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# Fuzzy Optimization with Robust Logistic Membership Function: A Case Study In For Home Textile Industry

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**Abstract:** Many engineering optimization problems can be considered as linear programming problems where the all or sum of the parameters involved are linguistic in nature. These can only be quantified using fuzzy sets. The aim of this paper is to solve fuzzy linear programming problem where the parameters involved are fuzzy numbers with logistic membership functions. To explore the applicability of the present study a numerical example is considered determine monthly production planning and profit of home-textile group. To solve this problem LINGO Software is used. 

## 1. INTRODUCTION

A decision situation related to human aspect, in fact, has only a little to do with the absolute attributes – certainty and precision – which are not present in human cognition, perception, reasoning and thinking. There are many issues and things that can only be defined by vague and ambiguous predicates. Thus it has become clear that formal math-analytical modeling of a real decision situation does not reflect the pervasiveness of human perception, cognition and mutual interaction with the outside world [7].

We often encounter difficulty that not all of the parameters for solving real decision problem are exactly known. The main problem in such cases is the problem of information acquisition and modelling them with proper stress [3]. The concept of a fuzzy set was introduced by Zadeh to represent or manipulate data and information possessing non-statistical uncertainties. Extensive development of the theories fuzziness has to some extent, attempted to break this impasse [3, 7, 20].

Bellman and Zadeh [1] introduced the basis of most fuzzy optimization problems, in which both objectives and constraints in an ill-defined situation are represented by fuzzy sets. The theory of Fuzzy Linear Programming (FLP) was first developed for solving imprecise or vague problems in the field of artificial intelligence, especially in reasoning and modeling linguistic terms. In solving fuzzy decision making problems, the earlier work came from [1], then from some symmetric [22] and non-symmetric models [5, 10, 11, 12, 15, 16]. The Diet Problem in chicken farm was successfully solved by using interactive FLP approach [2]. Blending Problem was solved using FLP approach and satisfactory trade-off between cost and quality had been achieved [4] and [9]. The financial problem was solved in [8]. The objective of this problem is to decide a maximum return by investing on security bonds. Zeleny [21] has proposed a simplified procedure for optimum design of system in a fuzzy environment. This design problem can be applied in many ways, such as inventory problem, just in time problem and waste management problem. FLP approach has been used with good level of satisfaction even though the constraints and objectives are fuzzy. Watada [17] has proposed one form of logistic membership function to overcome difficulties in using linear membership function in solving fuzzy decision making problem. Non-linear logistic membership function was presented by Vasant [18, 19].

In this paper a methodology to solve fuzzy linear programming problem with logistic membership is considered. In section 2 the basic model is defined and the fuzzy inequality relations are demonstrated in subsection 2.1. Subsection 2.2 is dealt with the fuzzy objective function and its crisp equivalent system. Subsection 2.3 considered the decision methodology. In section 3 a numerical example of home-textile group is considered to illustrate the present contribution. The concluding remarks are made in final section 4.

## 2. INVESTIGATION OF THE MODEL

A conventional linear programming problem is given by

$$\begin{aligned}&\text{Maximize } \; Cx\\ &\text{Subject to } \quad Ax \le b, \quad x \ge 0.\end{aligned} \tag{1}$$

in which the components of *1×n* vector *C*, *m×n* matrix *A* and *n×1* vector *b* are all crisp parameters and *x* is n-dimensional decision variable vector. This problem system (1) may be redefined in fuzzy environment with the re-elaborated structure as follows:

$$\text{Maximize } \sum_{j=1}^{n} \tilde{c}_j x_j$$

$$\text{Subject to } \quad \sum_{j=1}^{n} \tilde{a}_{ij} x_j \underset{\sim}{\le} b_i, \quad i = 1,2\cdots m \tag{2}$$

$$\mu_{\tilde{c}_j} = \begin{cases} 1 & \text{if } c_j \le c_j^a \\ \dfrac{B}{1 + Ce^{\alpha\left(\frac{c_j - c_j^a}{c_j^b - c_j^a}\right)}} & \text{if } c_j^a \le c_j \le c_j^b \\ 0 & \text{if } c_j \ge c_j^b \end{cases}$$

$$\mu_{\tilde{a}_{ij}} = \begin{cases} 1 & \text{if } a_{ij} \le a_{ij}^a \\ \dfrac{B}{1 + Ce^{\alpha\left(\frac{a_{ij} - a_{ij}^a}{a_{ij}^b - a_{ij}^a}\right)}} & \text{if } a_{ij}^a \le a_{ij} \le a_{ij}^b \\ 0 & \text{if } a_{ij} \ge a_{ij}^b \end{cases}$$

All fuzzy data $\tilde{c}_j \equiv \tilde{S}(c_j^a, c_j^b)$ and $\tilde{a}_{ij} \equiv \tilde{S}(a_{ij}^a, a_{ij}^b)$ are fuzzy variables having the logistic membership functions [23] as shown in Figure 1 and described by the above formulae.

The following points are to be clarified up when we replace system (1) by system (2),

(i) Specification of fuzzy inequality relations and methodology to obtain its crisp equivalents.
(ii) The interpretation 'maximization' in logistic type objective functions.

### 2.1. Conversion of ith resource constraint.

Using Zadeh's extension principle the left side of

Thus, $f_i(.)$ can be written as,

$$f_i\left(\sum_{j=1}^{n} a_{ij} x_j\right) = \begin{cases} 1 & \text{if } \sum_{j=1}^{n} a_{ij} x_j \le \sum_{j=1}^{n} a_{ij}^a x_j \\ \dfrac{B}{1 + Ce^{\alpha\left(\frac{\sum_{j=1}^{n} a_{ij} x_j - \sum_{j=1}^{n} a_{ij}^a x_j}{b_i - \sum_{j=1}^{n} a_{ij}^a x_j}\right)}} & \text{if } \sum_{j=1}^{n} a_{ij}^a x_j \le \sum_{j=1}^{n} a_{ij} x_j \le b_i \\ 0 & \text{if } \sum_{j=1}^{n} a_{ij} x_j \ge b_i \end{cases} \tag{5}$$

Here, $\sum_{j=1}^{n} a_{ij}^a x_j$ is the infimum of $\sum_{j=1}^{n} a_{ij} x_j$ at membership grade 1.

Now, (3) may be simplified as follows:

$$\left.\sum_{j=1}^{n} \tilde{a}_{ij} x_j \right|_{\varepsilon} = X_i^{\varepsilon} \quad \text{(say)}$$

$$\Rightarrow \frac{B}{1 + Ce^{\alpha\left(\frac{X_i^{\varepsilon} - \sum_{j=1}^{n} a_{ij}^a x_j}{\sum_{j=1}^{n} a_{ij}^b x_j - \sum_{j=1}^{n} a_{ij}^a x_j}\right)}} = \varepsilon$$

$$\Rightarrow (B - \varepsilon) = \varepsilon C e^{\alpha\left(\frac{X_i^{\varepsilon} - \sum_{j=1}^{n} a_{ij}^a x_j}{\sum_{j=1}^{n} a_{ij}^b x_j - \sum_{j=1}^{n} a_{ij}^a x_j}\right)}$$

$$\Rightarrow X_i^{\varepsilon} = \sum_{j=1}^{n} a_{ij}^a x_j + \frac{\sum_{j=1}^{n} a_{ij}^b x_j - \sum_{j=1}^{n} a_{ij}^a x_j}{\alpha} \log\left(\tfrac{B-\varepsilon}{\varepsilon C}\right)$$

Thus, $X_i^{\varepsilon} \le b_i$

$$\Rightarrow \sum_{j=1}^{n} a_{ij}^a x_j + \frac{\sum_{j=1}^{n} a_{ij}^b x_j - \sum_{j=1}^{n} a_{ij}^a x_j}{\alpha} \log\left(\tfrac{B-\varepsilon}{\varepsilon C}\right) \le b_i$$

Therefore the system (3) and (4) may be written with an equivalent system as

$$\begin{cases} \sum_{j=1}^{n} a_{ij}^a x_j + \dfrac{\sum_{j=1}^{n} a_{ij}^b x_j - \sum_{j=1}^{n} a_{ij}^a x_j}{\alpha} \log\left(\tfrac{B-\varepsilon}{\varepsilon C}\right) \le b_i & (6) \\ \dfrac{B}{1 + Ce^{\alpha\left(\frac{\sum_{j=1}^{n} a_{ij} x_j - \sum_{j=1}^{n} a_{ij}^a x_j}{b_i - \sum_{j=1}^{n} a_{ij}^a x_j}\right)}} \rightarrow Max & (7) \end{cases}$$

### 2.2. Conversion of fuzzy objective function

Let D is the aspiration of the objective function, which may be determined by maximizing $\sum_{j=1}^{n} c_j^b x_j$ , subject to $\left.\sum_{j=1}^{n} \tilde{a}_{ij} x_j \right|_{\varepsilon} \le b_i, \forall i$ and as this is the maximum extend of objective function at ε level. Similarly for minimization problem the minimum extent of objective function may be calculated by minimizing $\sum_{j=1}^{n} c_j^a x_j$ .

Applying same technique we may reformulate the problem as:

$$\begin{cases} \left.\sum_{j=1}^{n} \tilde{c}_j x_j \right|_{\varepsilon} \le D & (8) \\ f'(\sum_{j=1}^{n} c_j x_j) \to Max & (9) \end{cases}$$

where $f'(.)$ may be interpreted as the subjective assessment of $\sum_{j=1}^{n} c_j x_j$ with regard to the aspiration D as follows:

$$f'(\sum_{j=1}^{n} c_j x_j) = \begin{cases} 1 & \text{if } \sum_{j=1}^{n} c_j x_j > D \\ \dfrac{B}{1+Ce^{\alpha\left(\frac{D-\sum_{j=1}^{n} c_j x_j}{D-\sum_{j=1}^{n} c_j^a x_j}\right)}} & \text{if } \sum_{j=1}^{n} c_j^a x_j \le \sum_{j=1}^{n} c_j x_j \le D \\ 0 & \text{if } \sum_{j=1}^{n} c_j x_j < \sum_{j=1}^{n} c_j^a x_j \end{cases}$$

Thus,

$$\begin{cases} \sum_{j=1}^{n} c_j^a x_j + \dfrac{\sum_{j=1}^{n} c_j^b x_j - \sum_{j=1}^{n} c_j^a x_j}{\alpha} \log\left(\frac{B-\varepsilon}{\varepsilon C}\right) \le D & (11) \\ \dfrac{B}{1+Ce^{\alpha\left(\frac{D-\sum_{j=1}^{n} c_j x_j}{D-\sum_{j=1}^{n} c_j^a x_j}\right)}} \to Max & (12) \end{cases}$$

### 2.3. Final Formulation and Optimization

In finding compromise solution up to the DM's satisfaction, we now use Zadeh's min operator to combine the objective functions (4) and (9) and get a conventional problem as:

$$\begin{aligned} & Max \quad \lambda \\ & Subject\ to \\ & \quad \lambda \le 1, \\ & \quad \lambda \le f_i(\sum_{j=1}^{n} a_{ij} x_j), \forall i \\ & \quad \lambda \le f'(\sum_{j=1}^{n} c_j x_j) \\ & \quad \left.\sum_{j=1}^{n} \tilde{a}_{ij} x_j \right|_{\varepsilon} \le b_i, \quad \left.\sum_{j=1}^{n} \tilde{c}_j x_j \right|_{\varepsilon} \le D \\ & \text{and } \lambda, x_j \ge 0, \quad \forall j \end{aligned} \tag{13}$$

Equivalently (13) may be written as,

$$\begin{aligned} & Max \quad \lambda \\ & Subject\ to \\ & \quad \lambda \le 1, \\ & \quad \lambda \le \frac{B}{1+Ce^{\alpha\left(\frac{\sum_{j=1}^{n} a_{ij} x_j - \sum_{j=1}^{n} a_{ij}^a x_j}{b_i - \sum_{j=1}^{n} a_{ij}^a x_j}\right)}}, \forall i \qquad (10) \\ & \quad \lambda \le \frac{B}{1+Ce^{\alpha\left(\frac{D-\sum_{j=1}^{n} c_j x_j}{D-\sum_{j=1}^{n} c_j^a x_j}\right)}} \\ & \quad \sum_{j=1}^{n} a_{ij}^a x_j + \frac{\sum_{j=1}^{n} a_{ij}^b x_j - \sum_{j=1}^{n} a_{ij}^a x_j}{\alpha} \log\left(\frac{B-\varepsilon}{\varepsilon C}\right) \le b_i, \forall i \\ & \quad \sum_{j=1}^{n} c_j^a x_j + \frac{\sum_{j=1}^{n} c_j^b x_j - \sum_{j=1}^{n} c_j^a x_j}{\alpha} \log\left(\frac{B-\varepsilon}{\varepsilon C}\right) \le D \\ & \text{and } \lambda, x_j \ge 0, \quad \forall j \end{aligned}$$

## 3. NUMERICAL EXAMPLE

The profit for a unit of sheet sale is around 1.05 Euro; pillow case sale is around 0.3 Euro and sheet of a quilt sale is around 1.8 Euro. This firm thinks to sale "approximately 25.000 units of sheet, 40.000 units of pillow case and 10.000 units of sheet of a quilt". Monthly working capacity and required process time for the production of sheet, pillow case and sheet of a quilt are given in Table 1 [6].

In this view, let's determine monthly production planning and profit of home-textile group. $X_1$ represents the quantity of sheet that will be produced, $X_2$ represents the quantity of pillow case and $X_3$ represents the quantity of a sheet of a quilt.

Considering the profit figures with logistic membership functions as given in table 1 these define

around 1.05 $\equiv \tilde{S}(1.02,1.08)$,

around 0.3 $\equiv \tilde{S}(0.2,0.4)$,

around 1.8 $\equiv \tilde{S}(1.7,2.0)$.

Then the mathematical model of the above problem is

Maximize

$\tilde{S}(1.02,1.08)x_1+\tilde{S}(0.2,0.4)x_2+\tilde{S}(1.7,2.0)x_3$

subject to

$.0033x_1+.001x_2+.0033x_3 \le 208;$

$.056x_1+.025x_2+.1x_3 \le 4368;$

$.0067x_1+.004x_2+.017x_3 \le 520;$

$.01x_1+.01x_2+.01x_3 \le 780;$

$x_1 \ge 25000;$

$x_2 \ge 40000;$

$x_3 \ge 10000;$ (15)

which gives the optimal value of the objective function as 67203.88 for,

$x_1 = 29126.21, x_2 = 35000.00$ and $x_3 = 10873.79$

Using above aspiration level in equations (6) and (7) the problem becomes:

**Table 1** Required Process Time for sheet, pillow case and sheet of a quilt [6]

| **Departments** | **Required unit time(hour)** | | | **Working hours for a month** |
|---|---|---|---|---|
| | Sheet | Pillow case | Sheet of a quilt | |
| Cutting | 0.0033 | 0.001 | 0.0033 | 208 |
| Sewing | 0.056 | 0.025 | 0.1 | 4368 |
| Pleating | 0.0067 | 0.004 | 0.017 | 520 |
| Packaging | 0.01 | 0.01 | 0.01 | 780 |

Maximize

$\tilde{S}(1.02,1.08)x_1+\tilde{S}(0.2,0.4)x_2+\tilde{S}(1.7,2.0)x_3$

subject to

$.0033x_1+.001x_2+.0033x_3 \le 208;$

$.056x_1+.025x_2+.1x_3 \le 4368;$

$.0067x_1+.004x_2+.017x_3 \le 520;$

$.01x_1+.01x_2+.01x_3 \le 780;$

$x_1 \ge 25000;$

$x_2 \ge 40000;$

$x_3 \ge 10000;$ (15)

Let us set $B = 1, C = .001, \varepsilon = 0.2$ and $d = 13.8$

The aspiration of the objective function is being calculated as

Maximize $1.08x_1+0.4x_2+2.0x_3$

subject to

$.0033x_1+.001x_2+.0033x_3 \le 208;$

$.056x_1+.025x_2+.1x_3 \le 4368;$

$.0067x_1+.004x_2+.017x_3 \le 520;$

$.01x_1+.01x_2+.01x_3 \le 780;$

$x_1 \ge 25000;$

$x_2 \ge 40000;$

$x_3 \ge 10000;$

Maximise $\lambda$

subject to

$0 \le \lambda \le 1;$

$.0033x_1+.001x_2+.0033x_3 \le 208;$

$.056x_1+.025x_2+.1x_3 \le 4368;$

$.0067x_1+.004x_2+.017x_3 \le 520;$

$.01x_1+.01x_2+.01x_3 \le 780;$

$.x_1 \ge 25000;$

$x_2 \ge 40000;$

$x_3 \ge 10000;$

$\lambda + .001\lambda e^{13.8\eta} = 1;$

$(1.05 - 1.02\eta)x_1 + (.3 - .2\eta)x_2 + (1.8 - 1.5\eta)x_3 \ge (1 - \eta)67203.88;$

$1.0304x_1 + .2052x_2 + 1.752x_3 \le 67203.88;$

$1.0304x_1 + .2052 * x2 + 1.752 * x3 \le 53710;$

With the help of LINGO 10.0 we obtain the following results:

$\lambda = 0.5323011, x_1 = 27766.99, \ x_2 = 40000.00,$
$x_3 = 10233.01, \eta = 0.4911863$

## 4. CONCLUSION

A A modified methodology has been developed to solve FLP (Fuzzy Linear Programming) problem with logistic membership function. The decision maker's credibility level is well considered in this process. Currently this research in progress towards the development of a new methodology for industrial management problems using the evolutionary fuzzy-neural approach with multi-media interactive technology in addition to the standard human-computer work-load sharing.

## REFERENCES

R. E. Bellman & L. A. Zadeh, "Decision-making in a fuzzy environment", *Management Science*, 17, 4, 1970, pp. 141 – 164.

J. J. Buckley, Generalized and extended fuzzy sets with application*, Fuzzy Sets and Systems*, 25, 1988, pp. 159-174.

Debjani Chakraborty, "Optimization in imprecise and uncertain environment", Ph. D. Thesis (1995), IIT Kharagpur, India.

S. Chanas, "The use of parametric programming in fuzzy linear programming", *Fuzzy Sets and Systems*, 11, 1983, pp. 243-251.

A. N. S. Freeling, "Fuzzy sets and decision analysis", *IEEE Transactions on System, Man, and Cybernetics*, 10, 7, 1980, pp. 341-354.

Irfan Ertugrul and Aysegül Tus, "Interactive fuzzy linear programming and an application sample at a textile firm", *Fuzzy Optimization and Decision Making*, 6, 2007, pp. 29–49.

M. M. Gupta, "Cognition, perception and uncertainty", in: *Fuzzy logic in knowledge-based systems, decision and control*, Eds., M. M. Gupta and T. Yamakawa (Elsevier Science Publishers B.V., North Holland, 1988).

Lai, Y. J. and Hwang, C. L., *Fuzzy Mathematical Programming Methods and Applications*, Springer-Verlag, Berlin, 1992.

H. Nakayama, J. Nomura, K. Sawada and R. Nakajima, "An Application of Satisficing Trade-off Method to a Blending Problem of Industrial Materials", in: G. Fandel et al. (Eds.), *Large Scale Modelling and Interactive Decision Analysis*. (Springer, 1986), pp. 303-313.

C. V. Negoita and M. Sularia, "On fuzzy mathematical programming and tolerances in planning", *Economic Computer and Economic Cybernetic Studies and Researches*, 1, 1976, pp. 3-14.

C. V. Negoita and D. A. Ralescu, "On Fuzzy Optimization", *Kybernetes* , 6, 1977, pp. 139-196.

C. V. Negoita, "The current interest in fuzzy optimi-zation", *Fuzzy Sets and Systems*, 6, 1981, pp. 261-269.

H. Rommelfanger, "Interactive decision making in fuzzy linear optimization problems", *European Journal of Operational Research*, 41, 2, 1989, pp. 210–217.

H. Rommelfanger, "Inequality relation in fuzzy constraints and its use in linear fuzzy optimization", in: *The Interface Between Artificial Intelligence and OR in Fuzzy Environment*, Eds., J.L. Verdegay & M. Delgado, (Verlag TUV Reinland, Köln, 1989) 195 – 211.

H. Tanaka, T. Okuda and K. Asai, "On fuzzy mathematical programming," *Journal of Cybernetics*, 3, 1974, pp. 37-46.

H. Tanaka, T. Okuda and K. Asai, "A formulation of fuzzy decision problems and its application to an investment problem", *Kybernetes* , 5, 1976, pp. 25-30.

J. Watada, "Fuzzy Portfolio Selection and its Applications to Decision Making," *Tatra Mountains Mathematics Publication* 13, 1997, pp. 219–248.

P. Vasant and A. Bhattacharya, "Sensing degree of fuzziness in MCDM model using modified flexible S-curve MF", *International Journal of Systems Science*, 38, 4, 2007, pp. 279-291.

A. Bhattacharya and P. Vasant, "Soft-sensing of level of satisfaction in TOC product-mix decision heuristic using robust fuzzy-LP", *European Journal of Operational Research*, 177, 2007, pp.55-70.

L. A. Zadeh, "Fuzzy Sets", *Information and Control*, 8, 1965, pp. 338-353.

M. Zeleny, "Spontaneous social orders", *Interna-tional Journal of General Systems*, 11, 1985, pp. 117–131.

H. J. Zimmermann, "Application of fuzzy set theory to mathematical programming'', *Information Sciences*, 36, 1985, pp. 25–58.

P. Vasant, A. Bhattacharya, B. Sarkar and S. K. Mukherjee, "Detection of level of satisfaction and fuzziness patterns for MCDM model with modified flexiable S-curve MF", *Applied Soft Computing*, 7, 2007, pp. 1044-1054.