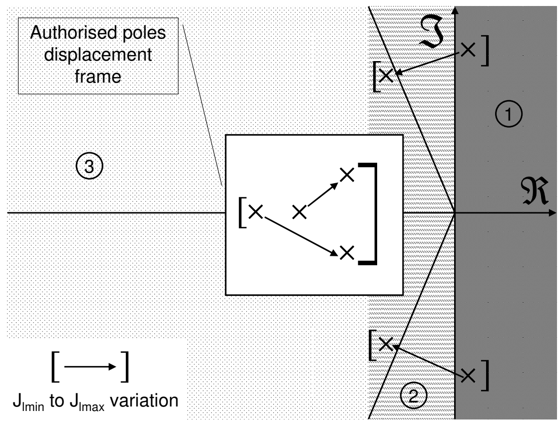



Fig. 4. Generic closed loop poles variation

specific disturbances which are characteristic of most of industrial issues. Controlling the load velocity $\omega_l$ through the motor torque $T_m$ consists in controlling transfer function (1) with the inertia value $J_l \in \left[J_{lmin} \;\; J_{lmax}\right]$.

$$\begin{cases} d_0 = K_{sh}(J_l + J_m) \\ d_1 = f_m f_l + J_l K_{sh} + J_m K_{sh} \\ d_2 = 2 J_l f_m \\ d_3 = J_l J_m \end{cases}$$

$$\frac{\omega_l(s)}{T_m(s)} = \frac{K_{sh}}{d_0 + d_1 s + d_2 s^2 + d_3 s^3} \tag{1}$$

Nevertheless, the inertia variation bring modification on system poles making the system behavior different in respect to inertia as shown in (Fig. 4). This problem does not resume as a stabilization problem (avoid zone1) but consist also to a robust problem limiting time response variation (reducing zone3) , overshoot (avoid zone2) and disturbance rejection.

Variations showed on (Fig. 4) reveal that two high modulus complex poles have their real part tending to positive value when inertia is decreasing. So to avoid any instability on inertia variation, to calculate controller, minimal inertia is chosen.

PID synthesis can cancel the instability phenomena but is limited for robust performances and a robust pole placement is not sufficient to be stable, fast and robust Shin and Huh [2000]

### 2.2 State space formulation

To use optimization algorithms based on mathematical principles a state space representation is requested. For the speed control problem, the speed behavior of both the load and machine must be used in the state vector. In fact, in speed control, using positions leads to an uncontrollable formulation. Thus, the third variable is position difference integrated from speeds difference (Fig. 2)

Then, the state vector is defined to be : $X = \left[\omega_m \ \omega_l \ \Delta\theta\right]^T$

And the 3-dimensional state space representation is given in (2) where $J_l$ is the varying load inertia.

$$\begin{cases} \dot{X} = \underbrace{\begin{pmatrix} -\frac{f_m}{J_m} & 0 & -\frac{K_{sh}}{J_m} \\ 0 & -\frac{f_l}{J_l} & \frac{K_{sh}}{J_l} \\ 1 & -1 & 0 \end{pmatrix}}_{A} X + \underbrace{\begin{pmatrix} \frac{1}{J_m} \\ 0 \\ 0 \end{pmatrix}}_{B} T_m \\ y = \underbrace{\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}}_{C} X \end{cases} \tag{2}$$

The controller must force the closed loop system to be faster as possible and keep it unsensitive to load variation. To elaborate the controller design, different strategies are possible and the load variation has to be taken into account. Robust control methods commonly add weight functions or inequalities to this system and try to provide best controller. The problem is solved with heavy computational programs and expert analyzes ($H_\infty$, LMI...). The deduced controller is not satisfactory and the methods to solve the complex problem are not self-evident therefore lighter formulation are applied here as the following one.

### 2.3 Linear Quadratic formulation

Instead of robust approach, or a direct robust pole placement, a Linear Quadratic method is performed in this paper. The criterion (3) is used and must be minimized with a state feedback $T_m = -KX$. This method always provides a stable closed loop system with wide phase margin Ferreti et al. [1998] and a simple controller to implement.

$$J = \int_0^\infty (X^T Q X + R T_m^2) dt \tag{3}$$

The controller gains $K$ are obtained by computing (4). Where $Q$ and $R$ are matrices to be defined and $P$ is the positive solution of Riccati equation (5).

$$K = R^{-1} B^T P \tag{4}$$

$$PA + A^T P - PBR^{-1} B^T P + Q = 0 \tag{5}$$

Equation (5) is well known and it is easy to find programs to solve this linear problem. The optimization is done by a function as *lqr()* in Matlab© or open source code.

The problem now is to define the $Q$ and $R$ matrices. Depending on the definition of these matrices, different controller gains and different closed loop behaviors are obtained. A pole placement, a minimal energy management or a minimum time constant can be imposed, etc. In the following part, the goal is to tune $Q$ and $R$ coefficients to keep in touch the effects on closed loop performances. Procedures experimented have to convert easily desired performance in requested parameters in the criterion definition.

## 3. CONTROL SYNTHESIS APPROACH

Three methods are discussed in this paper. The first one, a PID synthesised in a previous work Sou [2006] serving for comparison. The second one takes advantages of Linear Quadratic optimization but matrices are tuned by a trial and error algorithm. Finally, the third one blends previous criterion to a dominant poles placement to achieve quickly the synthesis.

### 3.1 Optimized PID

The most implemented regulator in industrial system has been selected for a first study. The point is to compare the new applied approach introduced using the best possible PID and show its limitations. In (6) are the PID formulation used and its coefficient's value. This regulator has been optimized in frequency domain. Detailed analyses through different methods (Genetic Algorithm, Iterative approach..) and calculation are given in Sou [2006]. the bounded limitations of torque, inertia variation were taken into account to assure optimizes performances by minimizing time response variation and limiting overshoot.

$$\begin{aligned} PID &= K_p \cdot [1 + \frac{1}{T_i s} + \frac{T_d s}{1 + a T_d s}] \\ K_p &= 1.3\, rd/(Nm.s) \qquad T_i = 0.19\, Nm/rd \\ T_d &= 0.00083\, rd/(s^2.Nm) \quad a = 0.5 \end{aligned} \tag{6}$$

This set of coefficients assures performances shown in the experimental part and the over-all variations are in (Table 1).

Of course, the wide range of performances implies that this regulator is not convenient for our needs. In the following par, a lot of cautions were made to designed a regulator with better performance and usable by industrial applications.

### 3.2 State feedback

A state feedback with the addition of an integral action was chosen now, as described in (Fig. 5)

The previous 3-dimensional state space formulation is augmented to describe the integral terms evolution. The resulting state system (7) has its integral action on load velocity. It results on a null steady state error. The state feedback gain are performed by the minimization of the criterion (3). This method on a mono-input system provides a minimal (60°) phase margin. That may allow more robust performances than PID.

$$X_{aug} = \left[\omega_m \ \omega_l \ \Delta\theta \ X_i\right]^T$$

$$\begin{cases} \dot{X}_{aug} = \underbrace{\begin{pmatrix} A & 0 \\ 0 \ -1 \ 0 & 0 \end{pmatrix}}_{A_{aug}} X_{aug} + \underbrace{\begin{pmatrix} B \\ 0 \end{pmatrix}}_{B_{aug}} T_m \\ y = \underbrace{\begin{pmatrix} C & 0 \end{pmatrix}}_{C_{aug}} X_{aug} \end{cases} \tag{7}$$

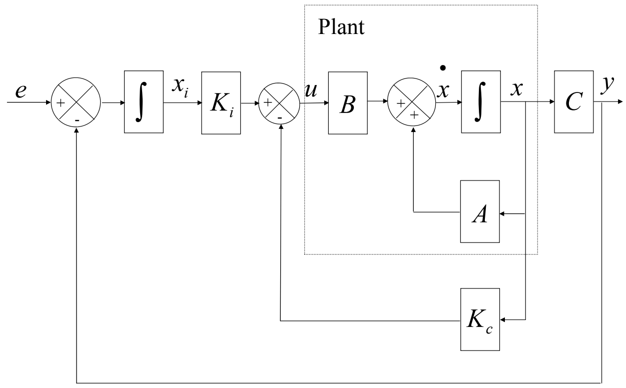



Fig. 5. LQI controller

To achieve our performance objectives, two different ways of $Q$ and $R$ tuning are now compared. The first one uses simplified matrices (only 3 degrees of freedom. The last one blends optimization and dominant poles placement.

*3.3 Limited Q and R synthesis (LQ3)*

A simplified form using only 3 DOF in $Q$ and $R$ is first presented. The sparse matrices used in this case are shown in (8). Each effect are identified and compared with PID coefficients in experimental part (4). Moreover, the 3 DOF presented in the state space formulation are equivalent to the 3 DOF of the PID polynomial form.

$$R = \gamma \quad Q = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta \end{pmatrix} \tag{8}$$

The major problem of this control design is to set up the correct weights in the quadratic function. Due to the few weight number, the tuning of $\alpha$, $\beta$ and $\gamma$ can be performed using trial and error method. This method checks requirements by executing off-line simulations. For this purpose, some important influences of parameters must be underlined :

- $\alpha$ imposes the constraints to the load velocity dynamics. High value makes the system slower. It is directly correlative with load speed overshoot.
- $\beta$ impacts on integral action dynamics and so on system dynamics. The higher it is, the smaller rising time will be.
- $\gamma$ is used to limit the maximum control input (motor torque) by choosing high value. Therefore, the setting and rising time become longer until dynamic changes from first to second order with an increasing overshoot (which must be less than 5% in industrial applications).

Nevertheless, the single weight variation knowledge is insufficient to assume a correct dynamic and time response. Obviously, the system must be stable on the over-all variation then trials allows us to specify weight rate (9) to be quickly close to the wanted regulator.

$$\begin{cases} \dfrac{\alpha}{\gamma} < 50 \\ \dfrac{\beta}{\gamma} < 20000 \end{cases} \tag{9}$$

So, the design method consists in the following algorithm :

(A) Stability purpose leads us to set up minimal inertia as reference : $J_l = J_{lmin}$.
(B) Using the supervised trial and error method and the knowledge of the weights characteristics, a set of LQ matrices is defined.
(C) Time response, poles placement and torque have to be analized on the over-all variation. Here the complete PMSM control is simulated to take into account inverter and current dynamic compatibility.
(D) If the robustness criterion is achieved (limited motor torque, time response variation and overshoot as less as possible), the parameters set is taken otherwise step back to (B).

After achieving this algorithm, a set of parameters has been calculated providing the regulator (10) :

$$\begin{aligned} R &= 10 \ Q = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 36 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 30000 \end{pmatrix} \\ K^T &= \begin{pmatrix} 0.426 & 1.662 & 122.872 & -54.772 \end{pmatrix} \end{aligned} \tag{10}$$

The responses with this regulator are shown in section 4. According to (Table 1), this regulator provides better performances than PID. This gain is quite good but this method has the same drawback than PID. Indeed, to optimize them, an algorithm must be executed with a lot of off-line iterations. That means, lots of conception times. The optimization is performed by a human being and it is possible that the user found a local and not a global optimum. The industrial world needs fast results method and uses classical engineering tools. This is the purpose of the last introduced method.

*3.4 Q and R synthesis and poles placement (LQPLPP)*

Most of regulator designers are used to place poles for controller since it allows them the knowledge of closed loop dynamic before simulation. Therefore, this new method blends the LQI criterion and traditional poles placement to reach a fast design method. Theoretically proven in Anderson and Moore [1989] and based on a $n$-dimensional standard state space formulation, the calculus needs the four following steps;

(A) First of all, choose the $(n-1)$ closed loop poles.
(B) Solve (11) where $A_{aug}$ and $B_{aug}$ are the system open loop matrix, $p_0(s)$ the open loop polynomial, $m(s)$ the desired closed loop polynomial and $d$ a column vector which have to be found.

$$d^T(pI_n - A_{aug})^{-1}B_{aug} = \frac{m(s)}{p_0(s)} \tag{11}$$

(C) Afterwards, matrices $Q$ and $R$ can be deduced from (12).

$$Q = \rho dd^T \quad R = 1 \tag{12}$$

Where $\rho$ is an iterative placement parameter.

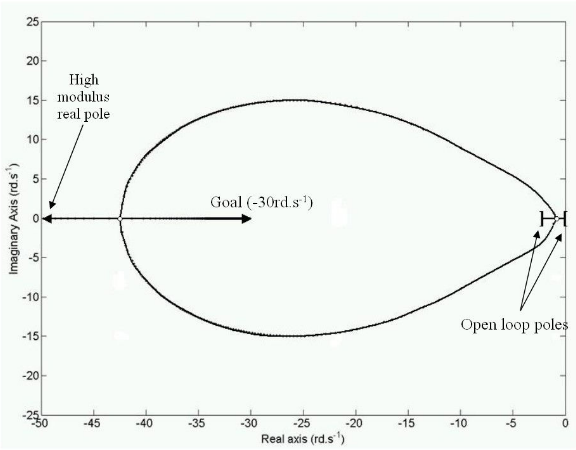



Fig. 6. LQI controller

(D) Finally, like previous methods, the regulator must be checked for both time response variation and motor torque limitation. If requirements are not achieved, step back to (A).

The placement parameter $\rho$ has important effects on the system. As high $\rho$ is, as closed are the closed loop poles from the desired placement. Otherwise, closed loop poles tend towards open loop poles.

The last pole which has not been chosen will be on real axis and its value grows up when $\rho$ grows up. So, $\rho$ is as much higher as necessary to have both, the dominant poles place and the last one five to ten time faster to be neglected in the desired behavior. Poles placement is selected obviously selected to obtain a firs order dynamic. This will minimize the overshoot with maximum inertia. Poles are one real dominant at $-30rd.s^{-1}$ and two complex with big modulus at $(-30 \pm 1700j)rd.s^{-1}$.

On (Fig. 6), the closed loop poles variation is drawn function of $\rho$. Along with $\rho$ increasing value, complex poles move not to their position but as closer as possible to allow a criterion minimization. Dominant poles become complex before splitting into two real poles, the chosen dominant one and the latest goes to infinite as foretold. Note that $\rho$ can take high value as in our example (10000). The main pole is strictly placed so $\rho$ is increased until the non placed pole become five to ten times faster to be non dominant.

The results provide the controller parameter (13):

$$\begin{aligned} \rho &= 4960 \ R = 1 \\ Q &= \begin{pmatrix} 0.00 & -0.11 & -3.82 & 3.30 \\ -0.11 & 4.72 & 159.67 & -137.97 \\ -3.82 & 159.67 & 5394.20 & -4661.12 \\ 3.30 & -137.97 & -4661.12 & 4027.67 \end{pmatrix} \\ K^T &= \begin{pmatrix} 0.523 & 1.796 & 183.515 & -63.464 \end{pmatrix} \end{aligned} \tag{13}$$

## 4. EXPERIMENTAL RESULTS

To check our assumptions and methods, an experimental setup shown on (Fig. 3) is used. Torque is provided by an actual PMSM actuator. The motor is supplied by a three-phase inverter controlled by a PMW law. All parameter are lister in Appendix A. This setup tries to reproduce the main industrial phenomena and consists in a variable load inertia created by an axis and two removable discs. So experiments are conduced and compared for a step reference tracking and a torque disturbance response.

To be noted, with minimal inertia and all controller, a sinusoidal disturbance of $50\,Hz$ is observable only when inertia is minimal. This disturbance is inducted by current PWM noises and amplified through inertia and stiffness resonance. When inertia is bigger, this noise is filtered.

Firstly, PID and LQ3 regulators are compared on (Fig. 7). Obviously PID controller at minimal inertia does not match requirement with an overshoot bigger than 5% on (Fig. 7(a)). However, it is the faster regulator. LQ3 controller is slower but match the requirements and has a smaller time response variation. The torque disturbance response is better for LQ3 controller (Fig. 7(b)). Indeed, the deviation is two time smaller. The minimal phase margin of the optimized method allow to keep overshoot and time variation slower. Moreover, the PID overshoot at minimal inertia is due to a saturated command. So LQ3 controller can be faster or may allow with less variation a disturbance torque from the start.

Then the two Linear Quadratic optimized regulators are compared on (Fig. 8). The two controllers have almost the same responses. LQPLPP is a little faster than LQ3 (Fig. 8(a)). $Q$ matrices is full and not sparse as LQ3 $Q$ matrices. So optimization have more parameter and reach a better solution. Last remark on deviation due to a disturbance torque, the deviation is less important for maximal inertia. The energy stocked by inertia smooth the response and minimize the deviation.

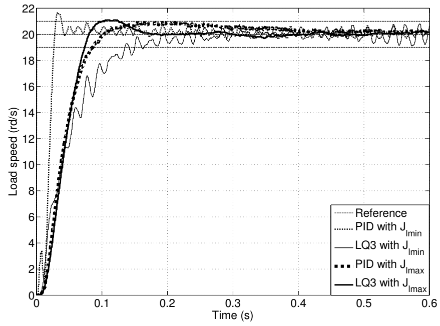
| | Minimal time response (ms) | Time response variation (%) | Maximal overshoot (%) | Disturbance error amplitude (%) |
|---|---|---|---|---|
| PID | 40 | 120 | 8.3 | 4 |
| LQPLPP | 70 | 63 | 5 | 1.7 |
| LQ3 | 73 | 92 | 5 | 1.7 |

Table 1. Methods performances

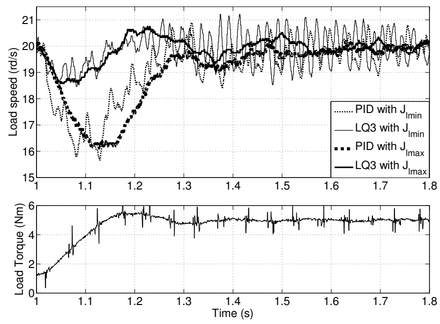
(Table 1) shows comparisons of each method under inertia variation so parametric robustness. In spite of its fastest response, PID has worse performances on the other criterion. Linear Quadratic controllers have almost the same performance. Finally between this two controllers, the main differences is the way they are synthesised. The linear quadratic criterion imposes minimal stability condition on the two cases. Poles placement provides quick tuning because it has less parameter "1" instead of "3" to tune with off-line iterative simulation. the time design economy is a big advantages for this method.

## 5. CONCLUSION

This paper describes the robust design and comparison of linear quadratic controller by two method in elastic load application with large load inertia variation driven by a PMSM motor. Alike other studies, PID do not achieve

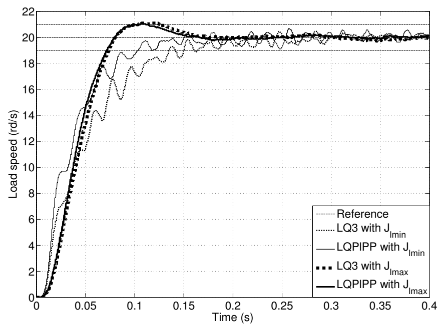



(a) $20rd.s{-}1$ step response

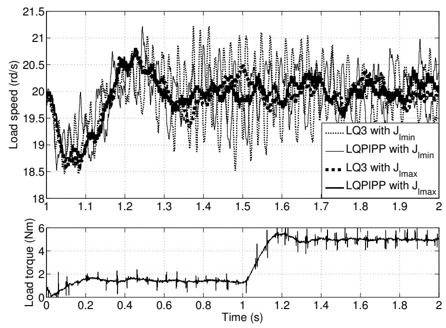



(b) Torque disturbance response

Fig. 7. Experimental comparison between PID and LQ3




(a) $20rd.s{-}1$ step response




(b) Torque disturbance response

Fig. 8. Experimental comparison between LQPLPP and LQ3

purpose. State space allows more complex controller as linear quadratic which has interesting characteristics in term of robustness and keep implementation simple for industrial application.

First optimization method has long time of development and may lead to local optimum. Contrarily, the last approach leads quickly to a solution which achieves the robustness goal thanks to a poles placement. This is more adapted to industrial time and complexity reduction goal. Experimentations driven on a test bench corroborate simulation and show up advantages of these methods.

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## Appendix A. PMSM AND LOAD PARAMETERS

| PMSM parameters | |
|---|---|
| $R_s = 0.6\,\Omega$ | $K_c = 0.83\,Nm/A$ |
| $L_c = 1.9\,mH$ | $f_m = 0.06 \cdot 10^{-3}\,Nm.s/rd$ |
| $pp = 4$ pair of poles | $J_m = 0.74 \cdot 10^{-3}\,kg.m^2$ |
| Load parameters | |
| $f_l = 8.5 \cdot 10^{-3}\,Nm.s/rd$ | $K_{sh} = 2000\,Nm/rd$ |
| $J_l \in \left[0.006\ 0.038\right]\,kg.m^2$ | |
| PI current control | |
| $K_p = 1.533\,V/Nm$ | $T_i = 0.0024\,V.s/Nm$ |