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Spacecraft attitude dynamics and control in the presence of large magnetic residuals ⋆

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Abstract: The attitude dynamics of a satellite with a large magnetic residual dipole is analysed and the effect of perturbations of the orbital parameters on stability is discussed. The analysis is the basis for the design of an attitude control strategy that minimizes the required control torque while satisfying an absolute pointing error constraint whenever direct compensation of magnetic torque is not achievable. The proposed strategy is validated in a case study.

1. INTRODUCTION

Space vehicles are usually equipped with attitude control systems for two different reasons. First of all, the open-loop attitude dynamics is never asymptotically stable, so the control system must ensure closed-loop stability for the controlled satellite. In addition, a satellite is subject to a number of disturbance torques affecting its motion, the effect of which must be suitably attenuated in order to achieve the desired level of pointing performance. External disturbance torques have different sources, such as gravity gradient, aerodynamics, solar radiation and residual magnetic dipoles (see, e.g., Hughes [1986], Wertz [1978]). All disturbance torques are in principle state-dependent, i.e., they depend on the spacecraft attitude. The design of *nominal* attitude controllers is usually performed on the basis of linearised models for the equations of rigid body angular dynamics, in which only gravity gradient torques are explicitly included, given the relatively simple nature of the underlying physics. Torques due to magnetic residuals, on the other hand, are normally dealt with as external disturbances, although their physical modelling is extremely simple, the only source of uncertainty being the amplitude and direction of the satellite's residual dipole vector. The main difficulty associated with the analysis of the effect of a magnetic residual dipole is associated with the time-variability in the dynamics introduced by the effect of the geomagnetic field. The effect of magnetic residual on the nonlinear attitude dynamics has been studied by Chen and Liu [2002]. When considering the linearised dynamics near a desired attitude, it turns out that periodic systems and control theory has been used extensively in the study of the design problem associated with magnetic attitude control, see for example Silani and Lovera [2005] and the references therein), but to the best knowledge of the Authors no specific attention has been dedicated to the open-loop linearised dynamics of a spacecraft with magnetic residuals or to the associated design issues.

In view of the above discussion, this paper aims at two objectives: studying the open loop behavior of the system and designing the attitude controller for a satellite with a large magnetic residual.

The paper is organised as follows: in Section 2 a description of the dynamics of a generic spacecraft is presented; a linearised dynamic model is derived and studied in Section 3. The associated control and guidance issues are finally discussed in Section 4, while Section 5 presents some simulation results obtained in a realistic case study.

2. SPACECRAFT MODEL

In this Section the nonlinear, time-varying equations of attitude motion for a spacecraft subject to a magnetic residual dipole are derived. To this purpose, the following coordinate frames are used. INE is the Earth-centered inertial frame; ORB is the orbital reference frame defined as follows. The origin of this frame moves with the center of mass of the satellite. The Z_r axis points toward the Earth; the X_r axis is in the orbit plane, perpendicular to Z_r, in the direction of the orbital velocity of the spacecraft. The Y_r axis completes the orthogonal system. Thus, ORB orbits around the inertial frame and its angular velocity in local coordinates for a circular orbit is $[0 \ \ -\omega_0 \ \ 0]^T$ where ω_0 is the orbital angular frequency. Finally, SAT is the body-fixed frame whose origin is the center of mass of the satellite and whose axes are aligned with the satellite principal axes. The origin of SAT is coincident with ORB origin.

The attitude dynamics model is a 7-dimensional system whose state is made of an attitude quaternion q and an angular velocity vector ω

$$x = [q_1 \ q_2 \ q_3 \ q_4 \ \omega_x \ \omega_y \ \omega_z]^T, \quad (1)$$

where the attitude is expressed with respect to the ORB frame, i.e., the quaternion $q = [q_1 \ q_2 \ q_3 \ q_4]^T$ represents the rotation from the orbital reference frame to the body fixed frame and $\omega = [\omega_x \ \omega_y \ \omega_z]^T$ is the relative angular velocity between the body fixed frame and the orbital reference frame, expressed in body coordinates. If the matrix Ω is defined as

$$\Omega(\omega) = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix},$$

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then the differential equation for the attitude kinematics, parameterised in terms of the attitude quaternion q, is given by

$$\dot{q} = \frac{1}{2}\Omega(\omega)q,$$

while the Euler equations can be used to express conservation of angular momentum in body fixed coordinates

$$\dot{h} + \omega_{sat} \times h = \tau_g + \tau_m + \tau_a,$$

where $h = I\,\omega_{sat}$ is the satellite angular momentum expressed in body coordinates, τ_g is the gravity gradient torque, τ_m the magnetic disturbance torque and τ_a the actuator torque. After a change of coordinates the dynamics can be written in ORB frame as

$$\begin{aligned} \dot{q}(t) &= \frac{1}{2}\Omega(\omega_{rel})q \\ I\dot{\omega}_{rel} &= I\left(A\begin{bmatrix}0\\-\omega_0\\0\end{bmatrix}\times\omega_{rel}\right) - \omega_{rel}\times A\begin{bmatrix}0\\-\omega_0\\0\end{bmatrix} + \\ &+\tau_g + \tau_m + \tau_a. \end{aligned} \tag{2}$$

In the above equations the gravity gradient and the magnetic torque still need to be specified. Gravity gradient modeling is a standard issue and it is treated in many references, for example Sidi [1997]. For a satellite with diagonal inertia matrix (i.e., $I = \mathrm{diag}(I_x, I_y, I_z)$), the gravity gradient is given by

$$\tau_g = 3\omega_0^2\begin{bmatrix}(I_z - I_y)\,A_{2,3}\,A_{3,3}\\ -(I_z - I_x)\,A_{1,3}\,A_{3,3}\\ -(I_x - I_y)\,A_{1,3}\,A_{2,3}\end{bmatrix}. \tag{3}$$

The magnetic torque, on the other hand, can be expressed as

$$\tau_m = m \times b_{sat},$$

where m and b_{sat} are, respectively, the satellite residual dipole and the Earth's magnetic field expressed in body coordinates. For the purpose of stability analysis a dipole approximation of the Earth's magnetic field suffices. The dipole equations, when assuming no Earth rotation and no orbit precession, are

$$b_{orb} = \frac{\mu_m}{a^3}\begin{bmatrix}\cos(\omega_o t)s_{i_m}\\ -c_{i_m}\\ 2\sin(\omega_0 t)s_{i_m}\end{bmatrix},$$

where i_m is the inclination of the satellite's orbit with respect to the magnetic equator, a is the orbit's semimajor axis and μ_m is the field's dipole strength. The above assumptions hold if the orbital period of the satellite is small compared to the Earth rotation period: Earth rotation can be treated as a slowly varying parameter affecting the magnetic inclination of the orbit. In conclusion, the magnetic torque is computed as

$$\tau_m = m \times A(q)\frac{\mu_m}{a^3}\begin{bmatrix}\cos(\omega_o t)s_{i_m}\\ -c_{i_m}\\ 2\sin(\omega_0 t)s_{i_m}\end{bmatrix}. \tag{4}$$

The magnetic torque is periodically time varying; this renders the final model time-periodic.

Substituting (3) and (4) into (2) one gets the nonlinear, time varying periodic system

$$\dot{x} = f(x, t, \tau_a). \tag{5}$$

3. OPEN LOOP STABILITY ANALYSIS

In this Section the stability of the reference Earth-pointing attitude is studied. In the present work we will assume that the residual dipole lies on the $x - y$ plane of the satellite, so that it can be parametrized as a norm and direction on the plane, yielding

$$m(m_0, \gamma) = \begin{cases} m_x = m_0 c_\gamma \\ m_y = m_0 s_\gamma \\ m_z = 0. \end{cases}$$

It is possible to linearize equation (5) in the neighborhood of the reference attitude, characterized by null pointing error and expressed by the vector $x_0 = [0\ 0\ 0\ 1\ 0\ 0\ 0]^T$. For small perturbations, the state vector can be reduced to 6 variables by dropping the kinematic equation $\dot{q}_4$ and substituting it with a unit norm constraint; moreover it is convenient to rearrange the state variables as

$$x = [q_2\ \omega_y\ q_1\ q_3\ \omega_x\ \omega_z]^T. \tag{6}$$

Consequent linearization yields

$$\dot{\delta x} \approx f(x_0, t, 0) + A(t)\delta x + B\tau_a, \tag{7}$$

where $A(t) = \left.\frac{\partial f(x,t,\tau_a)}{\partial x}\right|_{x=x_0,\tau_a=0}$ and $B = \left.\frac{\partial f(x,t,\tau_a)}{\partial \tau_a}\right|_{x=x_0}$, and

$$f(x_0, t, 0) = \begin{bmatrix} 0 \\ \frac{\mu_m}{I_y a^3}\left(-2m_0 c_\gamma \sin(\omega_0 t)s_{i_m}\right) \\ 0 \\ 0 \\ \frac{\mu_m}{I_x a^3}\left(2m_0 s_\gamma \sin(\omega_0 t)s_{i_m}\right) \\ \frac{\mu_m}{I_z a^3}\left(-m_0 c_\gamma c_{i_m} - m_0 s_\gamma \cos(\omega_0 t)s_{i_m}\right) \end{bmatrix}. \tag{8}$$

Matrix $A(t)$ can be divided into four blocks

$$A(t, m_0, \gamma) = \left[\begin{array}{c|c} A_p(t, m_0, \gamma) & A_{coupl12}(t, m_0, \gamma) \\ \hline A_{coupl21}(t, m_0, \gamma) & A_{y+r}(t, m_0, \gamma) \end{array}\right] \tag{9}$$

where the pitch, the coupled yaw-roll dynamics and their couplings are highlighted. With the above definitions, the following expressions are obtained

$$A_p(t, m_0, \gamma) = \begin{bmatrix} 0 & \frac{1}{2} \\ -6\omega_0^2\sigma_y - \frac{2\mu_m m_0 c_\gamma s_{i_m}\cos(\omega_0 t)}{a^3 I_y} & 0 \end{bmatrix},$$

$$A_{coupl12}(t, m_0, \gamma) = \begin{bmatrix} 0 & 0\ 0\ 0 \\ -\frac{2}{a^3 I_y} m_0 \mu_m c_\gamma c_{i_m} & 0\ 0\ 0 \end{bmatrix},$$

$$A_{coupl21}(t, m_0, \gamma) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{2}{a^3 I_x} m_0 \mu_m s_\gamma s_{i_m}\cos(\omega_0 t)) & 0 \\ \frac{4}{a^3 I_z} m_0 \mu_m s_\gamma s_{i_m}\cos(\omega_0 t)) & 0 \end{bmatrix},$$

where $\sigma_x \equiv (I_y - I_z)/I_x$, $\sigma_y \equiv (I_x - I_z)/I_y$ and $\sigma_z \equiv (I_y - I_x)/I_z$, and finally

$$B = P\begin{bmatrix} O_{3\times 3} \\ I^{-1} \end{bmatrix},$$

where P is a suitable permutation matrix taking into account the reordering of the state variables. The following considerations are due:

$$A_{y+r}(t, m_0, \gamma) = \begin{bmatrix} 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ -8\omega_0^2\sigma_x + \frac{2m_0\mu_m s_\gamma c_{i_m}}{a^3 I_x} & 0 & 0 & \omega_0(1-\sigma_x) \\ \frac{4}{a^3 I_z} m_0\mu_m c_\gamma s_{i_m} \sin(\omega_0 t)) & -2\omega_0^2\sigma_x + \frac{2m_0\mu_m}{I_z a^3}(s_\gamma c_{i_m} - c_\gamma s_{i_m}\cos(\omega_0 t)) & \omega_0(\sigma_z - 1) & 0 \end{bmatrix}$$

1) In general, the reference attitude is not an equilibrium. Equation (8) describes the torques acting on the satellite when it is in the reference attitude. For the case of $m_x = m_z = 0$, Figure 1 depicts the maximum magnetic torque acting on the satellite in the reference attitude for different i_m values and over an orbit period. When $i_m = 0$ the reference is an equilibrium state and the residual torque increases with i_m.

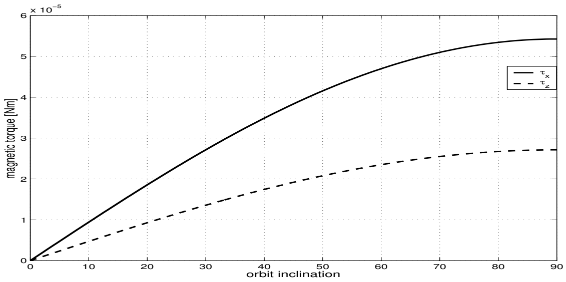

Fig. 1. Maximum residual torque components per unit dipole for the case of $m_x = m_z = 0$ and an orbit altitude of 250 km.

2) The dynamic matrix, $A(t)$, is a time-periodic matrix with period equal to the orbital period T.

3) As is well known (see, e.g., Sidi [1997]), in the absence of residual dipole the satellite dynamics show a natural decoupling between pitch and roll-yaw motion. When taking the residual dipole into account, the system's dynamics retains a block triangular structure provided that the inclination γ of the residual dipole in the $x-y$ plane takes very specific values: in particular, the structure is lower triangular for $\gamma = \pm 90°$ and upper triangular for $\gamma = 0°$ and $\gamma = 270°$. In both cases a decoupling between the pitch dynamics and the roll-yaw dynamics arises; moreover in the case of $\gamma = \pm 90°$ the pitch and roll-yaw dynamics turn out to be time-invariant while only the coupling from roll-yaw to pitch is time-varying (this simplification can be exploited for control design purposes - see Section 4).

4) The analytical expression for the open-loop dynamics (9) provides a guideline for the definition of a scalar parameter to be used in measuring the importance of the residual dipole. This concept can be formalised by letting

$$\mu := max\left\{\frac{m_0}{I_x}, \frac{m_0}{I_y}, \frac{m_0}{I_z}\right\}, \quad [A\ kg^{-1}]. \tag{10}$$

μ is an indicator of the impact of magnetic residual dipole on the dynamics of the spacecraft.

Having written the parametrized dynamic matrix of the system it is now possible to use LTP systems theory to study the stability characteristics of the open-loop system as a function of the residual dipole. In this study, the so-called periodic Schur decomposition (see Bojanczyk et al. [1992], Lust [1997]) has been employed. As a first step we prove that the configuration with $\gamma = -90°$ is the *'most stable'*. This is shown by the position of the characteristic multipliers as the angle γ increases from -90° to 90°. Figure 2 shows the characteristic multipliers as a function of the residual dipole angle for a LEO satellite with $\mu = 0.14$. The multipliers start off from the unit circle with $\gamma = -90°$ and move away from it as the residual dipole tends to align to y; it should be noted that the effect on stability of the orientation of the residual dipole is more and more significant as μ increases.

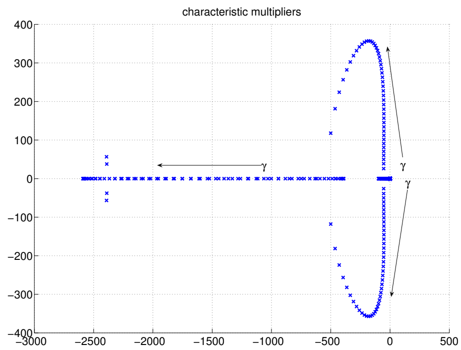

Fig. 2. Characteristic multipliers of matrix (9), for a LEO satellite with $\mu = 0.14$, as the residual dipole rotates in the $x - y$ plane.

It has already been noted that another important parameter is orbital inclination. For the $\mu = 0.14$ case, Figure 3 depicts the characteristic multipliers loci for two different configurations, as the orbital inclination varies between 30° to 50°. In the left hand plot the residual dipole is taken in the $-y$ direction; in the right hand one $\gamma = -80$ is assumed. The advantages of having the dipole in the $-y$ direction are clear: in this case the inclination of the orbit does not affect stability of the system as the characteristic multipliers move on the unit circle. Finally, though details

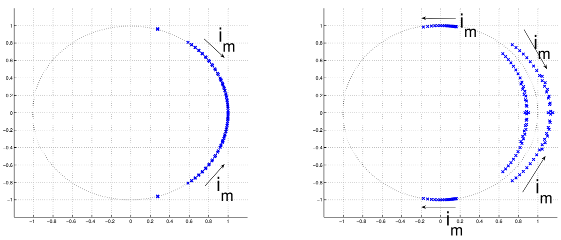

Fig. 3. Characteristic multipliers of matrix (9), for a LEO satellite with $\mu = 0.14$. Orbit inclination from 30 to 50 degrees are considered. In the left hand plot $\gamma = -90$; in the right hand one $\gamma = -80$.

are omitted for brevity, it can be seen that if the stability analysis is carried out by varying the orbit altitude or the dipole magnitude it is apparent that they do not play a role in the position of the characteristic multipliers.

4. CONTROLLER DESIGN

In this Section we turn to the issues associated with control design. In particular, we will first discuss the issue of robustness with respect to the orientation of the magnetic residual dipole, and subsequently we will turn to the problem of dealing with the nominal magnetic disturbance torque (8).

4.1 Stabilisation of attitude dynamics

As mentioned in the Introduction, in the practice of attitude control systems design magnetic residuals are conventionally treated as external disturbances. However, the results of the previous Section seem to indicate that a residual dipole with adverse characteristics may lead to a significantly unstable satellite and, as a consequence, to a more critical design problem. The two apparently contradictory views can be reconciled by noting that the instability brought in by magnetic residuals is associated with the modulation of the geomagnetic field, which is intrinsically very slow. Therefore, while it is expected that a wider controller bandwidth will be needed in order to dominate the effect of magnetic residuals and stabilise the satellite, such a bandwidth increase might not be as dramatic as expected from the inspection of the characteristic multipliers loci of Figure 2, *provided that the available actuators can ensure the necessary control authority.* In order to investigate in which situation the uncertainty associated with the direction of the magnetic residual dipole vector can actually give rise to stability problems, we have considered a satellite with $\mu = 0.28$ and designed two different controllers in the nominal $\gamma = -90$ configuration. The first control law is of the form $\tau_a = Kx$, where the gain K has been designed using LQ control theory, with the aim of achieving a closed loop settling time of about 150 seconds on each axis. The second control law, on the other hand, assumes the use of magnetic torquers as actuators and is of the form

$$\tau_a = \frac{1}{b^T b} S(b) S^T(b) K x, \quad S(b) = \begin{bmatrix} 0 & b_z & -b_y \\ -b_z & 0 & b_x \\ b_y & -b_x & 0 \end{bmatrix},$$

(see, e.g., Sidi [1997], Silani and Lovera [2005] for details on magnetic attitude control), and the gain K has been chosen in order to ensure closed-loop stability of the nominal configuration, with a settling time of about 1500 seconds on each axis. The effect on the spectral radius of the closed-loop dynamics of the spacecraft is illustrated in Figures 4 and 5 for the wide bandwidth controller and for the magnetic one, respectively. It is apparent that in the first case no significant problems arise because of the uncertainty associated with the orientation of the residual dipole. In the second case, on the other hand, the rotation of the dipole vector can lead to a loss of stability in the closed-loop system.

4.2 Disturbance management

Besides giving rise to periodic perturbations of the linearised attitude dynamics, the presence of a magnetic residual dipole leads to the nominal disturbance torques given by equation (8). Such disturbances lead to very different situations in terms of the design of the attitude control system depending on the actual *source* of the residual dipole. The simplest situation is the one arising when the residual dipole is due to the magnetisation of components of the satellite platform (solar arrays, batteries, power electronics...). In this case, suitable schemes for disturbance estimation and compensation can be implemented, such as the ones presented in Pittelkau [1993], Lovera et al. [2002]. The idea is to actively compensate the disturbance torque, provided that the available actuators allow for such an explicit compensation. Indeed, the most critical situation corresponds to a case in which the residual dipole is too large to be explicitly compensated, and/or it is necessary for the operation of the payload. In such a situation (which might occur in missions designed to run particle physics experiments) the problem of reducing the needed propellant is particularly compelling. If the magnetic disturbances are instantaneously greater than the torque that can be exerted by the actuators, one has to deal with the fact that some degree of pointing error is unavoidable. Therefore the goal is to find means to optimize the actuator usage while satisfying the pointing requirements.

Actuator usage reduction can be achieved by exploiting the magnetic disturbance to one's favor. The net effect of the magnetic torque is to align the residual dipole with the local direction of the geomagnetic field. As shown in Section 3, this phenomenon is undamped, but a controller may be devised to damp this dynamics and considerably reduce the control torques. One way to implement this solution is to track a moving attitude reference computed

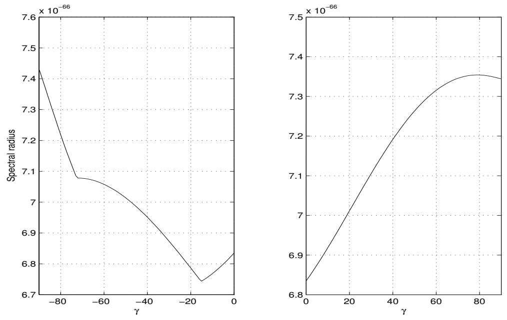

Fig. 4. Spectral radius for a LEO satellite with $\mu = 0.14$, controlled with wide bandwidth feedback, as the residual dipole rotates in the $x - y$ plane.

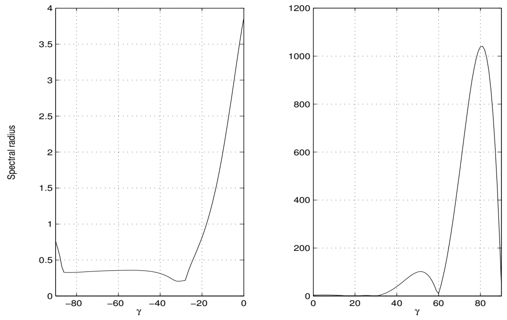

Fig. 5. Spectral radius for a LEO satellite with $\mu = 0.14$, controlled with magnetic actuators, as the residual dipole rotates in the $x - y$ plane.

so to keep the residual dipole aligned with the local direction of the magnetic field. It should be pointed out that this strategy cannot always be applied; self-alignment may not meet the pointing requirements for the specific mission; nevertheless the idea can still be employed to reduce control action. The rationale is to compute the reference attitude so to minimize the magnetic interaction, while satisfying the pointing requirements. This can be achieved by following the perfect alignment whenever it yields acceptable errors and by staying on the error constraint boundary for the rest of the orbit. The field alignment condition is a two degrees of freedom constraint; the remaining degree of freedom is a rotation around the y axis (the one aligned with the field) which is chosen to minimize the pointing error. If the aforementioned case with the residual dipole in the -y direction is assumed and the pointing error specification is given in terms of Absolute Pointing Error (APE), the field aligned reference frame can be computed as in Algorithm 1.

input: threshold
while *in scientific mode* **do**
  measure/estimate local magnetic field $\mathbf{b}$;
  $\mathbf{y} := -\mathbf{b}$;
  measure/estimate local zenith/nadir $\mathbf{r}$;
  $\mathbf{k} := \frac{\mathbf{y}\times\mathbf{r}}{|\mathbf{y}\times\mathbf{r}|}$;
  rotate $\mathbf{y}$ of 90° around $\mathbf{k}$ and call the vector $\mathbf{z}$;
  complete the reference frame0 ($\mathbf{x}$,$\mathbf{y}$,$\mathbf{z}$) ; `/* frame0 is aligned with the magnetic field */`
  APE := acos($\mathbf{r}\cdot\mathbf{z}$) ;
  if *APE < threshold* **then**
    frame_set := frame0 ; `/* magnetic alignment is achievable */`
  else
    θ := APE - threshold;
    rotate frame0 of θ around $\mathbf{k}$ and call it frame1;
    frame_set := frame1;
  end
  feed frame_set to the three-axis controller;
end

Algorithm 1: Moving reference generation.

5. SIMULATION RESULTS

In this Section, the results obtained in the design of an attitude control law for a LEO satellite characterised by a large magnetic residual dipole are presented and discussed. An approximately spherical satellite with $\mu = 0.14$ is considered. The satellite orbit is designed to decay from an altitude of 500 km to to 250 km while the orbit inclination varies from 30° to 50°. The mission requires a maximum APE of 15°.

The simulations have been carried out using an object-oriented environment for satellite dynamics (see Lovera [2006] for details) developed using the Modelica language. For the purpose of the present study, a full nonlinear simulation of the coupled orbital and attitude dynamics has been performed. The simulation of the space environment has been performed on the basis of the JGM-3 spherical expansion for the geopotential as a gravitational model, of the Harris-Priester model for the atmospheric density distribution (see Montenbruck and Gill [2000] for details) and of the International Geomagnetic Reference Field (IGRF, see Wertz [1978]) for the Earth's magnetic field. Disturbance torques due to gravity gradient (including J_2 effects), magnetic residual dipole and solar radiation pressure (computed using the solar coordinates formulas given in Montenbruck and Gill [2000]) have been taken into account in the simulation.

In order to check the feasibility of the perfect alignment strategy the APE for different orbital inclinations is computed and plotted in Figure 6.

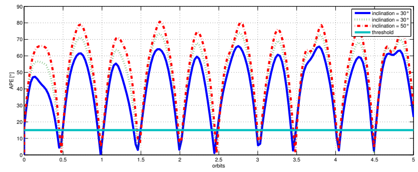

Fig. 6. Absolute pointing error for different orbit inclinations under perfect magnetic field alignment.

The following remarks can be made:

- The pointing error meets the specifications only for a small fraction of the orbit (around 14% of the orbit). The duration of this period mainly depends on the orbital inclination.
- The greater the inclination is, the greater the pointing errors are.
- Altitude does not have any effect on the pointing error.

Although the self-aligning controller does not meet the mission specifications, the moving reference solution can be adopted in order to reduce the control action. The strategy described in Section 4 is implemented with an acceptable APE of 15°. This choice allows to identify an upper bound on the achievable torque reduction. In the final implementation it may be necessary to reduce the threshold to account for closed loop tracking errors and errors in the magnetic field measure/estimate.

The results for the case of a low altitude orbit with a 50° inclination are thoroughly described in the following. Note that a normalized scale was used for confidentiality reasons; it is still possible, however, to appreciate the differences between the two control strategies. As previously described, the 50° orbit is the worst case; it will be shown that this remains true also with the moving reference attitude. A period of 24 hours is simulated and the pointing errors and required torque are depicted. In order to better appreciate the advantages of the proposed solution, the results are compared to the ones of a three axis state feedback controller (referred to as fixed reference controller). Figure 7 depicts the APE for the perfect alignment with the magnetic field and the moving reference case. It is clear that when the perfect alignment satisfies the pointing error specifications, the alignment is followed; while during the rest of the orbit the attitude that minimizes the product $\tau = m \times b$ with the APE constraint of 15° is the one that yields the maximum acceptable error. Figure 8 shows the disturbance torques that need to be canceled by the actuators to achieve perfect tracking of the commanded reference. The moving reference is superior under two aspects. The new strategy requires less torque around the x axis than the fixed reference strategy and the satellite

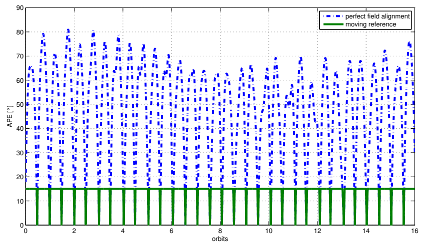

Fig. 7. Absolute pointing error.

is not subject to any disturbance around the z axis. Both strategies do not suffer from magnetic disturbance around the y axis.

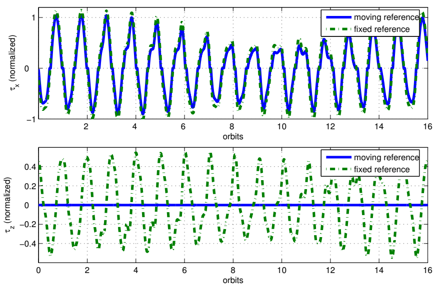

Fig. 8. Magnetic torques to be compensated.

Finally the norm of the total control torque is discussed; this variable is particularly important because it allows to evaluate the needed propellant and the savings introduced by the new control strategy, whenever thrusters are used. Figure 9 shows this variable.

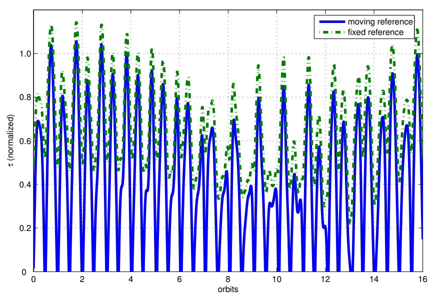

Fig. 9. Norm of actuation torque.

Table 1 summarizes the results of the moving reference strategy. The table shows the time spent in magnetic alignment, the reduction of the maximum torque with respect to the fixed reference strategy in the same orbital conditions and an estimate of the propellant reduction that the moving reference strategy allows with respect to the fixed reference. It can be noted that the savings introduced

inclination	10°	20°	30°	40°	50°
% time in field alignment	47%	26%	16%	12%	10%
% max torque reduction	34%	21%	15%	11%	7%
% prop. reduction	66%	55%	45%	39%	36%

Table 1. Moving reference strategy summary.

by the moving reference are of considerable magnitude and are inversely proportional to the orbit's inclination.

6. CONCLUDING REMARKS

A model of the attitude dynamics for a spacecraft with a large magnetic residual has been derived and the stability of the linearised, open-loop dynamics has been analysed. The results of the analysis have guided the design of a closed loop strategy useful whenever the magnetic disturbance cannot be directly compensated. The controller is based on the tracking of a moving reference computed so as to minimize the magnetic disturbance while satisfying the pointing requirements. The devised policy is applied to a case study and it is shown that this controller can reduce the amount of needed propellant up to 45% (with respect to the fixed reference controller).

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