Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

A New Proportional Controller for Nonlinear Bilateral Teleoperators ⋆

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Abstract: One of the major breakthroughs in the problem of control of bilateral teleoperators with guaranteed stability properties has been the use of scattering signals to transform the transmission delays into a passive transmission line. Under the reasonable assumption that the human operator and the contact environment define passive (force to velocity) maps, stability of the overall system is then ensured. This robust and physically appealing scheme, first proposed by Anderson and Spong, has ever since dominated the field. In this paper we propose two novel teleoperation schemes, based on a simple P-Like controller. These schemes do not make use of the scattering or wave variables. Moreover, under the classical assumption of passivity of the terminal operators plus a gravity compensation term, we can ensure position coordination of the master and the slave.

Keywords: Telerobotics, Time-delay, Robot control, Lyapunov functions.

1. INTRODUCTION

The communication channel that connects the master and the slave manipulators, in bilateral teleoperation, often involves large distances or imposes limited data transfer between the local and the remote sites. Such situations can result in substantial delays between the time a command is introduced by the operator and the time the command is executed by the remote robot. This time-delay affects the overall stability of the system (Sheridan [1993]).

Anderson and Spong [1989] proposed, in a groundbreaking work, to send the scattering signals in order to transform the transmission delays into a passive (virtual) transmission line. The transmission line is then interconnected with the master and slave robots, which define passive force to velocity operators, while the human operator and the contact environment constitute the terminations to the transmission line. Since power–preserving interconnection of passive systems is again passive $\mathcal{L}_2$–stability of the overall system is ensured under the reasonable assumption that the human operator and the environment define passive (force to velocity) maps. This robust and physically appealing scheme has ever since dominated the field. [1] See Arcara and Melchiorri [2002] and Hokayem and Spong [2006] for two interesting survey articles focused on control of teleoperators.

Position Coordination or Position Drift has been one of the major drawbacks of the basic scattering/wave variable method (i.e., the position of the slave does not converge to the position of the master). The work of Chopra et al. [2006] shows that this objective is achieved adding a term, proportional to the delayed position error, on the basic scattering transformation. Chopra and Spong [2005] have proposed that if teleoperation is viewed as a synchronization problem, where the objective is to synchronize the master and slave *velocities*, asymptotic stability of the system can be achieved, however, this scheme does not guarantee position coordination. In their work is also shown that if using an adaptive Slotine–Li scheme, then, position coordination can be achieved. In a recent publication (Lee and Spong [2006]) it is claimed that a Proportional Plus Delayed Derivative scheme, that does not require scattering transformations, yields also a stable operation including the position coordination. Unfortunately, the stability analysis hinges upon unverifiable assumptions on the human and contact environment operators, namely, that they define $\mathcal{L}_\infty$–stable maps *from velocity to force*. Notice that even the simplest scenario of a linear spring–damper system does not verify this assumption because the transfer function from velocity to force contains a derivative operator that is not $\mathcal{L}_\infty$–stable.

Namerikawa and Kawada [2006] have considered a *symmetric* teleoperator and they propose a modification to the scattering–based scheme of Chopra et al. [2006] to

⋆ This work has been partially supported by the spanish CICYT projects: DPI2005-00112 and DPI2007-63665, the FPI program with reference BES-2006-13393, and also by the mexican CONACyT grant-169003.

[1] See de Rinaldis et al. [2006] for the application of the dual idea, that is, transform a real transmission line into pure delays, a classical problem of electrical systems.

overcome the need of adding a (twice delayed) term $\dot{\mathbf{q}}_s(t-2T)$ in the slave robots force. Mimicking the derivations of the stability proof in Chopra et al. [2006]. They claimed that the closed–loop system is Lyapunov stable and that velocities and velocity errors asymptotically converge to zero. In their work it is claimed that the controller imposes no restriction on the damping injection, but this seems to be unreliable because their condition (23) exactly coincides with the one given in Chopra et al. [2006]. It is worth mentioning that the Lyapunov–like functions of Namerikawa and Kawada [2006] and the one used in the present paper, (7), are not the same. The former contains an additional term that brings along in the derivative a negative square of the velocity errors, see equation (16) in Namerikawa and Kawada [2006] and it relies on the scattering transformation.

In this paper we prove that indeed it is possible to achieve stable behavior of teleoperators with a simple P–like scheme under the classical assumption of passivity of the terminal operators, providing additional damping (via velocity feedback) to both manipulator subsystems. Two schemes are considered: 1) controlling the master and the slave with the (delayed) position errors and 2) the slave controlled with the same position error and the master with the delayed slave's force. In both cases we prove that all signals remain bounded and that the velocities belong to $\mathcal{L}_2$ for any passive external interaction. (Furthermore, velocities converge to zero if the forces applied by the human and the environment are bounded.) It is also proved that if adding a gravity compensation (and a mild assumption on the inertia matrices) we achieve position coordination. The main contributions of this paper are gathered in Proposition 1 and Proposition 2 which are an extension to our prior work in Nuño et al. [2007].

The paper is arranged as follows: Section 2 presents the dynamic models for the teleoperator; Section 3 analyzes the first scheme, P-like controller for both manipulators; the results on controlling the master with force feedback and the slave with a P-like controller are outlined in Section 4; finally we present some simulations for both schemes in Section 5 followed by the conclusions and future work, Section 7, of this work.

2. MODELING THE N–DOF TELEOPERATOR SYSTEM

The master and the slave are modeled as a pair of n–degree of freedom (DOF) serial links with revolute joints. Their corresponding nonlinear dynamics are described by

$$\begin{aligned}\mathbf{M}_m(\mathbf{q}_m)\ddot{\mathbf{q}}_m + \mathbf{C}_m(\mathbf{q}_m,\dot{\mathbf{q}}_m)\dot{\mathbf{q}}_m + \mathbf{g}_m(\mathbf{q}_m) &= \boldsymbol{\tau}_m - \boldsymbol{\tau}_h \\ \mathbf{M}_s(\mathbf{q}_s)\ddot{\mathbf{q}}_s + \mathbf{C}_s(\mathbf{q}_s,\dot{\mathbf{q}}_s)\dot{\mathbf{q}}_s + \mathbf{g}_s(\mathbf{q}_s) &= \boldsymbol{\tau}_e - \boldsymbol{\tau}_s, \qquad (1)\end{aligned}$$

where $\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \mathbf{q}_i \in \mathbb{R}^n$ are the acceleration, velocity and joint position, respectively. $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ are the inertia matrices, $\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$ the coriolis and centrifugal effects, defined via the Christoffel symbols of the first kind, $\mathbf{g}_i \in \mathbb{R}^n$ the vectors of gravitational forces, $\boldsymbol{\tau}_i \in \mathbb{R}^n$ are the control signals and $\boldsymbol{\tau}_h \in \mathbb{R}^n$, $\boldsymbol{\tau}_e \in \mathbb{R}^n$ are the forces exerted by the human operator and the environment interaction, respectively. $i = m$ for the master and $i = s$ for the slave.

In order to analyze the behavior of the teleoperator we use the following well–known properties of the dynamical model for robotic manipulators with rotational joints [2]:

- P1 Skew-Symmetric property of the Inertia Matrix. $\dot{\mathbf{M}}_i(\mathbf{q}_i) = \mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) + \mathbf{C}_i^\top(\mathbf{q}_i,\dot{\mathbf{q}}_i)$.
- P2 $\exists\ k_{u_i} \in \mathbb{R}^+$ such that $U_i(\mathbf{q}_i) \geq k_{u_i}$ where $U_i(\mathbf{q}_i)$ is the potential energy of the manipulator that satisfies
$$\frac{\partial U_i(\mathbf{q}_i)}{\partial \mathbf{q}_i} = \mathbf{g}_i(\mathbf{q}_i).$$
- P3 $\exists\ \alpha_i, \beta_i \in \mathbb{R}^+$ such that $\alpha_i I \geq \mathbf{M}_i(\mathbf{q}_i) \geq \beta_i I$.
- P4 For all $\mathbf{q}_i, \mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, $\exists\ k_{c_i} \in \mathbb{R}$ such that $|\mathbf{C}_i(\mathbf{q}_i,\mathbf{x})\mathbf{y}| \leq k_{c_i}|\mathbf{x}||\mathbf{y}|$, where $|\cdot|$ is the Euclidean norm.

We assume that the time-delay imposed by the communication channel is constant on each direction, but it may differ from one to another. the total round trip time-delay is equal to $T_m + T_s \geq 0$. Also, following standard considerations, we assume the human operator and the environment define passive (force to velocity) maps, that is, there exists $\kappa_i \in \mathbb{R}^+$ s.t. $\forall t \geq 0$,

$$\int_0^t \dot{\mathbf{q}}_m^\top(\sigma)\boldsymbol{\tau}_h(\sigma)d\sigma \geq -\kappa_m;\ -\int_0^t \dot{\mathbf{q}}_s^\top(\sigma)\boldsymbol{\tau}_e(\sigma)d\sigma \geq -\kappa_s \qquad (2)$$

3. CONTROL VIA PROPORTIONAL POSITION ERRORS PLUS DAMPING INJECTION

In this section we propose that the forces applied on both sides are proportional to the position errors between the master and the slave plus a damping injection term. The control laws are then given by [3]

$$\begin{aligned}\boldsymbol{\tau}_m &= K_m[\mathbf{q}_s(t-T_s) - \mathbf{q}_m] - B_m\dot{\mathbf{q}}_m \\ \boldsymbol{\tau}_s &= K_s[\mathbf{q}_s - \mathbf{q}_m(t-T_m)] + B_s\dot{\mathbf{q}}_s\end{aligned} \qquad (3)$$

where K_m, K_s, B_m and B_s are positive constants.

Before going through the stability result we present a lemma that will be instrumental for the analysis without proof. The proof for this lemma is established with a direct application of Young's and Schwartz's inequalities. The interested reader may refer to Chopra et al. [2006] for a version of the proof.

Lemma 1. For any vector signals $\mathbf{x}, \mathbf{y}$ and any $T, \alpha > 0$ we have

$$2\int_0^t \mathbf{x}^\top(s)\int_0^T \mathbf{y}(s-\sigma)d\sigma ds \leq \alpha\|\mathbf{x}\|_2^2 + \frac{T^2}{\alpha}\|\mathbf{y}\|_2^2, \qquad (4)$$

where $\|\cdot\|_2$ is the $\mathcal{L}_2$ norm of the signal.

Proposition 1. Consider the teleoperator system (1) controlled by (3) with $\boldsymbol{\tau}_h, \boldsymbol{\tau}_e$ verifying (2). Fix the damping injection and proportional gains such that

$$2B_mB_s > (T_m^2 + T_s^2)K_mK_s, \qquad (5)$$

Then:

(i) Velocities and position error of the teleoperator are bounded. (i.e., $\dot{\mathbf{q}}_i,\ \mathbf{q}_m - \mathbf{q}_s \in \mathcal{L}_\infty$) and moreover $\dot{\mathbf{q}}_i \in \mathcal{L}_2$.

[2] These properties can be found on advanced robotics books like Kelly et al. [2005] and Spong et al. [2005].

[3] To avoid cluttering the notation we will omit the time argument of all signals except for the case when it appears delayed.

(ii) Assume additionally that

A1. The human operator stands still and the slave robot is not in contact with the environment (i.e. $\boldsymbol{\tau}_h(t) \equiv 0$ and $\boldsymbol{\tau}_e(t) \equiv 0$).

A2. A gravity compensation term is added to the controllers, that is,

$$\begin{aligned}\boldsymbol{\tau}_m &= K_m[\mathbf{q}_s(t-T_s) - \mathbf{q}_m] - B_m\dot{\mathbf{q}}_m + \mathbf{g}_m(\mathbf{q}_m) \\ \boldsymbol{\tau}_s &= K_s[\mathbf{q}_s - \mathbf{q}_m(t-T_m)] + B_s\dot{\mathbf{q}}_s - \mathbf{g}_s(\mathbf{q}_s).\end{aligned} \quad (6)$$

A3. The terms $\frac{\partial^2 M_i^{jk}}{\partial q_i^r \partial q_i^l}$ are bounded.

Under these conditions, the master and slave velocities asymptotically converge to zero and position coordination is achieved, that is

$$\lim_{t\to\infty} |\mathbf{q}_m(t) - \mathbf{q}_s(t-T_s)| = 0.$$

Proof. Consider the following non–negative function

$$\begin{aligned}V(\mathbf{q}_i,\dot{\mathbf{q}}_i) &= \frac{1}{2}\dot{\mathbf{q}}_m^\top \mathbf{M}_m(\mathbf{q}_m)\dot{\mathbf{q}}_m + \frac{K_m}{2K_s}\dot{\mathbf{q}}_s^\top \mathbf{M}_s(\mathbf{q}_s)\dot{\mathbf{q}}_s + \\ &+ \frac{K_m}{2}|\mathbf{q}_m - \mathbf{q}_s|^2 + \int_0^t (\dot{\mathbf{q}}_m^\top \boldsymbol{\tau}_h - \frac{K_m}{K_s}\dot{\mathbf{q}}_s^\top \boldsymbol{\tau}_e)d\sigma + \\ &+ U_m(\mathbf{q}_m) + \frac{K_m}{K_s}U_s(\mathbf{q}_s) - k_{u_m} - k_{u_s} + \kappa_m + \frac{K_m}{K_s}\kappa_s\end{aligned} \quad (7)$$

Using (2) and the properties P1, P2 of the robot manipulators, we obtain

$$\dot{V} = \dot{\mathbf{q}}_m^\top[\boldsymbol{\tau}_m + K_m(\mathbf{q}_m - \mathbf{q}_s)] - \frac{K_m}{K_s}\dot{\mathbf{q}}_s^\top[\boldsymbol{\tau}_s + K_s(\mathbf{q}_m - \mathbf{q}_s)]$$

substituting the control laws $\boldsymbol{\tau}_i$ and noting that

$$\mathbf{q}_i(t-T_i) - \mathbf{q}_i(t) = -\int_0^{T_i} \dot{\mathbf{q}}_i(t-\sigma)d\sigma \quad (8)$$

we get

$$\begin{aligned}\frac{1}{K_m}\dot{V} &= -\frac{B_m}{K_m}|\dot{\mathbf{q}}_m|^2 - \frac{B_s}{K_s}|\dot{\mathbf{q}}_s|^2 - \\ &- \dot{\mathbf{q}}_m^\top \int_0^{T_s} \dot{\mathbf{q}}_s(t-\sigma)d\sigma - \dot{\mathbf{q}}_s^\top \int_0^{T_m} \dot{\mathbf{q}}_m(t-\sigma)d\sigma.\end{aligned} \quad (9)$$

We will now invoke Lemma 1 to obtain a bound on the integral of $\dot{V}$. Towards this end, we integrate (9) from 0 to t and apply Lemma 1 to the third and fourth right hand terms. This yields

$$\begin{aligned}V(t) - V(0) \leq &- \left[B_m - \frac{K_m}{2}\left(\alpha_m + \frac{T_m^2}{\alpha_s}\right)\right]\|\dot{\mathbf{q}}_m\|_2^2 - \\ &- K_m\left[\frac{B_s}{K_s} - \frac{1}{2}\left(\alpha_s + \frac{T_s^2}{\alpha_m}\right)\right]\|\dot{\mathbf{q}}_s\|_2^2\end{aligned} \quad (10)$$

Note that $B_m > \frac{K_m}{2}[\alpha_m + \frac{T_m^2}{\alpha_s}]$ and $B_s > \frac{K_s}{2}[\alpha_s + \frac{T_s^2}{\alpha_m}]$ have a positive solution for α_m and α_s if $2B_mB_s > (T_m^2 + T_s^2)K_mK_s$. That, and the nonnegativity of V allow us to conclude that $\dot{\mathbf{q}}_i \in \mathcal{L}_2$. Furthermore, since V is bounded, from (7) and Properties P2, P3, we can find that $\dot{\mathbf{q}}_i,\ \mathbf{q}_m - \mathbf{q}_s \in \mathcal{L}_\infty$, thus, part (i) of Proposition 1 is proved.

We now proceed to prove (ii). First, we repeat the calculations done above with the new function $\tilde{V}(\mathbf{q}_i,\dot{\mathbf{q}}_i)$,

$$\begin{aligned}\tilde{V} &= \frac{1}{2}\dot{\mathbf{q}}_m^\top \mathbf{M}_m\dot{\mathbf{q}}_m + \frac{K_m}{2K_s}\dot{\mathbf{q}}_s^\top \mathbf{M}_s\dot{\mathbf{q}}_s + \frac{K_m}{2}|\mathbf{q}_m - \mathbf{q}_s|^2 + \\ &+ \int_0^t (\dot{\mathbf{q}}_m^\top \boldsymbol{\tau}_h - \frac{K_m}{K_s}\dot{\mathbf{q}}_s^\top \boldsymbol{\tau}_e)d\sigma + \kappa_m + \frac{K_m}{K_s}\kappa_s,\end{aligned} \quad (11)$$

where we have removed the terms associated to the potential energy, which satisfies the bound (10). Then, we will prove that $\dot{\mathbf{q}}_i$ are uniformly continuous and, since they belong to $\mathcal{L}_2$, will converge to zero. Note that

$$\mathbf{q}_m - \mathbf{q}_s(t-T_s) = \mathbf{q}_m - \mathbf{q}_s + \mathbf{q}_s - \mathbf{q}_s(t-T_s) \quad (12)$$

and

$$\mathbf{q}_s - \mathbf{q}_s(t-T_s) = \int_0^{T_s} \dot{\mathbf{q}}_s(t-\sigma)d\sigma \leq T_s^{\frac{1}{2}}\|\dot{\mathbf{q}}_s\|_2 \quad (13)$$

this bound is obtained applying Schwartz inequality. From (12), (13), and the facts that $\dot{\mathbf{q}}_s \in \mathcal{L}_2$ and $\mathbf{q}_m - \mathbf{q}_s \in \mathcal{L}_\infty$, we conclude that $\mathbf{q}_m - \mathbf{q}_s(t-T_s) \in \mathcal{L}_\infty$. Doing similar computations, we can also show that the signal $\mathbf{q}_m(t-T_m) - \mathbf{q}_s \in \mathcal{L}_\infty$.

Now, under Assumptions A1 and A2 the teleoperator dynamics (1) take the form

$$\begin{aligned}\ddot{\mathbf{q}}_m &= -\mathbf{M}_m^{-1}[(B_m + \mathbf{C}_m)\dot{\mathbf{q}}_m - K_m[\mathbf{q}_s(t-T_s) - \mathbf{q}_m]] \\ \ddot{\mathbf{q}}_s &= -\mathbf{M}_s^{-1}[(B_s + \mathbf{C}_s)\dot{\mathbf{q}}_s - K_s[\mathbf{q}_s - \mathbf{q}_m(t-T_m))]],\end{aligned} \quad (14)$$

where the arguments of $\mathbf{M}_i$ and $\mathbf{C}_i$ are omitted for simplicity. From the derivations above, and invoking Properties P3 and P4, we see that $\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$, which together with $\dot{\mathbf{q}}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ proves the claim that $\dot{\mathbf{q}}_i \to 0$.

From (14) and convergence of speeds we note that the claim of position coordination will be established if we can prove that $\ddot{\mathbf{q}}_i \to 0$. Towards this end, we will prove uniform continuity of these signals and use Barbălat's Lemma. Differentiating (14) we recover two types of terms: one consisting of $\frac{d}{dt}\mathbf{M}_i^{-1}$ times a bounded signal and the second one the product of $\mathbf{M}_i^{-1}$ times the derivative of the term in brackets. For the first term we have

$$\frac{d}{dt}\mathbf{M}_i^{-1} = -\mathbf{M}_i^{-1}\dot{\mathbf{M}}_i\mathbf{M}_i^{-1} = -\mathbf{M}_i^{-1}(\mathbf{C}_i + \mathbf{C}_i^\top)\mathbf{M}_i^{-1},$$

which is bounded because of Properties P3 and P4. The derivative of the term in brackets is also bounded under Assumption A.3.[4] Consequently, $\frac{d}{dt}\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$ and $\ddot{\mathbf{q}}_i$ are uniformly continuous. Because of continuity of these signals the integral exists and is given by

$$\int_0^t \ddot{\mathbf{q}}_i(\sigma)d\sigma = \dot{\mathbf{q}}_i(t) - \dot{\mathbf{q}}_i(0)$$

Taking the limit as $t \to \infty$ and using the fact that $\dot{\mathbf{q}}_i \to 0$ we get $\int_0^\infty \ddot{\mathbf{q}}_i(\sigma)d\sigma = -\dot{\mathbf{q}}_i(0)$, which is clearly bounded. Barbălat's Lemma then allows to conclude that $\ddot{\mathbf{q}}_i \to 0$ as required. This completes the proof of Proposition 1. ◁

4. CONTROL VIA PROPORTIONAL POSITION ERROR AND FORCE FEEDBACK PLUS DAMPING INJECTION

In this section we prove that we can also control the teleoperator by reflecting the force generated at the slave to the master while controlling the slave with a proportional position error term with damping injected to both manipulators.

[4] Assumption A3 ensures the terms $\frac{\partial^2 C_i^{jk}}{\partial q_i^r \partial q_i^l}$ are bounded.

Proposition 2. Consider the teleoperator system (1) controlled by

$$\begin{aligned}\boldsymbol{\tau}_m &= \boldsymbol{\tau}_s(t-T_s) - B_m\dot{\mathbf{q}}_m \\ \boldsymbol{\tau}_s &= K_s(\mathbf{q}_s - \mathbf{q}_m(t-T_m)) + B_s\dot{\mathbf{q}}_s\end{aligned} \quad (15)$$

Fix the damping injection and proportional gains such that

$$\frac{B_m}{K_s} > \frac{K_s}{B_s^{'}}(T_m^2+T_s^2) + \frac{1}{2}\left(\frac{B_s}{K_s} + (T_m+T_s)^2 + 1\right) \quad (16)$$

where $B_s^{'} = B_s - \epsilon$, for some $\epsilon > 0$.

(i) Velocities and position error of the teleoperator are bounded. (i.e., $\dot{\mathbf{q}}_i$, $\mathbf{q}_m - \mathbf{q}_s \in \mathcal{L}_\infty$) and moreover $\dot{\mathbf{q}}_i \in \mathcal{L}_2$.

(ii) Assume additionally that

A1. The human operator stands still and the slave robot is not in contact with the environment (i.e. $\boldsymbol{\tau}_h(t) \equiv 0$ and $\boldsymbol{\tau}_e(t) \equiv 0$);

A2. A gravity compensation term is added to the controllers, that is,

$$\boldsymbol{\tau}_m = \boldsymbol{\tau}_s^{'}(t-T_s) - B_m\dot{\mathbf{q}}_m + \mathbf{g}_m(\mathbf{q}_m) \quad (17)$$

$$\boldsymbol{\tau}_s = \underbrace{K_s(\mathbf{q}_s - \mathbf{q}_m(t-T_m)) + B_s\dot{\mathbf{q}}_s}_{\boldsymbol{\tau}_s^{'}} - \mathbf{g}_s(\mathbf{q}_s)$$

A3. The terms $\frac{\partial^2 M_i^{jk}}{\partial q_i^r \partial q_i^l}$ are bounded.

Under these conditions, the master and slave velocities asymptotically converge to zero and position coordination is achieved, that is

$$\lim_{t\to\infty} |\mathbf{q}_m(t) - \mathbf{q}_s(t-T_s)| = 0.$$

Proof. Let us propose the following non-negative function

$$\begin{aligned}V(\mathbf{q}_i,\dot{\mathbf{q}}_i) &= \frac{1}{2}\dot{\mathbf{q}}_m^\top \mathbf{M}_m(\mathbf{q}_m)\dot{\mathbf{q}}_m + \frac{1}{2}\dot{\mathbf{q}}_s^\top \mathbf{M}_s(\mathbf{q}_s)\dot{\mathbf{q}}_s + \\ &+ \frac{K_s}{2}|\mathbf{q}_m - \mathbf{q}_s|^2 + \int_0^t (\dot{\mathbf{q}}_m^\top \boldsymbol{\tau}_h - \dot{\mathbf{q}}_s^\top \boldsymbol{\tau}_e)d\sigma + \\ &+ U_m(\mathbf{q}_m) + U_s(\mathbf{q}_s) - k_{u_m} - k_{u_s} + \kappa_m + \kappa_s \end{aligned} \quad (18)$$

Using (2) and the properties P1, P2 of the robot manipulators, and evaluating $\dot{V}$ along the system trajectories we obtain

$$\dot{V} = \dot{\mathbf{q}}_m^\top[\boldsymbol{\tau}_m + K_s(\mathbf{q}_m - \mathbf{q}_s)] - \dot{\mathbf{q}}_s^\top[\boldsymbol{\tau}_s + K_s(\mathbf{q}_m - \mathbf{q}_s)]$$

substituting the control laws (15), we get

$$\begin{aligned}\dot{V} =& - B_m|\dot{\mathbf{q}}_m|^2 - B_s|\dot{\mathbf{q}}_s|^2 - K_s\dot{\mathbf{q}}_m^\top[\mathbf{q}_s - \mathbf{q}_s(t-T_s)] + \\ &+ B_s\dot{\mathbf{q}}_m^\top\dot{\mathbf{q}}_s(t-T_s) - K_s\dot{\mathbf{q}}_s^\top[\mathbf{q}_m - \mathbf{q}_m(t-T_m)] + \\ &+ K_s\dot{\mathbf{q}}_m^\top[\mathbf{q}_m - \mathbf{q}_m(t-(T_m+T_s))].\end{aligned}$$

Now, we use (8) to replace the inner products, of the terms in brackets, by their integrals. Then, we apply the bound

$$2\dot{\mathbf{q}}_m^\top\dot{\mathbf{q}}_s(t-T_s) \leq |\dot{\mathbf{q}}_m|^2 + |\dot{\mathbf{q}}_s(t-T_s)|^2$$

to the term $B_s\dot{\mathbf{q}}_m^\top\dot{\mathbf{q}}_s(t-T_s)$, integrate $\dot{V}$ and invoke Lemma 1 with the constants $\alpha_m = \frac{B_s^{'}}{2K_s}$, $\alpha_s = \frac{2T_s^2K_s}{B_s^{'}}$, and $\alpha = 1$ for the last right hand term. This yields

$$\begin{aligned}V(t) - V(0) \leq& -\frac{1}{2}\epsilon\|\dot{\mathbf{q}}_s\|_2^2 - \\ -& \left(2B_m - \frac{2K_s^2}{B_s^{'}}(T_m^2+T_s^2) - B_s - K_s - K_s(T_m+T_s)^2\right)\|\dot{\mathbf{q}}_m\|_2^2.\end{aligned}$$

It is easy to show that condition (16) ensures that the term in parenthesis in the right hand side of the inequality is positive, hence, nonnegativity of V proves that $\dot{\mathbf{q}}_i \in \mathcal{L}_2$. The rest of the proof follows *verbatim* the steps of the proof of Proposition 1 with

$$\begin{aligned}\ddot{\mathbf{q}}_m &= -\mathbf{M}_m^{-1}\left[(B_m + \mathbf{C}_m)\dot{\mathbf{q}}_m - \boldsymbol{\tau}_s^{'}(t-T_s)\right] \\ \ddot{\mathbf{q}}_s &= -\mathbf{M}_s^{-1}\left[(B_s + \mathbf{C}_s)\dot{\mathbf{q}}_s - K_s(\mathbf{q}_m(t-T_m) - \mathbf{q}_s)\right].\end{aligned}$$

◁

5. SIMULATIONS

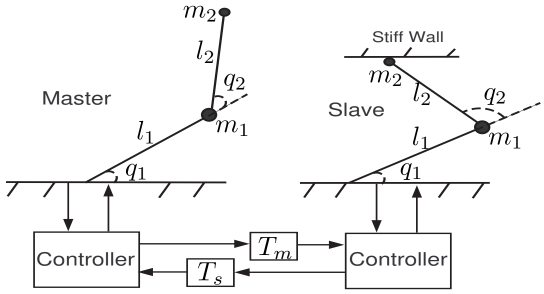


Fig. 1. Simulations scheme.

In this section a simulation of the aforementioned teleoperator scheme is presented. The master and the slave are modeled as a pair of 2 DOF serial links (see Fig. 1). The corresponding nonlinear dynamics follow (1). The inertia matrix $\mathbf{M}_i(\mathbf{q}_i)$ is given by

$$\mathbf{M}_i(\mathbf{q}_i) = \begin{bmatrix}\alpha_i + 2\beta_i\cos(q_{2_i}) & \delta_i + \beta_i\cos(q_{2_i}) \\ \delta_i + \beta_i\cos(q_{2_i}) & \delta_i\end{bmatrix}$$

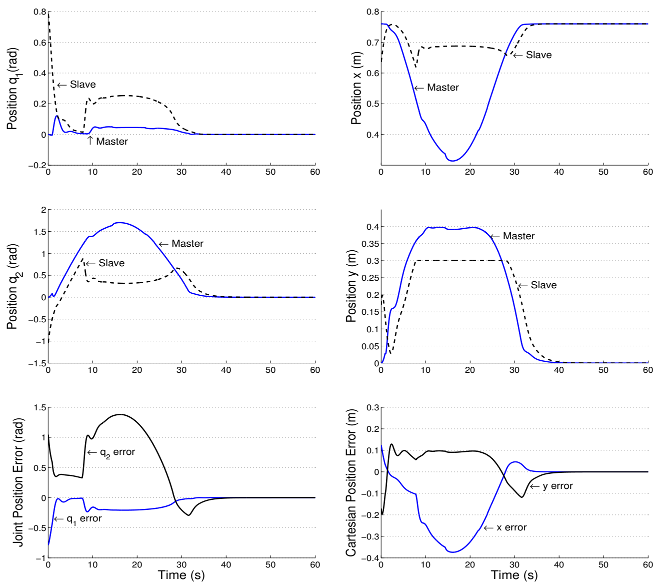
q_{k_i} is the articular position of each link with $k \in \{1,2\}$, $\alpha_i = l_{2_i}^2 m_{2_i} + l_{1_i}^2(m_{1_i} + m_{2_i})$, $\beta_i = l_{1_i}l_{2_i}m_{2_i}$ and $\delta_i = l_{2_i}^2 m_{2_i}$. The lengths for both links l_{1_i} and l_{2_i}, in each manipulator, are $0.38m$. The mass of each link correspond to $m_{1_m} = 3.9473$kg, $m_{2_m} = 0.6232$kg, $m_{1_s} = 3.2409$kg and $m_{2_s} = 0.3185$kg, respectively. These values are the same of those used in Lee and Spong [2006]. Coriolis and centrifugal forces are modeled as the vector $\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i$ which are

$$\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i = \begin{bmatrix}-\beta_i\sin(q_{2_i})\dot{q}_{2_i}^2 - \beta_i\sin(q_{2_i})\dot{q}_{1_i}\dot{q}_{2_i} \\ \beta_i\sin(q_{2_i})\dot{q}_{1_i}^2\end{bmatrix}$$

$\dot{q}_{1_i}$ and $\dot{q}_{2_i}$ are the respective revolute velocities of the two links. The gravity effects ($\mathbf{g}_i(\mathbf{q}_i)$) for each manipulator are represented by

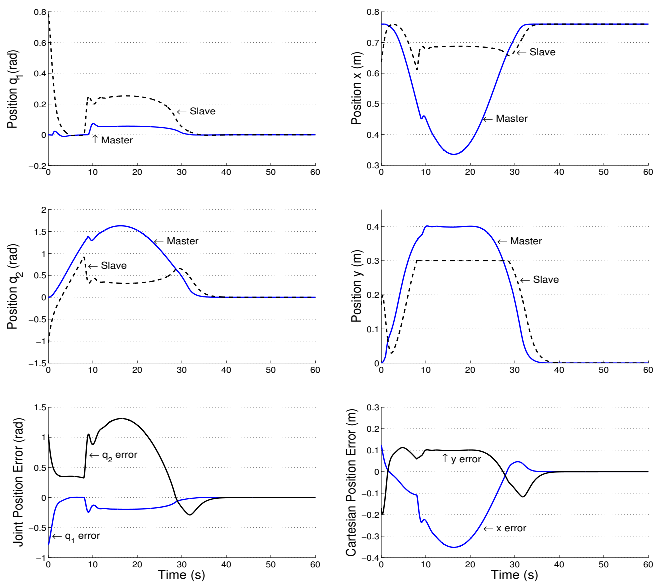
$$\mathbf{g}_i(\mathbf{q}_i) = \begin{bmatrix}\frac{1}{l_{2_i}}g\delta_i\cos(q_{1_i}+q_{2_i}) + \frac{1}{l_{1_i}}(\alpha_i - \delta_i)\cos(q_{1_i}) \\ \frac{1}{l_{2_i}}g\delta_i\cos(q_{1_i}+q_{2_i})\end{bmatrix}$$

At this point, it should be addressed that the human exerts a force on the master manipulator's tip, and the slave interaction with the environment is also measured at the manipulator's tip. Hence, for the simulations the following expressions are used $\boldsymbol{\tau}_h = \mathbf{J}_m^\top(\mathbf{q}_m)\mathbf{f}_h$ and $\boldsymbol{\tau}_e = \mathbf{J}_s^\top(\mathbf{q}_s)\mathbf{f}_e$, ($\mathbf{J}_i^\top(\mathbf{q}_i)$ is the Jacobian transposed of the robot manipulator). The controllers for these simulations are given by (6) and (17). For the first controller the gains fulfill (6), and are given by $K_m = 1.3$, $K_s = 1.7$, $B_m = 1.1$ and $B_s = 2.1$. For the second $K_s = 1.7$, $B_m = 6$ and



a) Joint space b) Cartesian space

Fig. 2. Simulation of the teleoperator system of Proposition 1 with $T_m = 0.7s$ and $T_s = 0.9s$.



a) Joint space b) Cartesian space

Fig. 3. Simulation of the teleoperator system of Proposition 2 with $T_m = 0.7s$ and $T_s = 0.9s$.

$B_s = 2.1$ which follow (17). The time-delay is set for the forward path to $T_m = 0.7s$ and for the backward to $T_s = 0.9s$. In order to evaluate the stability of the proposed scheme, a high stiff wall ($20000\ \frac{N}{m}$) at the cartesian coordinate, $y = 0.3m$, has been included in the environment. The initial conditions for the master and the slave differ one from the other, $\mathbf{q}_m(0) = [0,0]^T$ and $\mathbf{q}_s(0) = [-1/3\pi, -1/3\pi]^T$. Both controllers have been simulated with the same circumstances. The simulation has been carried out using MatLab SimuLink $^{\text{TM}}$.

The first scheme (P-like controller at both sites) is depicted in Figure 2, it is composed by the joint (part a) and cartesian (part b) space measures. Analyzing the plots we can clearly see that: the master and slave initial positions are different; the slave reaches the high stiff wall, located at $y = 0.3m$, around 8s and leaves it at 29s; and, around the 40s the position error converges to 0. Also note that because we used a non-scattering like scheme there are not undesired reflections nor oscillations.

The simulations for the scheme presented in Section 4 are shown in Figure 3, it is also composed by the joint (part a) and cartesian (part b) space measures. The main difference between these results and the previous is that, when the slave interacts with the wall it induces a small oscillation to the master, around the 10th second.

6. EXPERIMENTS

In order to verify the theoretical results two experiments have been carried out with an experimental test-bed that mainly consists of two direct-drive two DOF nonlinear manipulators. These manipulators are made of aluminium and are actuated by two pairs of Compumotor DM1015-B brushless DC motors. Optical encoders are used to measure the joint position, the joint velocity is digitally estimated and filtered. Two JR3 force-torque sensors, located at the manipulators end-effectors, are used to measure the force interaction with the human operator and environment, respectively. The controllers are implemented using WinCom 3.3 that enables Simulink$^{\text{TM}}$ models to interact with external hardware in real time. The sampling time is set to 4ms. An aluminium wall is located at one side of the slave in order to test the stability while interacting with an stiff environment. This setup is depicted in Figure 4.

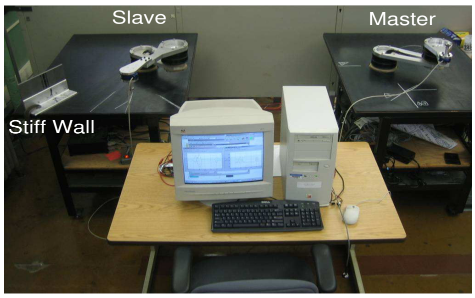
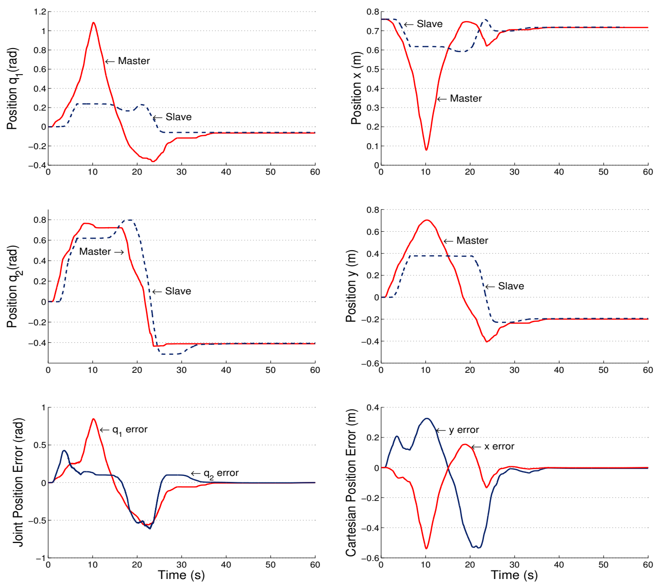


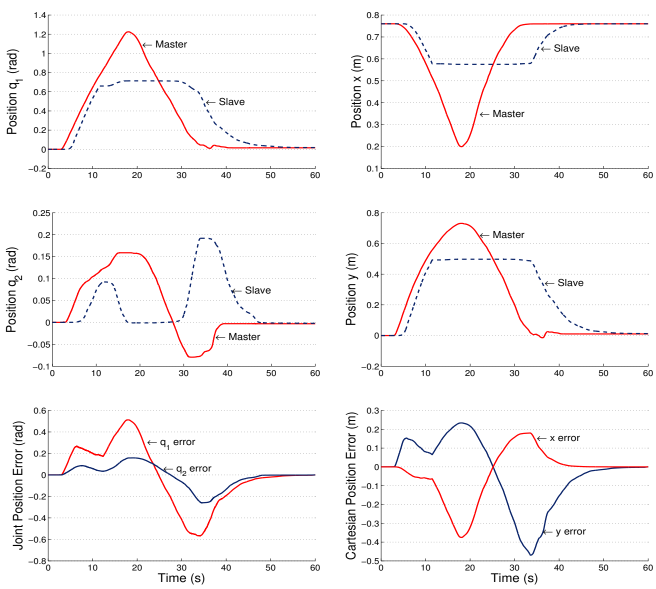
Fig. 4. Experimental teleoperator located at the Coordinated Science Laboratory, University of Illinois at Urbana Champaign.

Both experiments experienced a total time-delay of 1.6s ($T_m = 0.7$s and $T_s = 0.9$s), also, an aluminium wall was located (in cartesian coordinates) at $y = 0.39$m from $x = 0.5$ to 0.8m, for the first experiment, and at $y = 0.5$m from $x = 0.5$ to 0.8m, for the second, respectively. The first experiment results are depicted in Fig. 5, and its controller corresponds with the one in Proposition 1, the gains for this controller were set to: $Ks = 8$, $Bs = 6.5$, $Km = 15$ and $Bm = 12.5$. The second experiment was carried out using the statement of Proposition 2, shown in Fig. 6, the controller's gains are: $Ks = 8$, $Bs = 6.5$ and $Bm = 18$. In both experiments it is clearly seen that the position error is bounded, and moreover it converges to zero when the human does not move the master.



a) Joint space b) Cartesian space

Fig. 5. Experiments for the controller in Proposition 1 with $T_m = 0.7s$ and $T_s = 0.9s$.



a) Joint space b) Cartesian space

Fig. 6. Experiments for the controller in Proposition 2 with $T_m = 0.7s$ and $T_s = 0.9s$.

7. CONCLUSIONS AND FUTURE WORK

In this paper we have shown that it is possible to control a bilateral teleoperator with simple P-like schemes —obviating the need for scattering transformations and passivity considerations, and moreover, these schemes provide position error convergence. As shown in the proofs the key ingredient is the inclusion of damping that should "dominate" the proportional gains—see (5) and (16)—to ensure that the velocities are in $\mathcal{L}_2$. It is easy to see that when time-delay increases instability may arise, condition (5) and (16) inject damping to overcome this situation, thus, overdamped responses may be obtained for the sake of position tracking.

We may add that, in order to set the controller gains the time-delay should be known in advance. This assumption is not an issue nowadays, the time-delay can be easily known with some measure software tools (e.g. ping–like programs). Due to the increase in the use of Internet communications and its ubiquitous nature, the future of the control schemes aforementioned is to analyze the teleoperator dynamics under the influence of variable time-delays.

ACKNOWLEDGEMENTS

The first author gratefully acknowledges the hospitality of the people at the CSL at the University of Illinois at Urbana Champaign, especially to Mark W. Spong. The authors would like to thank Erick Rodríguez-Seda for his valuable help with the experiments, and the comments of the anonymous reviewers, that helped to improve this work.

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