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RATIONAL APPROXIMATION AND IDENTIFICATION OF DISTRIBUTED PARAMETER SYSTEMS

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Abstract: The unified infinite dimensional model structure which assumes on its base to develop the method and algorithms of systems rational approximation and identification is proposed for distributed parameter systems with discrete inputs and outputs. The considered truncated realization converges to infinite-dimensional non-rational model of system for nuclear type operators. Approximation is represented by series expansion on independent basis functions which are fundamental solutions of ordinary differential equations. The using of Jordan realization have succeeded in creation of iterative identification algorithm admitting sequential model reconstruction by separate parts consisting of one or several modes.

Keywords: distributed parameter systems, modeling, approximation, iterative identification, Jordan realization, uncertainty, Green function.

1. INTRODUCTION

There are many approaches and methods dealing with parameter identification problem for distributed parameter systems. In some of these approaches the model structure is selected as a boundary value problem for partial differential equations (PDE). At the same time there are many publications where models based on finite-dimensional approximation or finite element discretization of PDE are considered. A rather comprehensive treatment of the identification problem for distributed parameter systems has been reviewed for example in (Banks and Kunish, 1989).

The rational approximation and identification of the infinite dimensional systems are considered in this paper. It is proposed to apply the universal approach for model structure selection which generalizes the using of input – output relations for wide class of finite and infinite-dimensional systems including various PDE systems by means of Green function and unified standardized functions. Such model structure makes possible truncated rational approximation convergence to the original system with nuclear type operator (Glover, 1988, Makila, 1991).

Typically orthogonal basis functions are employed for optimal approximation of stable systems and there are many publications devoted to this problem (see, e.g. Wahlberg, 1994; Makila, 1990, Heuberger, 1995, Van Den Hof et.al., 1995). When approximate model is reconstructed using identification on the base of input- output data corrupted by experimental errors or noises there arises additional obstacle concerning model complexity and well-posedness (Makila, 1991). In order to cope with this problem the specific iterative identification algorithm which permits to find rational approximation agreeing with uncertainty in available data is developed. The idea of the offered approach is to approximate systems by means of series expansion with respect to independent basis functions which are fundamental solutions of finite-dimentional ordinary differential equations. Then identification is reduced to approximation of experimentally obtained outputs by means of such finite series with unknown not only expansion coefficients but also eigenvalues which are parameters of basis functions. The problem in such treatment may be constructively solved if rational approximation will be taken in the form of Jourdan realization. Applying of such realization allows using selected data sampling and modal analysis to restore model by its separate parts for which we use the term “submodels” because they contain one or several system modes. Jourdan form provides also other preferences which were utilized in proposed algorithm.

2. THE PROBLEM STATEMENT

For modeling, identification and control of the complex real system it is required at first to choose the appropriate model structure. It is very hard to do this without a priori information and general knowledge about the plant. If the object is distributed in space and its parameters or

characteristics are time-dependent, then we deal with the space-time system which can be characterized by scalar function in the simplest case and by vector function that depends on both the spatial variables z defined in some area F including the boundary and time on semiaxis $(t \ge t_0)$ in general case. As a rule the processes in such system are also determined by the boundary and initial conditions. Further linear systems will be considered only that is also a priori information. In this case the unified standard form of model structure is universal enough and well matched with existing methods of modeling and identification. The idea of such standardization was offered by Butkovsky and the wide class of systems with distributed and lumped parameters was collected and structured in standard form (Butkovsky, 1979).

The main characteristic of system in standard form is the Green function or the impulse response function. The Green function is called also as the influence function or the source function. Along with the scalar or vector variables $w(z,t)$ that characterize the space-time system state let's introduce the standardized function $f(z,t)$ that is interpreted as generalized function and by means of which any external volume, boundary or initial influence (inputs) to the system can be written down in the unified form. If the Green function $H(z,\zeta,t,\tau)$ and the standardized function $f(z,t)$ are known the system state is determined by equation

$$w(z,t) = \int_{t_0}^{t} \int_{\overline{F}} H(z,\zeta,t,\tau) f(\zeta,\tau) d\zeta \, d\tau \,, \qquad (1)$$

where $\overline{F}$ is the closure of the set F.
The structure (1) includes the static and quazystatic models. At last case the time is simply a parameter. Then (1) is represented as

$$w(z) = \int_{\overline{F}} H(z,\zeta) f(\zeta) d\zeta \qquad (2)$$

Systems with lumped parameters are the special case of (1)

$$w(t) = \int_{t_0}^{t} H(t,\tau) f(\tau) d\tau \qquad (3)$$

The equation (3) is well-known Cauchy formula

$$w(t) = \Phi(t,t_0) w_0 + \int_{t_0}^{t} \Phi(t,\tau) u(\tau) d\tau \qquad (4)$$

where $\Phi(t,\tau)$ is the transition matrix of linear differential equations system which generates the Green function. The generalized standardized function $f(\tau)$ is expressed as

$$f(\tau) = u(\tau) + w_0 \delta(\tau - t_0),$$

where δ is generalized Dirac function.

In vector case

$$w(z,t) = col(w_1(z,t), w_2(z,t), \ldots, w_k(z,t))$$

and

$$f(z,t) = col(f_1(z,t), f_2(z,t), \ldots, f_L(z,t)) .$$

So

$$w_k(z,t) = \int_{t_0}^{t} \int_{\overline{F}} \sum_{\ell=1}^{L} H_{\ell k}(z,\zeta,t,\tau) f_\ell(\zeta,t) d\zeta \, d\tau, \quad k = \overline{1,K} \qquad (5)$$

In wide-spread case when system is shift-invariant the Green function has expression

$$H(z,\zeta,t,\tau) = H(z,\zeta,t-\tau).$$

For practical using it is often enough to know not the field $w(z,t)$ but it's local or integral characteristics which can be measured or estimated. Represent them as the output variables $y(t) = col((y_1(t), y_2(t), \ldots, y_M(t))$. For local measurements we have $y_m(t) = w(z_m, t)$ whereas in integral case $y_m(t) = \int_{\overline{F}} \psi_m(z) w(z,t) dz$ where $\psi_m(z)$ is weight function defined on the set $F_m \subseteq \overline{F}$ and $\psi_m(z) \equiv 0$ when $z \in \overline{F} / F_m$. The set F_m can be proper subset of $\overline{F}$ or coincide with it.

Since $f(z,t)$ is generalized function the boundary or other singular action is defined by its structure. However the time dependence $f(z,t)$ is connected as a rule with external lumped parameters in the form $f(z,,t) = \sum_{r=1}^{R} u_r(t) f_r(z)$, where $u_r(t)$ defines the variables of the external action to the system and $f_r(z)$ describes its distributed influence. For example, in system with electromagnetic processes the currents in winding are external lumped inputs and spatial characteristics are defined by configuration of coils winding.

As a result for such systems with the finite number of inputs and outputs (widespread case in practice) the model structure can be written as

$$y_m^{(r)}(t) = \int_{t_0}^{t} H_{mr}(t,\tau) u_r(\tau) d\tau, \quad m = \overline{1;M}, \; r = \overline{1;R}, \qquad (6)$$

where

$$H_{mr}(t,\tau) = \int_{\overline{F}} \psi_m(z) dz \int_{\overline{F}} H(z,\zeta,t,\tau) f_r(\zeta) d\zeta .$$

Here $\psi_m(z)$ are considered as generalized functions that provide all measurements including pointwise in standardizing form.

It should be noted that $H_{mr}(t,\tau)$ are generated by the same Green function. Consequently the time characteristics of elements $H_{mr}(t,\tau)$ should be the same or complement each other. Thus the weight functions $\psi_m(z)$ and $f_r(\zeta)$ will determine the observability or controllability properties of relevant generalized degrees of freedom.

3. RATIONAL APPROXIMATION

Linear time-invariant systems under definite assumptions such that model (6) can be written in the convolution form with non-rational (infinite-dimension case) or rational (finite-dimensional system) transfer matrix functions $G(s)$ will be considered. Such systems for wide enough class of matrix-valued transfer functions are quarters of linear operators (matrices) *A, B, C, D* mapping between different

infinite-dimensional (or finite-dimensional) linear vector spaces, so that $G(s) = D + C(sI - A)^{-1}B$. At first, this class includes the systems with operators of nuclear type that induces bounded Hankel operator (Glover et al., 1988) which singular values $\sigma_j(\sigma_j \geq 0)$ satisfy $\sum_{j=1}^{\infty}\sigma_j < \infty$. It is known that the Hankel operator Γ is nuclear if $G(s)$ corresponding matrix-valued transfer function $C(sI-A)^{-1}B$ can be represented in the form

$$G_{mr}(s) = \sum_{i=1}^{\infty} g_{mi}^{(r)} \mathrm{Re}(\xi_i)(s-\xi_i)^{-1}, \quad G(s) = [G_{mr}(s)] \qquad (7)$$

under assumptions that complex numbers are located in the left half-plane (stable systems), singular values are different and $\sum_{i=1}^{\infty}\left|g_{mi}^{(r)}\right| \leq C\|\Gamma\|_N$, where $C = const$, and $\|\Gamma\|_N = \sum_{i=1}^{\infty}\sigma_i$ is a nuclear norm of Γ.

In principle this result is also spread to the case of multiple singular values by using the chain of Schmidt pairs (Adamjan et al, 1971). However only simple singular values case will be considered here.

The series (7) uniformly converges in *Re s >0* and in the nuclear norm of associated Hankel operators. The impulse response matrix $H(t)$ of time-invariant system in the case of nuclear operator Γ consists of the elements $H_m^{(r)}(t)$ that can be expressed by decomposition

$$H_{mr}(t) = \sum_{i=1}^{\infty} g_{mi}^{(r)} \mathrm{Re}(\xi_i) e^{\xi_i t} \qquad (8)$$

The functions $H_{mr}(t)$ are continuous almost everywhere on the semiaxis $t \geq t_0$ and satisfy

$$|H(t)| = \sup_{\substack{\|\ell\|=1 \\ \|\eta\|=1}} \left\{ \left| \langle H(t)\ell, \eta \rangle \right|, \ \ell \in C^R, \eta \in C^M \right\} \leq M\|\Gamma\|_N / t \qquad (9)$$

for all $t \geq t_0$ and M=*const*.

Accordingly with (Glover et al., 1988) series (7) converges in H_∞, H_2 and H_N; convergence of impulse response matrix $H(t)$ for expansion (8) in norms L^1, H^∞, Hankel, L^2 is provided by the classical closure theorem (see, e.g. Makila, 1991). Therefore the transfer matrix with elements

$$G_{mr}^n(s) = \sum_{i=1}^{n} g_{mi}^{(r)} \mathrm{Re}(\xi_i) \cdot (s - \xi_i)^{-1} \qquad (10)$$

or corresponding impulse response matrix that consists of elements

$$H_{mr}^n(t) = \sum_{i=1}^{n} g_{mi}^{(r)} \mathrm{Re}(\xi_i) \cdot e^{\xi_i t} \qquad (11)$$

can be selected as truncated model of system. Here convergence

$$\|G_{mr}^n(s) - G_{mr}(s)\|_N \to 0, \quad \|H_{mr}^n(t) - H_{mr}(t)\|_N \to 0$$

for $n \to \infty$, is guarantied where N is one of aforementioned norms. In fact, formula (10) describes the stationary finite-dimensional dynamical system with transfer matrix-valued elements written in the form of partial fractions decomposition. Each term in (10) and analogously in (11) corresponds to the general degree of freedom or mode of the system. Expressions (10) and (11) are equivalent to

$$H^n(t) = C_n e^{A_n t} B_n, \ G^n(s) = C_n (sI - A_n)^{-1} B_n \ , \qquad (12)$$

where matrices A_n, B_n and C_n define the finite-dimensional state-space system, i.e.

$$\frac{dx}{dt} = A_n x + B_n u$$
$$y = C_n x \qquad (13)$$

The dimensions of vectors y and u are equal to M and R respectively. The normal output realization (Glover et. al., 1988) and balanced realization are usually utilized in approximation theory. The truncated system (C_n, A_n, B_n) under such realizations coincides with (11) but finding the equations which link elements of matrices C_n, A_n, B_n with the eigenvalues ξ_i and coefficients $g_{mi}^{(r)}$ is nontrivial task.

Connection between models (13) and (11) becomes simple enough if we take the Jordan form of realization instead of balanced or normal output realization. In this case the matrix A_n can be written as $A_n = \mathrm{diag}\{S_1, S_2, ..., S_P\}$ where diagonal block

$$S_p = \begin{bmatrix} \alpha_p & -\beta_p \\ \beta_p & \alpha_p \end{bmatrix} \text{ and } \xi_p = \alpha_p \pm j\beta_p .$$

The matrices C_n and B_n are also expressed in the block form

$$C_n = [C_1, C_2, ..., C_P] \text{ and } B_n^T = [B_1^T, B_2^T, ..., B_P^T],$$

where

$$C_p^T = \begin{bmatrix} c_{1p}^c & c_{2p}^c & \cdots & c_{Mp}^c \\ c_{1p}^s & c_{2p}^s & \cdots & c_{Mp}^s \end{bmatrix}, \ B_p = \begin{bmatrix} b_{p1}^c & b_{p2}^c & \cdots & b_{pR}^c \\ b_{p1}^s & b_{p2}^s & \cdots & b_{pR}^s \end{bmatrix}$$

and T is the transpose operation. So (11) can be written as

$$H_{mr}^n(t) = \sum_{p=1}^{P} (g_{mp}^{(r1)} \cdot \cos\beta_p t + g_{mp}^{(r2)} \sin\beta_p t) e^{\alpha_p t} \ , \qquad (14)$$

where

$$g_{mp}^{(r1)} = c_{mp}^c b_{pr}^c + c_{mp}^s b_{pr}^s, \ g_{mp}^{(r2)} = c_{mp}^s b_{pr}^c - c_{mp}^c b_{pr}^s .$$

Moreover, it is suitable to select the observable normal Jordan realization if one of columns of each matrix C_p is assumed $c_{ip}^c = 0, \ c_{ip}^s = 1 \quad i = \overline{1;M}$ or controllable normal Jordan realization relying $b_{pi}^c = 0, \ b_{pi}^s = 1 \quad i = \overline{1;R}$ in each block B_p. In the first case equations connecting the coefficients of decomposition (14) with elements of matrices C_p and B_p are

$$b_{pr}^c = g_{ip}^{(r_2)}, b_{pr}^s = g_{ip}^{(r_1)} \ (r = \overline{1;R}, \ p = \overline{1;P}),$$
$$c_{mp}^c = \frac{1}{2R} \sum_{r=1}^{R} \frac{g_{mp}^{(r_1)} - g_{mp}^{(r_2)}}{g_{rp}^{(r_1)}}, \qquad (15)$$
$$c_{mp}^s = \frac{1}{2R} \sum_{r=1}^{R} \frac{g_{mp}^{(r_1)} + g_{mp}^{(r_2)}}{g_{ip}^{(r_2)}} \quad (m = \overline{1;M}, \ p = \overline{1;P}, \ m \neq i)$$

and in second case expressed as

$$c_{mp}^c = g_{mi}^{(r_1)}, c_{mp}^s = g_{mi}^{(r_2)} \ (m = \overline{1;M}, \ p = \overline{1;P}, \ p \neq i);$$

$$b_{pr}^{c}=\frac{1}{M}\sum_{m=1}^{M}\frac{g_{mp}^{(r_1)}g_{mi}^{(r_1)}+g_{mp}^{(r_2)}g_{mi}^{(r_2)}}{g_{mi}^{(r_1)^2}+g_{mi}^{(r_2)^2}},$$
$$b_{pr}^{s}=\frac{1}{M}\sum_{m=1}^{M}\frac{g_{mp}^{(r_1)}g_{mi}^{(r_2)}-g_{mp}^{(r_2)}g_{mi}^{(r_1)}}{g_{mi}^{(r_1)^2}+g_{mi}^{(r_2)^2}} \qquad (16)$$

If the system has one input and many outputs then it is reasonable to use the observable normal Jordan realization and when the system has one output and many inputs it is reasonable to use controllable normal Jordan realization. In these cases expansion coefficients and elements of matrices of the state space model have the unique coupling. In general case for systems with many inputs and outputs we have the overdetermined system of coupling equations and so solutions (15) and (16) were calculated by LSM that led to the averaging procedure for definite set of coefficients. Besides the real eigenvalues in decomposition (14) are the special case that is received at $\beta_p=0,\ c_{mp}^{s}=b_{pr}^{s}=0,$ i.e. $g_{mr}^{(r_2)}\equiv 0,\ \cos\beta_p t\equiv 1$. Consequently the model dimension or number of its degrees of freedom is equal to $n=P_1+P_2$, where P_1 corresponds to the complex eigenvalues and P_2 to the real ones respectively.

Therefore (14) is approximate structure for elements of impulse response matrix that is written as decomposition $H_{mr}^{n}(t)$ with respect of independent basis functions which are fundamental solutions of the equivalent linear system of differential equations. The Laplace transformation of (14) $[LH_{mr}^{n}(t)](s)$ leads to rational transfer function that is represented as partial fraction singular decomposition with n poles $s=\alpha_p\pm j\beta_p,\ p=\overline{1;P}$ in the left half-plane.

4. ITERATIVE IDENTIFICATION

Orthogonal basis functions have employed the effective tool for the purpose of system approximation and identification. It is established fact that every stable system has a unique series expansion in terms of such basis and a finite-length series of such expansion can serve as an approximate model. However it is easily understandable that the accuracy of the approximation will be essentially dependent on the choice of basis functions. If the dynamics of the basis generating system and the dynamic system to be modeled are closed, we will have the fast convergence. So it is proposed for approximation and iterative identification to apply independent but no necessarily orthogonal basis which is close to eigenfunctions of the considering system.

The impulse response functions in (14) are expressed as decompositions of independent functions that are fundamental solutions of (13). Here aren't known not only expansion coefficients $g_{mp}^{(r1)}$ and $g_{mp}^{(r2)}$, but eigenvalues parameters α_p and β_p also. If eigenvalues are known then optimal rational approximation would be analogous to the orthonormal basis case. So it is offered to estimate at first $\hat{\alpha}_p$ and $\hat{\beta}_p$ and after that find the optimal values of $\hat{g}_{mp}^{(r1)}$ and $\hat{g}_{mp}^{(r2)}$. The developed methods in system identification can be applied for realizing of this approach. The measurements obtained from experiments are available data. If it is required to establish the rational approximation of known boundary-valued problem for PDE with finite number of external inputs and outputs then relevant data can be obtained from computational simulation. For real and virtual plants identification problem remains the same. The input can include different signals but it should be informative and excite all significant system modes. To establish the informatiability conditions let's consider at first the exciting influence

$$u_r(t)=\sum_{j=1}^{J}(u_{rj}^{(s)}\sin\omega_j t+u_{rj}^{(c)}\cos\omega_j t) \qquad (17)$$

applied to every separate input. Input (17) allows to form variety different influences. Relevant response at the *m*-th output is

$$y_m^{(r)}(t)=\sum_{p=1}^{P}(d_{mp}^{(r_1)}\sin\beta_p t+d_{mp}^{(r_2)}\cos\beta_p t)e^{\alpha_p t}-$$
$$-\sum_{j=1}^{J}(d_{mj}^{(r_3)}\sin\omega_j t+d_{mj}^{(r_4)}\cos\omega_j t), \qquad (18)$$

where

$$d_{mp}^{(r_1)}=\frac{g_{mp}^{(r1)}}{2}\sum_{j=1}^{J}\left[u_{rj}^{(s)}(\Delta_1-\Delta_2)+u_{rj}^{(c)}(\Delta_3+\Delta_4)\right]-$$
$$-\frac{g_{mp}^{(r_2)}}{2}\sum_{j=1}^{J}\left[u_{rj}^{(s)}(\Delta_3-\Delta_4)-u_{rj}^{(c)}(\Delta_1+\Delta_2)\right]$$
$$d_{mp}^{(r_2)}=\frac{g_{mp}^{(r_1)}}{2}\sum_{j=1}^{J}\left[u_{rj}^{(s)}(\Delta_3-\Delta_4)-u_{rj}^{(c)}(\Delta_1+\Delta_2)\right]+$$
$$+\frac{g_{mp}^{(r_2)}}{2}\sum_{j=1}^{J}\left[u_{rj}^{(s)}(\Delta_1-\Delta_2)+u_{rj}^{(c)}(\Delta_3+\Delta_4)\right]$$
$$d_{mj}^{(r_3)}=u_{rj}^{(s)}\sum_{p=1}^{P}\left[\frac{g_{mp}^{(r_1)}}{2}(\Delta_1+\Delta_2)-\frac{g_{mp}^{(r_2)}}{2}(\Delta_3+\Delta_4)\right]+$$
$$+u_{rj}^{(c)}\sum_{p=1}^{P}\left[\frac{g_{mp}^{(r_1)}}{2}(\Delta_3-\Delta_4)+\frac{g_{mp}^{(r_2)}}{2}(\Delta_1-\Delta_2)\right]$$
$$d_{mj}^{(r_4)}=u_{rj}^{(s)}\sum_{p=1}^{P}\left[\frac{g_{mp}^{(r_1)}}{2}(\Delta_3-\Delta_4)+\frac{g_{mp}^{(r_2)}}{2}(\Delta_1-\Delta_2)\right]- \qquad (19)$$
$$-u_{rj}^{(c)}\sum_{p=1}^{P}\left[\frac{g_{mp}^{(r_1)}}{2}(\Delta_1+\Delta_2)-\frac{g_{mp}^{(r_2)}}{2}(\Delta_3+\Delta_4)\right]$$

$$\Delta_1=\frac{\alpha_p}{\alpha_p^2+(\beta_p-\omega_j)^2},\qquad \Delta_2=\frac{\alpha_p}{\alpha_p^2+(\beta_p+\omega_j)^2},$$
$$\Delta_3=\frac{\beta_p-\omega_j}{\alpha_p^2+(\beta_p-\omega_j)^2},\qquad \Delta_4=\frac{\beta_p+\omega_j}{\alpha_p^2+(\beta_p+\omega_j)^2}.$$

The output (18) consists of both eigenfunctions which define transient and functions that classify the steady-state process which are caused by (17).

But it is preferred to select from (18) either forced motion or transient process and use them separately. To select, i.e., the forced vibration for stable systems it is enough to take the data for $t>>T$ where T is transient time. The optimal

approximation of $y_m^{(r)}(t)$ ($t>>T$) by truncated sum may be carried out by finite-frequency identification method. However in identification on the base of forced motion there exists the problem of model dimension selection. Apparently the iterative identification on the base of transient data is more preferable for reconstruction of approximate model. In order to select the transient behavior let's perform the multirepeat integration of $y_{mp}^{(r)}(t)$ along moving intervals $\left[t, t+\frac{2\pi}{\omega_j}\right]$. As a result we receive a new signal $\widetilde{y}_{mp}^{(r)}(t)$ that is equal to

$$\widetilde{y}_m^{(r)}(t) = \int_t^{t+\frac{2\pi}{\omega_1}} d\theta_1 \int_{\theta_1}^{\theta_1+\frac{2\pi}{\omega_2}} d\theta_2 \quad \ldots \quad \int_{\theta_{j-1}}^{\theta_{j-1}+\frac{2\pi}{\omega_j}} y_m^{(r)}(\theta_j)d\theta_j = \\ = \sum_{p=1}^{P}\left(\widetilde{d}_{mp}^{(r_1)} \sin\beta_p t + \widetilde{d}_{mp}^{(r_2)} \cos\beta_p t\right)e^{\alpha_p t} \tag{20}$$

Coefficients $\widetilde{d}_{m_p}^{(r_1)}$, $\widetilde{d}_{m_p}^{(r_2)}$ are expressed linearly via $d_{m_p}^{(r_1)}$, $d_{m_p}^{(r_2)}$ but in cumbersome way. After getting the signal $y_m^{(r)}(t)$ from experiments and calculating $\widetilde{y}_m^{(r)}(t)$ the nontrivial task of parameters P, α_p, β_p, $\widetilde{d}_{mp}^{(r_1)}$, $\widetilde{d}_{mp}^{(r_2)}$ evaluating from (20) should be solved.

Consider now the computation of $g_{mp}^{(r_1)}$ and $g_{mp}^{(r_2)}$ from (19). This task can be correctly solved if determinant

$$\left(\sum_{j=1}^{J}\left[u_{rj}^{(s)}(\Delta_1-\Delta_2)+u_{rj}^{(c)}(\Delta_3+\Delta_4)\right]\right)^2 + \\ +\left(\sum_{j=1}^{J}\left[u_{rj}^{(s)}(\Delta_3-\Delta_4)-u_{rj}^{(c)}(\Delta_1+\Delta_2)\right]\right)^2$$

isn't equal to zero. When parameters α_p, β_p are unknown then in general case it is difficult to estimate the amplitudes $d_{mp}^{(r_1)}$, $d_{mp}^{(r_2)}$ which excite all modes under consideration. It is quite possible that the same input can give excitation maximum for some modes while other modes will be on the level of disturbances.

So it is suggested to use signals that are simple for analysis and which are able to excite the modes in wide eigenvalues range. The rectangular impulse with amplitude u_{r_0} that is attached to the *r*-th input on the interval $[t_0, t_1]$ satisfies these conditions.

For iterative identification there can be used the free motion at $t \ge t_1$ that can be written as

$$y_m^{(r)}(t) = u_{r_0}\sum_{p=1}^{P}\sqrt{\frac{1+e^{2\alpha_p(t_1-t_0)}-2e^{\alpha_p(t_1-t_0)}\cos\beta_p(t_1-t_0)}{\alpha_p^2+\beta_p^2}}\times \\ \times\left\{g_{mp}^{(r_1)}\sin\left[\beta_p(t-t_1)+\theta\right]-g_{mp}^{(r_2)}\cos\left[\beta_p(t-t_1\right]+\theta\right\}e^{\alpha_p(t-t_1)} \tag{21}$$

or the transient behavior at $t \ge t_0$ and $t_1 = \infty$ equal to

$$y_m^{(r)}(t) = u_{r_0}\sum_{p=1}^{P}\frac{1}{\sqrt{\alpha_p^2+\beta_p^2}}\left\{g_{mp}^{(r_1)}(e^{\alpha_p(t-t_0)}\sin\left[\beta_p(t-t_0)+\theta\right]- \right. \\ \left. -\sin\theta)-g_{mp}^{(r_2)}(e^{\alpha_p(t-t_0)}\cos\left[\beta_p(t-t_0)+\theta\right]-\cos\theta)\right\} \tag{22}$$

According to (21) and (22) the amplitudes of exciting modes decreases inversely proportionally to α_p and β_p and increases directly proportionally to u_{r_0}. Besides it is reasonable to increase the width of impulse for small values of α_p. The significant eigenvalues that can be identified occupy in the plane (α_p, β_p) spectral domain that includes the modes with informative signals on the background of data errors. The most broadband is the input taken as "ideal" pulse $u_r(t) = u_{r_0}\delta(t-t_0)$ that gives the output

$$y_m^{(r)}(t) = \sum_{p=1}^{P}u_{r_0}\left[g_{mp}^{(r_1)}\cos\beta_p(t-t_0)+g_{mp}^{(r_2)}\sin\beta_p(t-t_0)\right]e^{\alpha_p(t-t_0)} \tag{23}$$

Consider now certain possible iterative scheme of approximate model reconstruction. It is suggested using analytical output (21), (22) or (23) and corresponding experimental data on the transient $[t_1, T_1]$ or $[t_0, T_1]$ to reconstruct model iteratively by determining at each iteration parameters of one or several modes. One should start with modes that give the signal to noise ratio enough large at the end of interval that is nearly *T*. Obviously modes with the smallest values of α_p will give main contribution there. When we come near to t_1 or t_0 the number of essentially informative modes will grow on account of large α_p. This will be used in iterative identification. The idea is to represent the model as aggregation of submodels. Each submodel will be reconstructed separately. Data on the subinterval nearly T allow to determine all unknown parameters of the first submodel. After that we evaluate the output of this submodel using (21), (22), or (23) and subtract it from $\bar{y}_m^{(r)}(t)$ that was obtained in experiment. Thus we find a new signal $\bar{y}_m^{(r)}(t) - y_{m_1}^{(r)}(t)$ which is admissible for estimation of second submodel parameters. This signal will have the own informative interval $[t_1, T_1]$ or $[t_0, T_1]$. Nearly T_1 we select subinterval for second submodel and perform the same actions as for first submodel. Subtracting signal $y_{m_2}^{(r)}(t)$ from $\bar{y}_m^{(r)}(t) - y_{m_1}^{(r)}(t)$ we define new T_2 and corresponding subinterval. Following third submodel may be identified. Such iterations are repeated until the signal $\bar{y}_m^{(r)}(t) - y_{m_1}^{(r)}(t) - \ldots - y_{m_q}^{(r)}(t)$ becomes indistinguishable on the noise background. Aggregation of all submodels which were identified will give the Jourdan realization of finite-dimensional approximation that is in consistency with the existing uncertainty.

Let's turn to algorithms of submodel parameters estimation. It is offered the scheme consisting of two stages. At the first stage all submodel's α_p and β_p should be determined. The second stage that is addressed to expansion coefficients evaluation can be realized by standard LS method. Since the second stage is the well-known task we describe only algorithms of submodel eigenvalues estimation. Different approaches were considered one of which consists in the following.

From $\widetilde{y}_{m_q}^{(r)}(t) = \bar{y}_m^{(r)}(t) - y_{m_1}^{(r)}(t) - \ldots - y_{m_q}^{(r)}(t)$ where $q \ge 0$,

$\tilde{y}^{(r)}_{m_0}(t) = \bar{y}^{(r)}_{m}(t)$ we form new signals

$$y^{(r)}_{m_{q1}}(t,\gamma) = \tilde{y}^{(r)}_{m_q}(t) + \tfrac{1}{\gamma}\frac{d\,\tilde{y}^{(r)}_{m_q}(t)}{dt}, \qquad (24)$$

$$y^{(r)}_{m_{q^2}}(t,\alpha,\beta) = \tilde{y}^{(r)}_{m_q} + \frac{2\alpha}{\alpha^2+\beta^2}\frac{d\,\tilde{y}^{(r)}_{m_q}(t)}{dt} + \frac{1}{\alpha^2+\beta^2}\frac{d^2\tilde{y}^{(r)}_{m_q}(t)}{dt^2}$$

and

$$y^{(r)}_{m_{q3}}(t,\gamma) = \tilde{y}^{(r)}_{m_q}(t) + \gamma\int_0^t \tilde{y}^{(r)}_{m_q}(\tau)d\tau, \qquad (25)$$

$$y^{(r)}_{m_{q4}}(t,\alpha,\beta) = \tilde{y}^{(r)}_{m_q}(t) + 2\alpha\int_0^t \tilde{y}^{(r)}_{m_q}(\tau)d\tau +$$

$$+(\alpha^2+\beta^2)\int_0^t (t-\tau)\,\tilde{y}^{(r)}_{m_q}(\tau)d\tau$$

where α, β, γ are varied. Let's take the sequences of sampled data $\left\{\tilde{y}^{(r)}_{m_{q_j}}(t_i,\gamma),\ i=\overline{1,N}\right\}$ for j = 1,3 and $\left\{\tilde{y}^{(r)}_{m_{q_j}}(t_i,\alpha,\beta), i=\overline{1,N}\right\}$ for j = 2,4 within the subinterial range of identifying submodel. Compose the Hankel matrices

$$Y^{(r)}_{m_{q_j}} = \begin{bmatrix} \tilde{y}^{(r)}_{m_{q_j}}(t_1) & \tilde{y}^{(r)}_{m_{q_j}}(t_2) & \cdots & \tilde{y}^{(r)}_{m_{q_j}}(t_\xi) \\ \tilde{y}^{(r)}_{m_{q_j}}(t_2) & \tilde{y}^{(r)}_{m_{q_j}}(t_3) & \cdots & \tilde{y}^{(r)}_{m_{q_j}}(t_{\xi+1}) \\ \tilde{y}^{(r)}_{m_{q_j}}(t_\eta) & \tilde{y}^{(r)}_{m_{q_j}}(t_{\eta+1}) & \cdots & \tilde{y}^{(r)}_{m_{q_j}}(t_{\xi+\eta-1}) \end{bmatrix}, \qquad (26)$$

$$\xi+\eta-1 \le N,\ j=\overline{1,4}.$$

Let the SVD of $Y^{(r)}_{mq_j}$ be given by

$$Y^{(r)}_{m_{q_j}} = U^{(r)}_{m_{q_j}} \sum{}^{(r)}_{m_{q_j}} V^{(r)^T}_{m_{q_j}} \qquad (27)$$

where $U^{(r)}_{mq_j}$ and $V^{(r)}_{mq_j}$ are orthogonal matrices and $\Sigma^{(r)}_{mq_j}$ is a diagonal matrix with the singular values in non-increasing order on the diagonal.

Let's analyze behavior of singular values when α,β,γ are varied. Result in either one singular value (for case with γ) or two (for α, β) tends to zero. Corresponding illustrations are represented on fig.1 and fig.2 where singular values behavior for case of real eigenvalues is shown. Fig.1 corresponds to differential and fig.2 - to integral transformations.

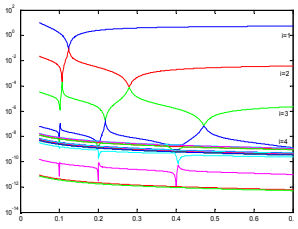

Fig. 1.

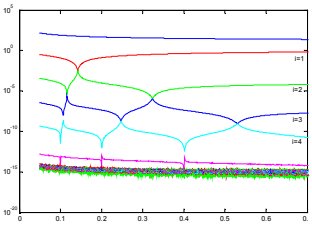

Fig. 2.

Such algorithm becomes more effective if it is a priori possible to find the rough estimation for eigenvalues or to point out affiliation interval for them. Sometimes this may be done directly from transient behavior on subinterval.

Remark 1. If we vary parameters α,β,γ there are rearranged not only singular values of submodel to be identified but also the singular values of the modes with weak response or disturbances. In many cases it is possible to estimate the eigenvalues more precisely by analyzing of minor singular values transformation.

Remark 2. Some outputs for different *m* and *r* may contain non-informative signal of definite modes due to their bad controllability or bad observability. So the complete model is compiled from all identifying modes with respect to $M \times R$ inputs-outputs.

5. CONCLUSION

Selective data choice for submodel identification is in certain meaning similar to orthogonalization procedure. So inspite of nonorthogonal series expansion due to specific sampling it becomes possible to realize iterative model reconstruction by separate parts.

The main dignity of the developed iterative identification method is that iterations tend to rational approximate model with parameters and basis functions which give minimal deviation between dynamics of each real system generalized degree of freedom and the model. Moreover, since iterations are terminated when all informative output signals of modes become exhausted we obtain the model dimension that is in full consistency with the errors in available data.

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