Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Asymptotic Rejection of General Periodic Disturbances in Nonminimum-Phase Nonlinear Output-Feedback Systems ⋆

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Abstract: This paper deals with asymptotic rejection of unmatched general periodic disturbances in nonminimum phase nonlinear output feedback systems. The steady state responses are defined for unstable systems subject to general periodic disturbances, and the generalized gain and phase are defined for stable systems subject to general periodic disturbances. Based on the new definitions, new results are obtained for the equivalent input disturbance and disturbance estimation. A L_p-convergent estimate of the equivalent input disturbance is incorporated in the control design to ensure the asymptotic rejection of unmatched general periodic disturbances while maintaining the stability of the nonlinear system.

1. INTRODUCTION

Asymptotic rejection of sinusoidal disturbances have been studied extensively in recent years (see Bodson et al. [1994], Bodson and Douglas [1997], Marino et al. [2003], Ding [2003]). A related problem is formulated as output regulation, where the output measurement contains the unknown disturbance by Isidori and Byrnes [1990], Huang and Rugh [1990], Isidori [1995]. Many periodic signals are not sinusoidal, and therefore cannot be modelled as an output of a finite-dimensional linear exosystem. Recently, a half-period integration method was proposed to characterize general periodic disturbances, and applied to asymptotic rejection of a class of general disturbances which have symmetric wave form in the half of the period, such as symmetric triangular wave, square wave etc. The half-period integration based disturbance rejection is demonstrated in a class of nonlinear output feedback systems which can be transformed to the output feedback form in Ding [2006c]. The disturbance rejection method proposed in Ding [2006c] has been extended to reject general disturbances whose wave patterns are described by odd functions in Ding [2007c]. With the introduction of integral phase shift, asymptotic rejection of half-period alternating disturbances which may have asymmetric half-period wave forms is achieved through the half-period integration operation with the integral phase shift by Ding [2007b]. In all the cases shown in Ding [2006c, 2007c,b], the disturbances are matched with the input, that is, the disturbances enter a system in the same channel as the input. In a more recent result (Ding [2007a]), asymptotic rejection of unmatched general periodic disturbance is reported for nonlinear systems in the output feedback form, based on the minimum phase assumption of the system.

In this paper, we consider asymptotic rejection of general periodic disturbances in a class of non-minimum phase systems. Even for disturbance-free case, there are not many results on the control design for the stability (see Ding [2001, 2006b]). Here, we concentrate on the disturbance rejection part, assuming that there exists a control design for the disturbance-free case. In the previous results shown in Ding [2006c, 2007c,b,a], the minimum phase assumption is essential for disturbance estimation and the calculation of the equivalent input disturbance. Therefore, we need to propose new methods for the equivalent input disturbance and disturbance estimation. We propose a definition of steady state response for unstable systems subject to periodic disturbances, and obtain a unique expression of this steady state response for stable and unstable systems. This steady state response of unstable systems is then used to solve the invariant manifold for the nonminimum phase nonlinear system, and then the calculation of the equivalent input disturbance. A new disturbance estimation method is proposed based on the generalized gain and phase for general periodic disturbances. The estimated disturbance converges to the equivalent disturbance in L_p sense, and the estimate is integrated with the control design to ensure the asymptotic rejection of the unmatched general periodic disturbance with the stability guaranteed for the closed-loop system. An example is included to demonstrate the proposed estimation and control algorithm.

2. PROBLEM FORMULATION

Consider a single-input-single-output nonlinear system which can be transformed into the output feedback form

$$\begin{aligned}\dot{x} &= A_c x + \psi(y) + bu + dw \\ y &= Cx\end{aligned} \tag{1}$$

with

⋆ This work was supported by EPSRC of UK under the grant EP/C500156/1.

$$A_c = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T,$$

$$b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_\rho \\ \vdots \\ b_n \end{bmatrix}, \; d = \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix}$$

where $x \in R^n$ is the state vector, $u \in R$ is the control, ψ is a known nonlinear smooth vector field in R^n with $\psi(0) = 0$, $w \in R$ is a periodical disturbance.

Similar to the definition of relative degree, we define the disturbance index ι as such that $d_i = 0$ for $i < \iota$ and $d_\iota \neq 0$.

Assumption 1. The disturbance can be expressed as

$$w(t) = a w_b(t + \phi) \tag{2}$$

where the unknown constants a and ϕ are referred to as amplitude and phase, and $w_b(t)$ is a known function satisfying the following

A1.1 $w_b(t + T) = w_b(t)$ with T, the known period.
A1.2 $w_b(t + \frac{T}{2}) = -w_b(t)$.
A1.3 For $t \in [0, T)$, the function $w_b(t)$ is bounded, and $w_b(t) \in \mathcal{C}^{\max\{\rho-\iota,0\}}$.

From A1.1 and A1.2, we have $w_b(\frac{T}{2}) = w_b(\frac{T}{2} - T) = w_b(-\frac{T}{2}) = -w_b(\frac{T}{2})$. Hence we can conclude $w_b(\frac{T}{2}) = 0$.

Remark 1. The condition A1.3 is to guarantee the existence of the equivalent input disturbances in the later part of the paper. In fact, this condition can be relaxed to include certain discontinuous periodic disturbances, such as the square wave disturbance shown in the example later in this paper.

The problem considered in this paper is to design a dynamic feedback control law u so that the overall system is stable and the unknown disturbance $w(t)$ is asymptotically rejected in the sense that $\lim_{t\to\infty} y(t) = 0$. The disturbance is first estimated and then the estimated disturbance is used for control design for disturbance rejection.

Assumption 2. The system has no invariant zeros on the imaginary axis, ie, the zeros of polynomial $\mathbf{B}(s) = \sum_{i=\rho}^{n} b_i s^{n-i}$ have non-zero real parts.

Remark 2. When the zeros of the polynomial $\mathbf{B}(s)$ have negative real parts, the zero dynamics of the system is unstable, and this is referred to as nonminimum phase system, following the definition of linear systems. It can be seen in the this paper, Assumption 2 does allow the system to have nonminimum phase zeros.

Control design of nonminimum phase nonlinear systems is a challenging problem itself. To concentrate on the disturbance estimation and rejection, we specify the conditions for the control design of the nonlinear systems when there is no disturbance.

Assumption 3. Consider the dynamic system

$$\begin{aligned} \dot{x} &= A_c x + \psi(y) + bu \\ y &= Cx \end{aligned} \tag{3}$$

There exists an output feedback controller

$$\dot{v} = f(v, y) \tag{4}$$

$$u = h(v, y) \tag{5}$$

such that the closed-loop control described under the state $\bar{z} = [x^T, v^T]^T$ is exponentially stable.

3. PRELIMINARY RESULTS

3.1 Steady State Responses of Linear Systems

The zero dynamics of (1) is linear. In this section we consider steady state responses of single-input linear systems to periodic inputs. Let

$$\dot{x} = Ax + bw \tag{6}$$

where $x \in R^n$ is the system state and A is a constant matrix and $b \in R^n$ is a constant vector and w is the periodic disturbance with period T.

When A is Hurwitz, we define the steady-state response as

$$x_s(t) = \lim_{N\to\infty} \int_{-NT}^{t} e^{A(t-\tau)} b w(\tau) d\tau \tag{7}$$

By direct evaluation, we have

$$\begin{aligned} &\int_{-NT}^{t} e^{A(t-\tau)} b w(\tau) d\tau \\ &= \int_{0}^{t} e^{A(t-\tau)} b w(\tau) d\tau + \sum_{i=1}^{N} \int_{-iT}^{-(i-1)T} e^{A(t-\tau)} b w(\tau) d\tau \end{aligned} \tag{8}$$

where N is a positive integer. Since $w(t)$ is a periodic function, we have

$$\begin{aligned} \int_{-iT}^{-(i-1)T} e^{-A\tau} b w(\tau) d\tau &= \int_{0}^{T} e^{-A(\tau - iT)} b w(\tau - iT) d\tau \\ &= e^{iAT} \int_{0}^{T} e^{-A\tau} b w(\tau) d\tau \end{aligned} \tag{9}$$

Hence, we have

$$\int_{-NT}^{t} e^{A(t-\tau)} b w(\tau) d\tau = \int_{0}^{t} e^{A(t-\tau)} b w(\tau) d\tau$$

$$+e^{At}\sum_{i=1}^{N} e^{iAT}\int_0^T e^{-A\tau}bw(\tau)d\tau \tag{10}$$

Taking a limit of (10) gives the following.

$$x_s(t) = \int_0^t e^{A(t-\tau)}bw(\tau)d\tau + e^{At}(I - e^{AT})^{-1}e^{AT}W_T \tag{11}$$

where

$$W_T = \int_0^T e^{-A\tau}bw(\tau)d\tau$$

and W_T is a constant vector in R^n.

If $-A$ is Hurwitz, ie, all the eigenvalues of A are with positive real parts, we define the steady state response as

$$x_s(t) = \lim_{N\to\infty}\int_{NT}^t e^{A(t-\tau)}bw(\tau)d\tau \tag{12}$$

The same result as shown in (11) can be obtained in a similar way.

If the system matrix has stable and unstable parts, we can introduce a linear transformation to separate the states into the stable and unstable parts. We have the following result for the steady state response of linear systems with stable and unstable modes. If A has eigenvalues with both positive and negative real parts, then there exists a transformation with a nonsingular matrix M

$$\begin{bmatrix} x_+ \\ x_- \end{bmatrix} = Mx \tag{13}$$

such that the system can be transformed to

$$\dot{x}_+ = A_+x_+ + b_+w \tag{14}$$

$$\dot{x}_- = A_-x_+ + b_-w \tag{15}$$

with A_+ having only the eigenvalues with positive real parts, and A_- with negative real parts. For the stable and unstable modes, we follow the definitions shown in (7) and (12) accordingly. The steady state response is then obtained as

$$x_s = M^{-1}\begin{bmatrix} x_{+s} \\ x_{-}s \end{bmatrix} \tag{16}$$

where x_{-s} and x_{+s} are obtained based on (11) with $\{A_-, b_-\}$ and $\{A_+, b_+\}$. It can be shown that the result given in (16) is the same as the result given in (11).

Therefore, based on the definitions of the steady state responses for the stable and the unstable modes, we have the following lemma to summarize the result.

Lemma 3.1 With the definitions shown in (7) and (12), the steady state response of (6) subject to the T-periodic general disturbance w is obtained as

$$x_s(t) = \int_0^t e^{A(t-\tau)}bw(\tau)d\tau + e^{At}(I - e^{AT})^{-1}e^{AT}W_T \tag{17}$$

We have another result for the steady state response.

Lemma 3.2 For a T-periodic input $w(t)$, the steady state response of (6) of is T-periodic. Furthermore, if $w(t + \frac{T}{2}) = -w(t)$, then $x_s(t + \frac{T}{2}) = -x_s(t)$.

Proof. Proof is omitted due to the page limit.

3.2 Generalized Gain and Phase

Consider a linear system

$$\dot{x} = Ax + bw \tag{18}$$

$$y = Cx \tag{19}$$

where x is the state variable, w is the general periodic disturbance, and y is the output of the system. If we only consider the steady state response of the system, then x_s and $y_s := Cx_s$ are T-periodic, based on the results shown in the previous subsection. If $w_b(t)$ satisfies the condition that $w_b(t) > 0$ for $t \in (0, \frac{T}{2})$, and $w_b(t) < 0$ for $t \in (T, \frac{T}{2})$, and the steady state output y_s crosses zero twice in one period, we design the phase shift of the system with respect to this particular wave form.

Let $y_s(t)$ be the steady state response of $w_b(t)$. The phase shift ϕ_L of the linear system $\{A, b, C\}$ is defined such that $y_s(-\phi_L) = 0$, $y_s(t - \phi_L) > 0$ for $t \in (0, \frac{T}{2})$, and $y_s(t - \phi_L) < 0$ for $t \in (T, \frac{T}{2})$.

The gain, a_L, of the linear system $\{A, b, C\}$ subject to the input w is defined as

$$a_L = \frac{\int_0^{\frac{T}{2}} |y_s(\tau)|d\tau}{\int_0^{\frac{T}{2}} |w_b(\tau)|d\tau} \tag{20}$$

Remark 3. The phase ϕ_L and the gain a_L defined above can be viewed as a natural generalization of the gain and the phase for frequency response. In fact, when the input is a sinusoidal function, ie, $w_b = \sin\omega t$, it can be easily checked that the phase and the gain defined above are given by $a_L = |G(j\omega)|$ and $\phi_L = \angle G(j\omega)$ where $G(j\omega) = C(j\omega I - A)^{-1}b$.

4. EQUIVALENT INPUT DISTURBANCE

To extract the zero dynamics, we introduce a partial state transformation for system (1)

$$z = \begin{bmatrix} x_{\rho+1} \\ \vdots \\ x_n \end{bmatrix} - \sum_{i=1}^{\rho} B^{\rho-i}\bar{b}x_i \tag{21}$$

where

$$B = \begin{bmatrix} -b_{\rho+1}/b_\rho & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -b_{n-1}/b_\rho & 0 & \dots & 1 \\ -b_n/b_\rho & 0 & \dots & 0 \end{bmatrix}, \quad \bar{b} = \begin{bmatrix} b_{\rho+1}/b_\rho \\ \vdots \\ b_n/b_\rho \end{bmatrix}$$

We will use the coordinates $(x_1, \dots, x_\rho, z)$ for the steady-state response of the system. We denote their corresponding steady state variables by $(\pi_1, \dots, \pi_\rho, \pi_z)$. It can be obtained that

$$\pi_z(t) = \int_0^t e^{B(t-\tau)} d_z w(\tau) d\tau + e^{Bt}(I - e^{BT})^{-1} e^{BT} W_T \tag{22}$$

where

$$d_z = \begin{bmatrix} d_{\rho+1} \\ \vdots \\ d_n \end{bmatrix} - \sum_{i=1}^{\rho} B^{\rho-i} \bar{b} d_i$$

with $W_T = \int_0^T e^{-B\tau} d_z w(\tau) d\tau$. Since the system output y does not contain the periodic disturbance, we have its corresponding steady state response $\pi_1 = 0$. From the system dynamics, we have, for $i = 1, \dots, \rho - 1$,

$$\pi_{i+1}(t) = \frac{d\pi_i(t)}{dt} - d_i w \tag{23}$$

Based on the state transformation introduced earlier, we can use its inverse transformation to obtain

$$\begin{bmatrix} \pi_{\rho+1} \\ \vdots \\ \pi_n \end{bmatrix} = \pi_z + \sum_{i=1}^{\rho} B^{\rho-i} \bar{b} \pi_i \tag{24}$$

Therefore the periodic trajectory in the state space is obtained as

$$\pi = [\pi_1, \dots, \pi_\rho, \pi_{\rho+1}, \dots, \pi_n]^T \tag{25}$$

Finally, the equivalent input disturbance μ is given by

$$\mu = \frac{1}{b_\rho}[\frac{d\pi_\rho(t)}{dt} - \pi_{z,1} - \sum_{i=1}^{\rho} r_i \pi_{z,i} - d_\rho w] \tag{26}$$

with $r_i = [B^{\rho-i}\bar{b}]_i$, for $i = 1, \dots, \rho$.

We can also define the basic wave form for μ. Since the relation of π and μ to w are linear, we can substitute w by $a w_b(t + \phi)$ in the calculation of π and μ in (22), (25), (24) and (26). This gives

$$\mu(t) = a\mu_b(t + \phi) \tag{27}$$

where $\mu_b(t)$ is obtained as

$$\mu_b(t) = \frac{1}{b_\rho}[\frac{d\pi_{b,\rho}(t)}{dt} - \pi_{b,\rho+1} - d_\rho w_b(t)] \tag{28}$$

Let $e = x - \pi$ denote the difference between the state variable x and the periodic trajectory π.

The periodic trajectory, π, plays a similar role as the invariant manifold in the set-up for the rejection of disturbances generated from linear exosystems. For this we have the following result.

Theorem 4.1 For the difference between the state variable x of (1) and the periodic trajectory given in (25), denoted by $e = x - \pi$, it satisfies the following equation

$$\begin{aligned} \dot{e} &= A_c e + \psi(y) + b(u - \mu) \\ y &= Ce \end{aligned} \tag{29}$$

where $\mu = a\mu_b(t + \phi)$ with μ_b given by (28). Furthermore, the equivalent disturbance μ is half-period alternating if w is.

Proof. From the construction of π shown in (23) and (24), it can be obtained that

$$\dot{\pi} = A_c \pi + b\mu + dw \tag{30}$$

Then (29) can be directly obtained from (30) and (1). From Lemma 3.2, we have that π_z is half-period alternating, and therefore for π as w is half-period alternating. This concludes the proof.

5. DISTURBANCE REJECTION

With the equivalent input disturbance μ, we have converted the system into the form of matched disturbance as shown in (29). But the results for the matched case such as the one shown in Ding [2006a] require the system to be minimum phase, we need to propose new disturbance rejection and control design. In the result presented in Ding [2006a], we require that the disturbance is half-period alternating and with the phase $\phi = 0$ indicating the point that the basic wave form starts from zero and then increase. For the equivalent input disturbance, this may not be the case even if the disturbance w_b holds this property. We introduce the offset phase shift ϕ_o to adjust the phase of the equivalent input disturbance such that $\mu_b(\phi_o) = 0$ and $\mu_b(\phi_o + \delta) > 0$ for a small positive real δ. Therefore we define

$$\bar{\mu}_b(t) := \mu_b(t + \phi_o) \tag{31}$$

For $\bar{\mu}_b(t)$, we have $\bar{\mu}_b(t + \frac{T}{2}) = -\bar{\mu}_b(t)$, and $\bar{\mu}_b(0) = 0$. The equivalent input disturbance can now be expressed as $\mu(t) = a\bar{\mu}_b(t + \phi - \phi_o)$. Since the phase ϕ is unknown, and needs to be estimated, we can absorb ϕ_o in ϕ with ϕ unknown, and therefore we can express $\mu(t) = a\bar{\mu}_b(t + \phi)$. With the definition of μ and $\bar{\mu}_b$, the problem that we consider in this paper is reformulated as the rejection of matched disturbance of the following system

$$\begin{aligned} \dot{e} &= A_c e + \psi(y) + b(u - \mu) \\ y &= Ce \end{aligned} \tag{32}$$

where $\mu = a\bar{\mu}_b(t + \phi)$ with ϕ and a unknown.

For the estimation of the unknow disturbance, we need the following assumption.

Assumption 4. The basic wave form for the equivalent input disturbance $\bar{\mu}_b$ satisfies:

A4.1 There exists a δ, $0 < \delta < \frac{T}{4}$, such that for $t \in (0, \delta)$, $\bar{\mu}_b(t) > K_b t^l$, and for $t \in (\frac{T}{2} - \delta, \frac{T}{2})$, $\bar{\mu}_b(t) > K_b(\frac{T}{2} - t)^l$ and with K_b and l are positive reals, and $\bar{\mu}_b(t) \geq K_b \delta^l$ for $t \in [\delta, \frac{T}{2} - \delta]$.
A4.2 For $t \in [0, T)$, the function $\bar{\mu}_b(t)$ is bounded, and has bounded derivatives except at a finite number of discontinuous points, where the left and right derivatives exist and are bounded.

For the disturbance estimation, the half-period integration operator $\mathcal{I}$ is needed which is defined below

$$\mathcal{I} \circ f(t) := \mathcal{I}(f(t)) = \int_{t-\frac{T}{2}}^{t} f(s) ds \tag{33}$$

The following filter is designed to extract the contribution in the state from the input and the output:

$$\dot{p} = (A_c + kC)p + \psi(y) + bu - ky \tag{34}$$

where $p \in R^n$, $k \in R^n$ is chosen so that $A_c + kC$ is Hurwitz.

An estimate of w is given by

$$\hat{\mu}(t) = \frac{\hat{a}}{a_L} \bar{\mu}_b(\hat{\phi}_1 - \phi_L) \tag{35}$$

where

$$\hat{a} = \frac{\mathcal{I} \circ |p_1 - y|}{\mathcal{I} \circ |\bar{\mu}_b(t)|} \tag{36}$$

$$\hat{\phi}_1(t) = \frac{1}{2}(\mathcal{I} \circ \text{sign}(p_1 - y) + \frac{T}{2})\text{sign}(p_1 - y) \tag{37}$$

and a_L and ϕ_L are the gain and phase shift of the linear system $\{(A_c + kC), b, C\}$ subject to the input w_b.

Theorem 5.1 If the disturbance w in (1) satisfies the conditions specified in Assumptions 1 and the corresponding equivalent input disturbance satisfies the conditions specified in Assumption 4, then the estimate given in (35) converges to the actual disturbance in L_p, ie., $\mu - \hat{\mu} \in L_p$ for $p = 1, 2$ and ∞.

Proof. Proof is omitted due to the page limit.

We shall show that disturbance rejection can be achieved by combining the feedback control designed in the case when there is absence of disturbance and the estimate of the disturbance.

Theorem 5.2 If the system (1) satisfies Assumptions 1 to 3, and the equivalent input disturbance satisfies Assumption 4, then the control input defined as

$$u = h(v, y) + \hat{\mu} \tag{38}$$

completely rejects the unknown disturbances and ensures the boundedness of the other variables in the system.

Proof. Proof is omitted due to the page limit.

6. EXAMPLE

Consider a nonlinear system in output feedback form

$$\begin{aligned} \dot{x}_1 &= x_2 - y^3 + u \\ \dot{x}_2 &= -u + w \\ y &= x_1 \end{aligned} \tag{39}$$

where $w = a w_b(t + \phi)$ is a periodic disturbance which satisfies Assumptions 1 with unknown a and ϕ. It is easy to see that the system (39) is in the format of (1) with $\phi(y) = [y^3 \ 0]^T$, $b = [1 \ -1]^T$ and $d = [0 \ 1]^T$. The system has a nonminimum phase zero at $s = 1$, and Assumption 2 is satisfied. Note that the disturbance is unmatched with the input as d is different from b.

The control design can be carried out for the example when there is no disturbance. In a similar way to the method presented in Ding [2001] for control design with one non-minimum-phase zero, we have the control design, with reference to (4) and (5), with $v \in R^2$,

$$\begin{aligned} f(v, y) &= \begin{bmatrix} -k_{r1} & k_{r1} + 1 \\ -k_{r2} & k_{r2} \end{bmatrix} v + \begin{bmatrix} y^3 \\ 0 \end{bmatrix} \\ &\quad - \begin{bmatrix} k_{r1} \\ k_{r2} \end{bmatrix} y + \begin{bmatrix} 0 \\ 1 \end{bmatrix} h(v, y) \end{aligned} \tag{40}$$

$$\begin{aligned} h(v, y) &= (1 + 3y^2)^{-1}[-v_1 - (d_3 + d_4(1 + 9y^4)) \\ &\quad \times (v_2 + d_1 v_1 + d_2 v_1 + y^3) \\ &\quad -(d_1 + d_2)(-k_{r1} v_1 + (k_{r2} + 1)v_2 + y^3) \\ &\quad +3y^2(v_2 + y^3)] \end{aligned} \tag{41}$$

where d_i, $i = 1$ to 4, are positive real design parameters, k_{r1} and k_{r2} the design parameters such that $\begin{bmatrix} -k_{r1} & k_{r1} + 1 \\ -k_{r2} & k_{r2} \end{bmatrix}$ is Hurwitz. The feedback control based on the pair f and g introduced above renders the closed loop system exponentially stable for the disturbance-free case, and therefore Assumption 3 is satisfied.

The equivalent input disturbance is obtained as

$$\begin{aligned} \mu(t) &= \int_0^t e^{t-\tau} w(\tau) d\tau \\ &\quad + e^t (1 - e^T)^{-1} e^T \int_0^T e^{\tau} w(\tau) d\tau \end{aligned} \tag{42}$$

In the simulation study, we used square wave disturbance with

$$w_b(t) = \begin{cases} 1 & \text{for } 0 < t < \frac{T}{2} \\ -1 & \text{for } \frac{T}{2} < t < T \\ 0 & \text{otherwise} \end{cases} \tag{43}$$

With the design parameters $k_1 = -2$, $k_2 = -1$ which implies $\lambda_1 = 1$, the basic wave form for the equivalent input disturbance is calculated as, for $0 \leq t < \frac{T}{2}$

$$\mu_b(t) = -1 + 2(1 - e^{\frac{T}{2}})^{-1}e^t \tag{44}$$

and for $\frac{T}{2} \le t < T$,

$$\mu_b(t) = 1 - 2e^{t-\frac{T}{2}} + 2(1 - e^{\frac{T}{2}})^{-1}e^t \tag{45}$$

The final control design is given by

$$u = h(v, y) + \hat{\mu} \tag{46}$$

Simulation study has been carried out for the estimation and control design shown in this example. The simulation results shown below are for the settings $T = 1$, $a = 1$, $k_1 = -3$, $k_2 = -2$, $k_{r1} = 5$, $k_{r2} = 2$, $d_1 = d_2 = d_3 = d_4 = 1$. The control input and the system output are shown in Figure 1, in which the output converges to zero with the input to asymptotically cancel the disturbance. Figure 2 shows the equivalent input disturbance and its estimate.

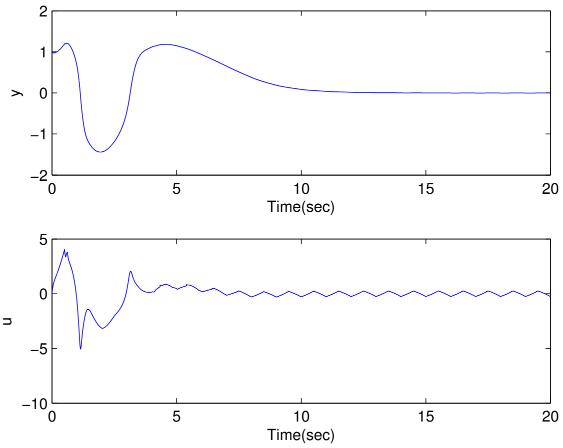

Fig. 1. The system input and output

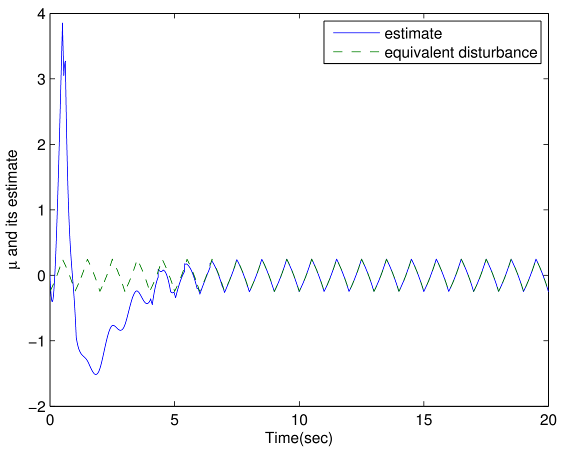

Fig. 2. The equivalent disturbance and its estimate

7. CONCLUSIONS

In this paper, we have proposed a design method for asymptotic rejection of unmatched general periodic disturbances in a class of nonminimum phase nonlinear output feedback systems. The proposed method is based on the introduction of steady state response of unstable systems subject to general periodic disturbances, and the introduction of the generalized gain and phase for general periodic disturbances. With a well defined format for equivalent input disturbance, a L_p-convergent estimate of the input disturbance can be obtained, which is essential to the proposed method. The proposed overall control design ensures asymptotic rejection of the disturbance and the overall stability of the system.

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