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Nonparametric Identification of the Nonlinear Element in Wiener Systems

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Abstract: We are considering the problem of identifying Wiener systems that includes memoryless nonlinearities. The focus is made on the determination of the system nonlinearity which is not necessarily invertible, smooth or parametric. To this end, a frequency approach is developed, that investigates the system output extrema. In the case where the nonlinearity is strictly monotonic, a simple experiment is performed involving the application of a sine signal. In the general case, such an experiment is repeated a few times with different amplitudes.

1. INTRODUCTION

An important research activity is devoted to the problem of nonlinear system identification based on Wiener models. Most of the proposed solutions have been developed supposing that the nonlinearity is a polynomial of known degree and the linear part is a transfer function of known order, see e.g. (Chou *et al.*, 1999), (Hasiewicz, 1987), (Hunter *et al.*, 1986), (Nordsjö, 2001), (Pajunen, 1992), (Voros, 1997) and (Wigren, 1993). The proposed identification algorithms has used iterative optimisation methods. But, these are shown to be efficient provided that the iterative process converges, e.g. see (Voros, 1997), (Wigren, 1993). Unfortunately, the convergence is not guaranteed except under restrictive conditions ((Wigren, 1993)). Frequency-type solutions have also been proposed, see e.g. (Gardiner, 1993). The idea is to apply repeatedly a sinusoidal input with different amplitudes and frequencies. Then, exploiting the polynomial nature of the nonlinearity, the input-output equation can be uniquely solved with respect to the unknown parameters. Nonparametric nonlinearities have in turn been dealt with using different approaches. In (Greblicki, 1992)-(Greblicki, 1997), the identification problem is coped with using stochastic tools. But, the input signal is assumed to be a white noise and the nonlinearity is supposed to be invertible. In (Bai, 2003) a frequency solution is proposed for noninvertible nonlinearities. However, that phase estimator is not generally consistent and, consequently, the consistency of the overall identification method is, in turn, not generally guaranteed, see (Giri et al., 2007).

In this paper, we are considering Wiener system identification in presence of not necessarily parametric, invertible and smooth nonlinearities. The focus is precisely made on the estimation of the nonlinearity, knowing that if this were available then the linear subsystem could be recovered using exiting methods (e.g. (Hu *et al*., 2005)). To this end, we will investigate the correlation between the extrema of the (unmeasured) internal signal $x(t)$ and those of the system output $y(t)$.

2. IDENTIFICATION PROBLEM STATEMENT

2.1 Class Identified Systems

We are considering nonlinear systems that can be described by the Wiener model (Fig.1), with a memoryless nonlinear element characterized by a piecewise continuous function $f(.)$. The above model is analytically described by the following equations:

$$x(t) = g(t) * u(t) \tag{1}$$

$$y(t) = f(x(t)) + v(t) \tag{2}$$

g(t) denotes the inverse Laplace Transform of *G(s)*; *x(t)* is a (non-measurable) internal signal; the noise $v(t)$ is a supposed to be a zero-mean stationary ergodic stochastic process.

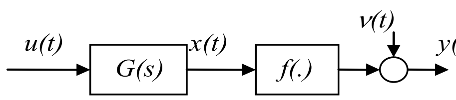

Fig. 1. Wiener model

2.2 Identification objective

Our purpose is to design an identification scheme that determines the function $f(x)$, in the interval $-U|G(j\omega)| \le x \le U|G(j\omega)|$, for a given couple

$U > 0, \omega > 0$. Since $x(t)$ is not measurable, the system identification should be fully based upon measurements of the input $u(t)$ and the output $y(t)$. Therefore, the considered identification problem does not have a unique solution: if the couple $\left(f(x), G(s)\right)$ represents a solution then, any couple of the form $\left(f(Kx), G(s)/K\right)$ is also a solution (where K is any nonzero real). Such a lack of uniqueness, will be exploited (in Section 3) to cope with the uncertainty on the amplitude of the internal signal $x(t)$.

3. BASIC MATHEMATICAL FACTS

3.1 Wiener Model Rescaling and Identification Problem Reformulation

All along this Section, the identified system is submitted to a given sine input:

$$u(t) = U \sin(\omega t) \quad (t \geq 0) \tag{3}$$

where the amplitude U>0 and the frequency $\omega > 0$ are kept constant. Let T be the corresponding period ($T = 2\pi/\omega$). Then, it follows from (1) that the internal signal turns out to be (in steady state) $x_u(t) = X_U \sin(\omega t - \varphi)$ with $X_U = U.\left|G(j\omega)\right|$ and $\varphi = -\arg\left(G(j\omega)\right)$. The resulting output signal is $y(t) = f(x_u(t)) + v(t)$. The above expression of $x_U(t)$ is preferably rewritten in the following form:

$$x_U(t) = X_U \cos(\omega(t - t_\varphi)) \text{ with: } \quad t_\varphi \overset{def}{=} \varphi/\omega + \pi/2 \tag{4}$$

$x_U(t)$ is not available since neither the amplitude X_U nor the phase φ are known. The first uncertainty can be coped with rescaling the system model (1) (making use of the fact that the model is not unique). Specifically, the focus will be made on the following rescaled models:

$$M^+(U) \overset{def}{=} \left(f_U^+, G_U^+(s)\right), M^-(U) \overset{def}{=} \left(f_U^-,\ G_U^-(s)\right) \tag{5}$$

$$\text{with:} \quad f_U^+(x) = f(X_U x), \qquad f_U^-(x) = f(-X_U x) \tag{6}$$

$$G_U^+(s) = \frac{1}{X_U} G(s), \quad G_U^-(s) = \frac{1}{-X_U} G(s). \tag{7}$$

The new models (5) also represent the system and generate respectively the following internal signals:

$$x^+(t) = \cos(\omega(t - t_\varphi^+)),\ x^-(t) = \cos(\omega(t - t_\varphi^-)) \tag{8}$$

where $t_\varphi^+ = t_\varphi$; $t_\varphi^- = t_\varphi + \pi/\omega$. That is, the new internal signals are independent on the amplitude U (contrarily to the signal $x_U(t)$ associated with the initial model (1)). Nevertheless, all models generate the same output, i.e.

$$y(t) = y_u(t) + v(t) \tag{9}$$

$$\text{with: } y_U(t) = f_U^+(x^+(t)) = f_U^-(x^-(t)) = f(x_U(t)) \tag{10}$$

In the light of the above observations, it is clear that the parameter t_φ turns out to be the only uncertain parameter. In the sequel, we seek identification of either f^+ or f^-. Notice that these are not distinguishable from the system input and output signals ($u(t)$, $y(t)$). So, it is not important which one of them will actually be determined.

3.2 Analysis of Internal and Output Signals Extrema

The identified system is submitted to the sine input (3) where U>0 and $\omega > 0$ are constant. First, let us investigate the correspondence between the extrema of $x^+(t)$ and $x^-(t)$, on one hand, and those of $y_U(t)$, on the other hand.

3.2.1 Correspondence between extrema of the internal signals and those of the undisturbed output:

It is clear that the global extrema of $x^+(t)$ and $x^-(t)$ occur at the instants:

$$t_i = t_\varphi + i\pi/\omega \quad (i=0,\ 1,\ 2,\ \ldots) \tag{11}$$

On the other hand, one gets from (10):

$$\frac{dy_U(t)}{dt} = \frac{df_U^+}{dx}(x^+(t))\frac{dx^+(t)}{dt} = \frac{df_U^-}{dx}(x^-(t))\frac{dx^-(t)}{dt} \tag{12}$$

Then, one has, for all integers i:

$$\frac{dy_U(t_i)}{dt} = 0 \tag{13}$$

That is, each extremum of $x^+(t)$ and $x^-(t)$ gives rise to an extremum of $y_U(t)$ and all these extrema occur at the same instants defined by (11), independently of the input magnitude. However, it is clear from (12) that $y_U(t)$ may well have other extrema occurring at different instants. These are simply the solutions of the equations:

$$\frac{df_U^+}{dx}(x^+(t)) = 0, \qquad \frac{df_U^-}{dx}(x^-(t)) = 0 \tag{14}$$

In the sequel, the focus will only be made on the extrema (of $x^+(t)$, $x^-(t)$, $y_U(t)$) that occur at the instants t_i defined by (11). Indeed, if one of these were identified it would be possible to determine the quantities $(t_\varphi^+, t_\varphi^-)$ and, equivalently, the unknown phase φ. The crucial issue is how to recognize the t_i's when only a recording of the (disturbed) output signal *y(t)* is available? The following subsection is a first step to answer such a question.

3.2.2 Characterization of internal signal extrema using the measured output:

First, notice that when the system is excited by the sine input defined by (3), the undisturbed output $y_U(t)$ is, in steady-state, periodic with period $T/m = 2\pi/m\omega$, for some (unknown) integer $m \geq 1$. Then, the effect of output noise can be removed resorting to the following specific filtering:

$$y(t,N) \overset{def}{=} \frac{1}{N}\sum_{k=0}^{N-1} y(t+kT) \text{ for } t \in [0,\, T) \qquad (15)$$

where $T = 2\pi/\omega$ and N is a sufficiently large integer. The averaged output thus defined is now related to the undisturbed output $y_U(t)$:

Proposition 3.1. Consider the system (1), submitted to Assumptions A1-A3, excited by the sinusoidal input (3) where $U>0$ and $\omega > 0$ are arbitrary but constant. Then, for t∈[0, T): $\lim_{N\to\infty} y(t,N) = y_U(t)$ (w.p.1). Consequently, one has for $t = t_0$ and $t = t_1$: $\frac{d}{dt}\left(\lim_{N\to\infty} y(t,N)\right) = 0$ (w.p.1), where the t_i's are defined by (11). □

Proof. As $y_U(t)$ is (in steady-state) periodic with period T/m, it follows from (9) that, for any real t and all integers k:

$$y(t+kT) = y_U(t) + v(t+kT) \qquad (16)$$

On the other hand, the ergodicity of $v(t)$ implies that, for any fixed *t*:

$$\lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} v(t+kT) = E(v(t+kT)) = 0 \text{ (w.p.1)} \qquad (17)$$

Combining this with (16), one gets, for all t:

$$\lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} y(t+kT) = y_U(t) \text{ (w.p.1)}$$

This proves the proposition □

It is thus established that, just as $y_U(t)$, the averaged output $y(t,N)$ has in turn two extrema in $[0,\, T)$, occuring at the instants t_i ($i = 0,1$) defined by (11). It may happen that $y(t,N)$ possesses other extrema at different instants. But, the extrema of interest are those occurring at the instants t_i. Now, the question is: how to recognize these instants when a recording of the undisturbed system output is available? To answer such a question, a procedure is described in Section 4 for the case of strictly monotonic functions $f(.)$. The general case is considered in section 5.

4. IDENTIFICATION OF MONOTONIC FUNCTIONS $f(.)$

The system is again submitted to a sine input $u(t) = U\sin(\omega t)$ where the amplitude $U>0$ and the frequency $\omega > 0$ are kept constant all along the present section. Furthermore, the function $f(.)$ is presently supposed to be either (strictly) increasing or (strictly) decreasing. It is clear that if $f(.)$ is increasing then $f_U^+(.)$ and $f_U^-(.)$ will, respectively, be increasing and decreasing and vice-versa. Whatever the precise situation, one has: $\frac{df_U^+}{dx}(x) \neq 0$ and $\frac{df_U^-}{dx}(x) \neq 0$ (for all *x*)

which yields, due to (12):

$$\frac{dy_U(t)}{dt} = 0 \Leftrightarrow \frac{dx^+(t)}{dt} = \frac{dx^-(t)}{dt} = 0$$

$$\Leftrightarrow t = t_i \;\; (i = 0,\, 1,\, 2,\, ...) \qquad (18)$$

Then, it follows from (18) that $y_U(t)$ has no other extrema than those occurring at the instants t_i. Furthermore, the monotony of $f_U^+(.)$ and $f_U^-(.)$ implies that these extrema are global, just as are the corresponding extrema of $x^+(t)$ and $x^-(t)$. Moreover, if the maximums of $y_U(t)$ occur at the instants t_{2j}, then its minimums will necessarily occur at the instants t_{2j+1}. Let τ_i (i = 0,1, 2,...) denote the instants where $y_U(t)$ really takes its maximums. Then, since $y_U(t)$ has the same period as $u(t)$, one has:

$$\tau_i - \tau_{i-1} = 2\pi/\omega \;\; (i = 1,2,...) \qquad (19)$$

The following proposition shows that, if one of the instants τ_i ($i = 0,\, 1,\, 2,\, ...$) were known then it would be possible to determine the model nonlinearity.

Proposition 4.1. Consider the system (1) submitted to the input signal (3) where $U>0$ and $\omega > 0$ are arbitrary but constant.

1) If $f(.)$ is strictly monotonic, there exists a Wiener model $M^* \in \{M^+(U), M^-(U)\}$ such that the maximums of the corresponding internal signal, say $x^*(t)$, occur at the instants τ_i ($i = 0,\, 1,\, 2,\, ...$).

2) More precisely, if $f(.)$ is increasing (resp. decreasing) then $M^* = M^+(U)$ and $x^*(t) = x^+(t)$ (resp. $M^* = M^-(U)$ and $x^*(t) = x^-(t)$).

3) Let f^* denote the nonlinearity associated to M^*. Then, one has for all t:

$$\lim_{N\to\infty} y(t,N) = f^*(x^*(t)) \text{ (w.p.1)} \qquad \square$$

Proof. The proof is omitted due to the limitation of the paper's length. □

Since y_U is not measurable, the τ_i 's have to be estimated from the filtered output $y(t,N)$. Making full use of Propositions 3.1 and 4.1, the following procedure is proposed to get estimates of the τ_i 's and, consequently, of the model nonlinearity.

Strictly Monotonic Nonlinearity Identification (MNI):

MNI-1. Apply a sine input $u(t)=U\sin(\omega t)$ to the nonlinear system of interest and get a recording of the output *y(t)* over a sufficiently large interval (the recording is preferably started in steady state).

MNI-2. Generate the averaged version $y(t,N)$ taking a sufficiently large value of *N*. Note τ^* any instant where $y(t,N)$ achieves its maximum (the next one is $\tau^*+2\pi/\omega$).

MNI-3. Let the internal signal $x^*(t)$ be a cosine (with period $2\pi/\omega$ and amplitude 1) that takes its maxima at τ^* and $\tau^*+2\pi/\omega$. That is $x^*(t)=\cos(\omega(t-\tau^*))$. Then, the parameterized curve $\left(x^*(t),y(t,N)\right)$, with $\tau^*\le t\le\tau^*+2\pi/\omega$ defines an estimate $\hat{f}_N(.)$ of the nonlinearity $f^*(.)$. The larger is *N* the better the estimate quality.

5. IDENTIFICATION OF NON MONOTONIC FUNCTIONS $f(.)$

In such a case, the main difficulty lies in the fact that the system output $y(t)$ (resulting from a sine input $u(t)=U\sin(\omega t)$) may possess other extrema than those produced by the internal signal ($x^+(t)$ or $x^-(t)$). Indeed, other extrema may occur, specifically there where equations (14) have solutions. Moreover, the set of instants where (14) is satisfied may be uncountable. As in Section 3, attention will be paid to the extrema of $y_U(t)$ that are associated to the internal signal. These extrema will be based used to solve the problem at hand, i.e. the determination of the system nonlinearity. For this reason, they will be referred to 'useful extrema'. Accordingly, the other extrema, if any, are called useless. Now, the question is: how to recognize the useful extrema when a (graphical) recording of the output *y(t)* is available?

To answer such a question let us analyze the effect that a change of the input amplitude will produce on signals and models (especially (5)-(6)). To this end, consider two sine inputs that only differ by their amplitudes:

$$u_1(t)=U_1\sin(\omega t),\quad u_2(t)=U_2\sin(\omega t) \tag{20}$$

where $U_1\neq U_2$. Let $\left(y_U^1(t),y_U^2(t)\right)$ and $\left(y_1(t),y_2(t)\right)$ denote the resulting undisturbed and disturbed outputs. Referring to the (initial) model $\left(f(.),G(s)\right)$, defined by equation (1), the internal signals turn out to be:

$$x_1(t)=X_1\sin(\omega t-\varphi),\quad x_2(t)=X_2\sin(\omega t-\varphi) \tag{21}$$

where $\varphi=-\arg\left(G(j\omega)\right)$ is independent of the input amplitude. Then, it readily follows that, the extrema of $x_1(t)$ and $x_2(t)$ take place at the same instants:

$$t=k_i\overset{def}{=}(2\varphi+\pi+2\pi i)/2\omega\ (i=0,1,2,\ldots) \tag{22}$$

i.e. they are not affected by a change of the input amplitude. Furthermore, one has:

$$\frac{dy_U^1(t)}{dt}=\frac{df}{dx}(x_1(t)).\frac{dx_1(t)}{dt}, \tag{23a}$$

$$\frac{dy_U^2(t)}{dt}=\frac{df}{dx}(x_2(t)).\frac{dx_2(t)}{dt} \tag{23b}$$

implying: $$\left.\frac{dy_U^1(t)}{dt}\right|_{t=k_i}=\left.\frac{dy_U^2(t)}{dt}\right|_{t=k_i}=0 \tag{24}$$

This yields the following statement:

Proposition 5.1. The output signals ($y_U^1(t)$, $y_U^2(t)$) generated by the system (1) in response to the inputs ($u_1(t)$, $u_2(t)$) exhibit the following features:

1) The useful extrema of $y_U^1(t)$ and $y_U^2(t)$ are achieved at the same instants i.e. these only depend on the frequency ω and not on the amplitude of the applied input. Consequently, a change on the input amplitude only produces (at the output) a change on the useful extrema amplitude. This is simply expressed saying that the useful extrema moves vertically when the input amplitude changes.

2) The useless extrema of $y_U^1(t)$ may not be (and generally are not) achieved at the same instants as those of $y_U^2(t)$ i.e. these instants depend on both the amplitude and frequency of the applied input. This observation is simply expressed saying that the useless extremum move horizontally (and they are the only to do so) when the amplitude of the applied input is changed. □

The above proposition is graphically illustrated by fig. 2 that shows the moving of both type of extrema when the non linearity is $f(x)=10x/(1+0.5x^2)$. The result of Proposition 5.1 is not immediately utilizable (to determine the instants of occurrence of the useful extrema), since it involves the non-measurable outputs ($y_U^1(t)$, $y_U^2(t)$). However, it can be made immediately utilizable (just as we did in Section 3) by simply substituting to ($y_U^1(t)$, $y_U^2(t)$) their filtered versions $(y_1(t,N), y_2(t,N))$. This is formally stated in the following procedure:

Nonmonotonic Nonlinearities Identification (NNI): first part

NNI-1. Apply successively 2 or more sine inputs ($u_1(t)$, $u_2(t)$, …) with different amplitudes but the same frequency. Get a recording of the resulting outputs $(y_1(t), y_2(t), ...)$ and generate their filtered versions $(y_1(t,N), y_2(t,N), ...)$ according to (15).

NNI-2. Compare the extrema of the filtered output signals and select all those that take place (in the different output recording) at the same instants (provided they are equally spaced). The extrema thus selected are the useful extrema; during the selection process make use of the fact that a useful extrema comes on each π/ω seconds, that is in each period one gets 2 (and only 2) useful extrema.

NNI-3. If necessary, take a large N or make one more experiment with a different amplitude and go back to step NNI-1.

Now to determine the nonlinearity, we will exploit again the data corresponding to the experiment made in NNI-1 corresponding to the largest input signal amplitude. For the selected experiment, the notations of Section 3 are resorted to i.e. the input $u(t) = U \sin(\omega t)$, the filtered output $y(t,N)$, the specific models ($M^+(U)$, $M^-(U)$), the resulting internal signals $x^+(t) = \cos(\omega(t - t_\varphi^+))$, $x^-(t) = \cos(\omega(t - t_\varphi^-))$. Let us point out some mathematical facts: First, note that in any time-interval of length $T = 2\pi/\omega$, there are two (and only two) useful extrema of $y_U(t)$ taking place in that interval. These may be (or not) of the same nature (minimum or maximum). Whatever the situation, two extrema (and only two) will be seen at any recording of $y(t,N)$, over a one period T. The first one is referred to 'reference extremum' and the corresponding instant is denoted τ_1. The second useful extremum is located at the instant $\tau_2 = \tau_1 + T$.

Proposition 5.2. There exists a Wiener model $M^* \in \{M^+(U), M^-(U)\}$ such that the corresponding internal signal $x^*(t)$ achieves its maxima (that are all equal to 1) at the instants $T_i \overset{def}{=} \tau_1 + iT$ $(i = 0, 1, 2, ...)$ □

Proof. the proof is omitted due to the paper's length. □

Based on the above result, we can continue the procedure ***NNI*** towards the identification of the nonlinearity f^* associated to the model M^*.

Nonmonotonic Nonlinearities Identification (NNI): second part

NNI-4. Let $y(t,N)$ be any one of the filtered outputs obtained in step **NNI-1**. Evaluate quickly from such a recording an estimate $\hat{\tau}_1$ of the instant τ_1.

NNI-5. Let $\hat{x}(t)$ be a cosine (with period $2\pi/\omega$ and amplitude 1) that takes its maximums at instants $\hat{T}_i = \hat{\tau}_1 + \frac{2\pi}{\omega} i$; (i = 1, 2, ...). Then, $\hat{x}(t)$ is an estimate of $x^*(t)$. Specifically, $\hat{x}(t) = \cos(\omega(t - \hat{\tau}_1))$. Furthermore, the parameterized curve $(\hat{x}(t), y(t,N))$, with $\hat{\tau}_1 \le t \le \hat{\tau}_1 + 2\pi/\omega$ defines an estimate $\hat{f}_N(.)$ of the nonlinearity $f^*(.)$. The larger is N the better the estimate quality.

Refined search of the extremum instant τ_1

The estimate $\hat{\tau}_1$ determined graphically may be improved using analytical tools. First, remark that one has, for any integer i and any real δ: $x(T_i - \delta) = x(T_i + \delta)$. Then, one has for all i and δ: $y_U(T_i - \delta) = f(x(T_i - \delta)) = f(x(T_i + \delta)) = y_U(T_i + \delta)$. That is, the output $y_U(t)$ that is periodic with period $T/m = 2\pi/m\omega$ (for some $m \ge 1$) presents in turn a symmetry with respect to the vertical axes passing by $t = T_i$ (*i=0, 1, 2, …*). Note also that the integer m can simply be determined using the fact that ω and the period $T/m = 2\pi/m\omega$ are both known. These observations will now be exploited to improve the estimate $\hat{\tau}_1$. To this end, let us introduce the following parameterized function:

$$s_\tau(t) \overset{def}{=} \begin{cases} 1 & \text{if} \quad \tau \le t < \tau + \pi/m\omega \\ -1 & \text{otherwhise} \end{cases} \tag{25}$$

where τ is any real such that $0 \le \tau \le \pi/m\omega$. Consider the following integral quantities:

$$J(\tau) \overset{def}{=} \int_0^{\frac{T}{m}} s_\tau(t) y_U(t) dt \qquad (T = 2\pi/\omega) \tag{26a}$$

$$I(\tau, N) \overset{def}{=} \int_0^{\frac{T}{m}} s_\tau(t) y(t,N) dt \tag{26b}$$

The symmetry of $y_U(t)$ with respect to the vertical axis passing by $t = T_1 = \tau_1$, implies that $J(\tau_1) = 0$. On the other hand, it follows using Proposition 3.1 that, for any fixed $\tau \in [0, \pi/m\omega)$: $I(\tau, N) \to J(\tau)$ as $N \to \infty$ (w.p. 1). From the above observations one gets:

$$\lim_{N \to \infty} I(\tau_1, N) = 0 \qquad \text{(w.p. 1)} \tag{27}$$

Therefore, the instant τ_1 can be determined searching the minimum of $|I(\tau, N)|$ with respect to τ, using usual iterative algorithms. Since the function $|I(\tau,N)|$ may have several (local) minima (due to others possible symmetries with respect to vertical axes passing by others instants), the search procedure should be initialized near the minimum of interest, namely τ_1. Indeed, the graphically obtained estimate $\hat{\tau}_1$ constitutes an appropriate initial value.

6. SIMULATION

The identification method developed in the previous sections will now be illustrated considering a Wiener system characterized by:

$$G(s)=\frac{12(s-1)}{(s+1)(s+2)}, \quad f(x)=\frac{10x}{1+0.5x^2} \qquad (28)$$

$v(t)$ is a sequence of uniform random number in $[-1,+1]$. The above system is successively submitted to sinusoidal inputs:

$u_1(t)=U_1\,sin(\omega t)$, $u_2(t)=U_2\,sin(\omega t)$, $U_1=1$, $U_2=0.8$, $\omega=\pi\ rad/s$. Note that, letting $u(t)=u_1(t)=U_1\,sin(\omega t)$, one gets for system (28):

$$f^+(x)=\frac{32.22x}{1+(1.611x)^2}, \; f^-(x)=-\frac{32.22x}{1+(1.611x)^2}$$

Following the NNI procedure, the averaged outputs $y_1(t,N)$ and $y_2(t,N)$ are generated, using (15) with $N=140$. The obtained signals are represented by fig. 2, and an estimate $\hat{\tau}_1$ (of τ_1), by a simple inspection. The obtained estimate has served to initialize the optimization procedure described in the end of Section 5. A refined estimate is thus obtained, namely $\hat{\tau}_1=62.8T/200$ (with $T=2\pi/\omega$). Then, as pointed out in ***NNI-5***, $\hat{x}(t)=cos(\omega(t-\hat{\tau}_1))$ turns out to be an estimate of $x^*(t)$ and the parameterized curve $(\hat{x}(t),y(t,N))$ defines an estimate $\hat{f}_N(.)$ of the nonlinearity $f^*(.)$; the obtained curve is plotted in fig. 3. These show that $\hat{f}_N(.)$ is an estimate of f_U^+.

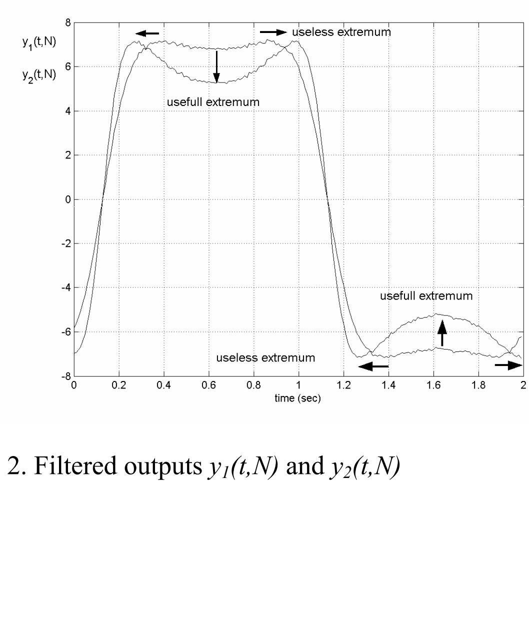

Fig. 2. Filtered outputs $y_1(t,N)$ and $y_2(t,N)$

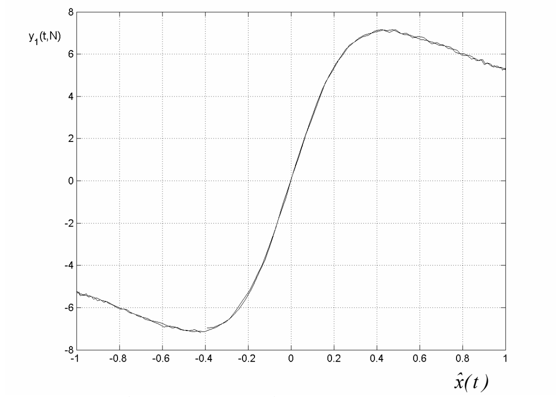

Fig. 3 The plot $(\hat{x}(t),y_1(t,N))$

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