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NONLINEAR H_∞ SYNCHRONIZATION FOR LUR'E SYSTEMS USING TIME-DELAY FEEDBACK CONTROL ⋆

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Abstract: This paper deals with the problem of nonlinear H_∞ synchronization for Lur'e systems using time-delay feedback control. Making use of a vector field modulation in the master system by a filtered binary valued message signal, applying a static output feedback control with time-delay to the slave system, and taking into account L_2-norm bounded noise in the channel, the master-slave synchronization is formulated as to minimize the L_2-gain from the exogenous input to a tracking error. Then a delay-dependent synchronization criterion is derived to analyze the error system. Sufficient conditions for the H_∞ synchronization and a feedback control with time-delay are obtained in terms of linear matrix inequality. Finally, the original message is recovered from the tracking error. Chua's circuit is given to illustrate the effectiveness of the proposed method.

Keywords: H_∞, synchronization, Lur'e system, Chua's circuit, time-delay

1. INTRODUCTION

Secure communication using chaotic Lur'e systems has received much attention during the last decade. Without taking into account a message signal in the synchronization, the master-slave synchronization for Lur'e systems has been addressed as to discuss the absolute stability of an error system, see for instance,Curran [1997], Liao [2003], Wu [1994] and references therein. When both a binary valued message signal and channel noise are considered in the synchronization scheme, methods of nonlinear H_∞ synchronization have been approached in order to recover a message signal Suykens [1997b,a]. The main idea of nonlinear H_∞ synchronization is to regard the message signal as a reference input and formulate the problem as to find a feedback controller such that the L_2-gain from the exogenous input to the tracking error is minimized or bounded by a prescribed level. On the other hand, due to the propagation delay frequently encountered in remote master-slave synchronization scheme, recently, there have been some research efforts to investigate the effect of time-delay on master-slave synchronization Cao [2005], Han [2007], He [2006], Liao [2003], Wu [2001]. However, to the best of authors' knowledge, study on nonlinear H_∞ synchronization for Lur'e systems with time-delay is still open and remains challenging, which motivates the present study.

In this paper, we will investigate the problem of nonlinear H_∞ synchronization for Lur'e systems with time-delay. Both a binary valued message signal and static output feedback control with time-delay will be considered in the master-slave synchronization scheme and the master-slave synchronization will be formulated as an H_∞ regulation. Applying the Lyapunov-krasovskii approach, a delay-dependent synchronization criterion will be derived to analyze the error system and, based on this, sufficient conditions for the synchronization and a solution to the time-delay feedback control will be given in terms of linear matrix inequality (LMI). Finally, we will use Chua's circuit to illustrate the effectiveness of the proposed method.

2. PRELIMINARIES AND PROBLEM STATEMENT

Consider the following master-slave synchronization scheme

$$\mathcal{R}: \begin{cases} \dot{\mu}(t) = A_\mu \mu(t) + B_\mu v(t), \\ r(t) = D_\mu \mu(t) + E_\mu v(t), \end{cases} \tag{1}$$

$$\mathcal{M}: \begin{cases} \dot{x}(t) = Ax(t) + B\varphi(Cx(t)) + Dr(t), \\ z_x(t) = Hx(t) + G\omega(t), \end{cases} \tag{2}$$

$$\mathcal{S}: \begin{cases} \dot{y}(t) = Ay(t) + B\varphi(Cy(t)) + u(t), \\ z_y(t) = Hy(t), \end{cases} \tag{3}$$

$$\mathcal{C}: \; u(t) = -K(z_x(t-\tau) - z_y(t-\tau)), \tag{4}$$

with master system $\mathcal{M}$, slave system $\mathcal{S}$, low pass filter $\mathcal{R}$, and controller $\mathcal{C}$. $x(t),\ y(t) \in R^n,\ \mu(t) \in R^{n_r}$ are state vectors; $z_x(t),\ z_y(t) \in R^l,\ r(t) \in R$ are output vectors; $\tau > 0$ is a constant time-delay; $v(t) \in R$ is a binary valued message signal; $\omega(t)$ denotes the channel noise; $A, B, C, D, A_\mu, B_\mu, D_\mu, E_\mu, H$, and G are constant matrices with appropriate dimensions; $\varphi(\cdot) : R^m \to R^m$ is a memoryless nonlinear vector valued function which is globally Lipschitz, $\varphi(0) = 0$, and suppose that the nonlinearity $\varphi(\cdot)$ is time invariant, decoupled, and satisfies a sector condition with $\varphi_i(\xi)$ belonging to a sector $[0, k]$, i.e.

$$\varphi_i(\xi)[\varphi_i(\xi) - k\xi] \le 0, \forall t \ge 0, \tag{5}$$

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for a given scalar $k > 0$. Defining $e(t) = x(t) - y(t)$, we have the error system

$$\begin{aligned}\dot{e}(t) &= Ae(t) + KHe(t-\tau) + B\eta(Ce(t), y(t)) \\ &\quad + Dr(t) + KG\omega(t-\tau), \qquad (6)\end{aligned}$$

where the initial condition of system (6) is $e(\theta) = \phi(\theta)$, $\forall\theta \in [-\tau, 0]$, $\phi(\theta)$ is a continuous vector valued function

$$\eta(Ce(t), y(t)) = \varphi(Ce(t) + Cy(t)) - \varphi(Cy(t)).$$

Let $C = \begin{pmatrix} c_1 & c_2 & \cdots & c_m \end{pmatrix}^T$, $c_i \in R^n$, $i = 1, 2, \cdots, m$. Suppose that $\eta(Ce(t), y(t))$ belongs to the sector $[0, k]$, i.e.

$$\eta_i(c_i^T e(t), y(t))[\eta_i(c_i^T e(t), y(t)) - kc_i^T e(t)] \le 0.$$

The output $z_x(t)$ is transmitted along the channel and the original message is not recovered from $e(t)$ but from taking the sign of a tracking error $z(t) = r(t) - \beta^T e(t)$ with $\beta = (1\ 0\ \cdots\ 0)^T$. We then have

$$\begin{cases} \dot{e}(t) = Ae(t) + KHe(t-\tau) + B\eta(Ce(t), y(t)) \\ \qquad + DD_\mu\mu(t) + DE_\mu v(t) + KG\omega(t-\tau), \\ \dot{\mu}(t) = A_\mu\mu(t) + B_\mu v(t), \\ z(t) = -\beta^T e(t) + D_\mu\mu(t) + E_\mu v(t), \\ e(\theta) = \phi(\theta),\ \mu(\theta) = 0,\ \forall\theta \in [-\tau, 0]. \end{cases} \qquad (7)$$

Denote

$$\begin{aligned}\xi^T(t) &= \begin{pmatrix} e^T(t) & \mu^T(t) \end{pmatrix}, \\ w^T(t) &= \begin{pmatrix} v^T(t) & \omega^T(t-\tau) \end{pmatrix}, \\ A_\xi &= \begin{pmatrix} A & DD_\mu \\ 0 & A_\mu \end{pmatrix},\ A_{\xi\tau} = \begin{pmatrix} KH & 0 \\ 0 & 0 \end{pmatrix}, \\ D_\xi &= \begin{pmatrix} DE_\mu & KG \\ B_\mu & 0 \end{pmatrix},\ B_\xi^T = \begin{pmatrix} B^T & 0 \end{pmatrix}, \\ H_\xi &= \begin{pmatrix} -\beta^T & D_\mu \end{pmatrix},\ G_\xi = \begin{pmatrix} E_\mu & 0 \end{pmatrix}.\end{aligned}$$

We rewrite (6) as

$$\begin{cases} \dot{\xi}(t) = A_\xi\xi(t) + A_{\xi\tau}\xi(t-\tau) + B_\xi\eta(Ce(t), y(t)) \\ \qquad + D_\xi w(t), \\ z(t) = H_\xi\xi(t) + G_\xi w(t), \\ \xi(\theta) = \tilde{\phi}(\theta),\ \forall\theta \in [-\tau, 0], \end{cases} \qquad (8)$$

where $\tilde{\phi}^T(\cdot) = \begin{pmatrix} \phi^T(\cdot) & 0 \end{pmatrix}$.

The goal in H_∞ master-slave synchronization is to find a matrix K such that system (8) is globally asymptotically stable with an L_2-gain bound γ, i.e., the system (8) with $w(t) = 0$ is globally asymptotically stable and, for any $w(t) \in L_2[0, \infty)$, the H_∞ performance $\|z(t)\|_2 \le \gamma\|w(t)\|_2$ is satisfied under condition $\tilde{\phi}(\theta) = 0$, $\forall\theta \in [-\tau, 0]$.

The following lemma is useful in deriving the synchronization criterion.

Lemma 1. Han [2005] For any constant matrix $M \in R^{n\times n}$, $M = M^T > 0$, scalar $\tau > 0$, and vector function $\dot{x} : [-\tau, 0] \to R^n$ such that the following integration is well defined, then

$$-\tau\int_{-\tau}^{0} \dot{x}^T(t+s)M\dot{x}(t+s)ds$$

$$\le \begin{pmatrix} x(t) \\ x(t-\tau) \end{pmatrix}^T \begin{pmatrix} -M & M \\ M & -M \end{pmatrix} \begin{pmatrix} x(t) \\ x(t-\tau) \end{pmatrix}.$$

3. MAIN RESULTS

In order to design controller $\mathcal{C}$ for the H_∞ synchronization, we first concentrate on the analysis of system (8). Choose a Lyapunov-Krasovskii functional candidate as

$$\begin{aligned}V(t) &= \xi^T(t)P\xi(t) + \int_{t-\tau}^{t} \xi^T(s)Q\xi(s)ds \\ &\quad + \int_{t-\tau}^{t} (\tau - t + s)\,\dot{\xi}^T(s)\,(\tau R)\,\dot{\xi}(s)ds, \qquad (9)\end{aligned}$$

where $P > 0$, $Q > 0$, and $R > 0$ are real $(n+n_r)\times(n+n_r)$ matrices. Applying Lemma 1, we have the following result.

Proposition 2. For given scalars $\gamma > 0$, $\tau > 0$, and a matrix K, system (8) is globally asymptotically stable with an L_2-gain bound γ if there exist real $(n+n_r)\times(n+n_r)$ matrices $P > 0$, $Q > 0$, $R > 0$, and a matrix $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_m) > 0$ such that

$$\Phi = \begin{pmatrix} (1,1) & (1,2) & (1,3) & PD_\xi & H_\xi^T & \tau A_\xi^T R \\ * & (2,2) & 0 & 0 & 0 & \tau A_{\xi\tau}^T R \\ * & * & -2\Lambda & 0 & 0 & \tau B_\xi^T R \\ * & * & * & -\gamma^2 I & G_\xi^T & \tau D_\xi^T R \\ * & * & * & * & -I & 0 \\ * & * & * & * & 0 & -R \end{pmatrix} < 0,$$

where

$$\begin{aligned}(1,1) &= PA_\xi + A_\xi^T P + Q - R, \\ (1,2) &= PA_{\xi\tau} + R, \\ (1,3) &= PB_\xi + kC_\xi^T\Lambda, \\ (2,2) &= -Q - R.\end{aligned}$$

Proof. Taking the derivative of $V(t)$ with respect to t along the trajectory of (8) yields

$$\begin{aligned}\dot{V}(t) &= \xi^T(t)(PA_\xi + A_\xi^T P + Q)\xi(t) \\ &\quad + 2\xi^T(t)PA_{\xi\tau}\xi(t-\tau) + 2\xi^T(t)PD_\xi w(t) \\ &\quad + 2\xi^T(t)PB_\xi\eta(C_\xi\xi(t), y(t)) \\ &\quad - \xi^T(t-\tau)Q\xi(t-\tau) + \dot{\xi}^T(t)(\tau^2 R)\dot{\xi}(t) \\ &\quad - \int_{t-\tau}^{t} \dot{\xi}^T(s)(\tau R)\dot{x}(s)ds. \qquad (10)\end{aligned}$$

For any $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_m) > 0$, from (10) we have

$$\begin{aligned}\dot{V}(t) &\le \xi^T(t)(PA_\xi + A_\xi^T P + Q)\xi(t) \\ &\quad + 2\xi^T(t)PA_{\xi\tau}\xi(t-\tau) + 2\xi^T(t)PD_\xi w(t) \\ &\quad + 2\xi^T(t)PB_\xi\eta(C_\xi\xi(t), y(t)) \\ &\quad - \xi^T(t-\tau)Q\xi(t-\tau) + \dot{\xi}^T(t)(\tau^2 R)\dot{\xi}(t) \\ &\quad - 2\sum_{i=1}^{m} \lambda_i\eta_i(c_{\xi i}^T\xi(t), y(t))(\eta_i(c_{\xi i}^T\xi(t), y(t))\end{aligned}$$

$$-kc_{\xi i}^T\dot{\xi}(t)) - \int_{t-\tau}^{t} \dot{\xi}^T(s)(\tau R)\dot{x}(s)ds. \qquad (11)$$

Using Lemma 1 to obtain

$$-\int_{t-\tau}^{t} \dot{\xi}^T(s)(\tau R)\dot{\xi}(s)ds \leq \begin{pmatrix} \xi(t) \\ \xi(t-\tau) \end{pmatrix}^T \begin{pmatrix} -R & R \\ R & -R \end{pmatrix} \begin{pmatrix} \xi(t) \\ \xi(t-\tau) \end{pmatrix}.$$

We then have

$$\begin{aligned}
&\dot{V}(t) \\
&\leq \xi^T(t)(PA_\xi + A_\xi^T P + Q)\xi(t) \\
&\quad +2\xi^T(t)PA_{\xi\tau}\xi(t-\tau) + 2\xi^T(t)PD_\xi w(t) \\
&\quad -\xi^T(t-\tau)Q\xi(t-\tau) + \dot{\xi}^T(t)(\tau^2 R)\dot{\xi}(t) \\
&\quad + \begin{pmatrix} \xi(t) \\ \xi(t-\tau) \end{pmatrix}^T \begin{pmatrix} -R & R \\ R & -R \end{pmatrix} \begin{pmatrix} \xi(t) \\ \xi(t-\tau) \end{pmatrix} \\
&\quad +2\xi^T(t)PB_\xi\eta(C_\xi\xi(t), y(t)) \\
&\quad +2k\eta^T(C_\xi\xi(t), y(t))\Lambda C_\xi\xi(t) \\
&\quad -2\eta^T(C_\xi\xi(t), y(t))\Lambda\eta(C_\xi\xi(t), y(t)). \qquad (12)
\end{aligned}$$

First, we analyze the global asymptotic stability of the system (8) with $w(t) = 0$. In the case of $w(t) = 0$, from (12) one obtains

$$\dot{V}(t) \leq \varsigma_1^T(t)\Theta\varsigma_1(t),$$

where

$$\varsigma_1^T(t) = \left(\xi^T(t) \ \xi^T(t-\tau) \ \eta^T(C_\xi\xi(t), y(t)) \right),$$

$$\Theta = \begin{pmatrix} (1,1) & (1,2) & PB_\xi + kC_\xi^T\Lambda + A_\xi^T(\tau^2 R)B_\xi \\ * & (2,2) & A_{\xi\tau}^T(\tau^2 R)B_\xi \\ * & * & -2\Lambda + B_\xi^T(\tau^2 R)B_\xi \end{pmatrix}$$

with

$$\begin{aligned}
(1,1) &= PA_\xi + A_\xi^T P + Q - R + A_\xi^T(\tau^2 R)A_\xi, \\
(1,2) &= PA_{\xi\tau} + R + A_\xi^T(\tau^2 R)A_{\xi\tau}, \\
(2,2) &= -Q - R + A_{\xi\tau}^T(\tau^2 R)A_{\xi\tau}.
\end{aligned}$$

In view of Schur complement, $\Phi < 0$ implies that $\Theta < 0$. Then we have $\dot{V}(t) < 0$ for all $\varsigma_1(t) \neq 0$, from which we conclude that the system (8) with $w(t) = 0$ is globally asymptotically stable.

Next, we consider the performance $\|z(t)\|_2 \leq \gamma \|w(t)\|_2$ for all $w(t) \in L_2[0, \infty)$ and a prescribed $\gamma > 0$ under condition $\phi(\theta) = 0, \forall\theta \in [-\tau, 0]$. Define

$$J_w = \int_0^\infty [z^T(t)z(t) - \gamma^2 w^T(t)w(t)]dt. \qquad (13)$$

From (12) and (13), we have

$$\begin{aligned}
J_w &= \int_0^\infty [z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t, \xi_t)]dt \\
&\quad - V(t)|_{t\to\infty} + V(t)|_{t=0} \\
&\leq \xi^T(t)(PA_\xi + A_\xi^T P + Q + H_\xi^T H_\xi)\xi(t) \\
&\quad +2\xi^T(t)PA_\xi\xi(t-\tau) - \xi^T(t-\tau)Q\xi(t-\tau) \\
&\quad +2\xi^T(t)(PD_\xi + H_\xi^T G_\xi)w(t) \\
&\quad +\dot{\xi}^T(t)(\tau^2 R)\dot{\xi}(t) + 2\xi^T(t)PB_\xi\eta(C_\xi\xi(t), y(t)) \\
&\quad + \begin{pmatrix} \xi(t) \\ \xi(t-\tau) \end{pmatrix}^T \begin{pmatrix} -R & R \\ R & -R \end{pmatrix} \begin{pmatrix} \xi(t) \\ \xi(t-\tau) \end{pmatrix} \\
&\quad -2\eta^T(C_\xi\xi(t), y(t))\Lambda\eta(C_\xi\xi(t), y(t)) \\
&\quad +2k\eta^T(C_\xi\xi(t), y(t))\Lambda C_\xi\xi(t) \left. -\gamma^2 w^T(t)w(t)\right) dt \\
&\quad - V(t)|_{t\to\infty} + V(t)|_{t=0}.
\end{aligned}$$

Observing that

$$V(t)|_{t\to\infty} \geq 0, V(t)|_{t=0} = 0,$$

we then have

$$J_w \leq \int_0^\infty \varsigma_2^T(t)\Psi\varsigma_2(t)dt,$$

where

$$\varsigma_2^T(t) = \left(\xi^T(t) \ \xi^T(t-\tau) \ \eta^T(C_\xi\xi(t), y(t)) \ w^T(t) \right),$$

$$\Psi = \begin{pmatrix} (1,1) & (1,2) & (1,3) & (1,4) \\ * & (2,2) & (2,3) & A_{\xi\tau}^T(\tau^2 R)D_\xi \\ * & * & (3,3) & B_\xi^T(\tau^2 R)D_\xi \\ * & * & * & (4,4) \end{pmatrix}$$

with

$$\begin{aligned}
(1,1) &= PA_\xi + A_\xi^T P + Q - R + H_\xi^T H_\xi \\
&\quad +A_\xi^T(\tau^2 R)A_\xi, \\
(1,2) &= PA_{\xi\tau} + R + A_\xi^T(\tau^2 R)A_{\xi\tau}, \\
(1,3) &= PB_\xi + kC_\xi^T\Lambda + A_\xi^T(\tau^2 R)B_\xi, \\
(1,4) &= PD_\xi + H_\xi^T G_\xi + A_\xi^T(\tau^2 R)D_\xi, \\
(2,2) &= -Q - R + A_{\xi\tau}^T(\tau^2 R)A_{\xi\tau}, \\
(2,3) &= A_{\xi\tau}^T(\tau^2 R)B_\xi, \\
(3,3) &= -2\Lambda + B_\xi^T(\tau^2 R)B_\xi, \\
(4,4) &= -\gamma^2 I + G_\xi^T G_\xi + D_\xi^T(\tau^2 R)D_\xi.
\end{aligned}$$

Using Schur complement, we have $\Psi < 0$ from $\Phi < 0$. Thus $J_w \leq 0$, which implies $\|z(t)\|_2 \leq \gamma \|w(t)\|_2$. This completes the proof.

We are now in the position to design controller $\mathcal{C}$ for the master-slave synchronization. Define

$$D_{\xi 0} = \left(D \ 0 \right), \ G_{\xi 0} = \left(0 \ G \right),$$

$$\hat{D}_\xi = \begin{pmatrix} DE_\mu & 0 \\ B_\mu & 0 \end{pmatrix}, \ K_\xi = \begin{pmatrix} K \\ 0 \end{pmatrix}.$$

Applying Proposition 2, system (8) is globally asymptotically stable with an L_2-gain bound γ if there exist positive definite matrices $P = \text{diag}(P_1, P_2)$ with $P_1 \in R^{n\times n}$ and $P_2 \in R^{n_r\times n_r}$, $Q \in R^{(n+n_r)\times(n+n_r)}$, $R \in R^{(n+n_r)\times(n+n_r)}$, $\Lambda = \text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_m)$, and matrix K_ξ with appropriate dimensions such that

$$\begin{pmatrix} (1,1) & (1,2) & (1,3) & (1,4) & H_\xi^T & \tau A_\xi^T R \\ * & (2,2) & 0 & 0 & 0 & (2,6) \\ * & * & -2\Lambda & 0 & 0 & \tau B_\xi^T R \\ * & * & * & -\gamma^2 I & G_\xi^T & (4,6) \\ * & * & * & * & -I & 0 \\ * & * & * & * & 0 & -R \end{pmatrix} < 0, \tag{14}$$

where

$$\begin{aligned}
(1,1) &= PA_\xi + A_\xi^T P + Q - R, \\
(1,2) &= PK_\xi D_{\xi 0} + R, \\
(1,3) &= PB_\xi + kC_\xi^T \Lambda, \\
(1,4) &= P(\hat{D}_\xi + K_\xi G_{\xi 0}), \\
(2,2) &= -Q - R, \\
(2,6) &= \tau D_{\xi 0}^T K_{\xi\tau}^T R, \\
(4,6) &= \tau(\hat{D}_\xi + K_\xi G_{\xi 0})^T.
\end{aligned}$$

Pre- and post-multiplying both sides of (14) with

$$\mathrm{diag}(I, I, I, I, I, PR^{-1})$$

and its transpose, letting $Y_\xi = PK_\xi$ with $Y = P_1 K$, yields

$$\begin{pmatrix} (1,1) & (1,2) & (1,3) & (1,4) & H_\xi^T & \tau A_\xi^T P \\ * & (2,2) & 0 & 0 & 0 & \tau D_{\xi 0}^T Y_\xi^T \\ * & * & -2\Lambda & 0 & 0 & \tau B_\xi^T P \\ * & * & * & -\gamma^2 I & G_\xi^T & (4,6) \\ * & * & * & * & -I & 0 \\ * & * & * & * & 0 & -PR^{-1}P \end{pmatrix} < 0. \tag{15}$$

Notice that (15) includes nonlinear term $-PR^{-1}P$ and, for any $R > 0$, $\varepsilon > 0$,

$$\begin{aligned} &-PR^{-1}P + 2\varepsilon P - \varepsilon^2 R \\ &= -(P - \varepsilon R)R^{-1}(P - \varepsilon R) \le 0. \end{aligned} \tag{16}$$

In light of Schur complement, we have (15) from (16) and

$$\begin{pmatrix} (1,1) & (1,2) & (1,3) & (1,4) & H_\xi^T & \tau A_\xi^T P & 0 \\ * & (2,2) & 0 & 0 & 0 & \tau D_{\xi 0}^T Y_\xi^T & 0 \\ * & * & -2\Lambda & 0 & 0 & \tau B_\xi^T P & 0 \\ * & * & * & -\gamma^2 I & G_\xi^T & (4,6) & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & 0 & -2\varepsilon P & \varepsilon R \\ * & * & * & * & * & * & -R \end{pmatrix} < 0, \tag{17}$$

where terms $(1,1)$, $(1,2)$, $(1,3)$, $(1,4)$, $(2,2)$, and $(4,6)$ are the same as those in (14) and (15). From the above discussion, we conclude the following result.

Proposition 3. For given scalars $\tau > 0$, $\gamma > 0$, and $\varepsilon > 0$, system (8) is globally asymptotically stable with an L_2-gain bound γ if there exist positive definite matrices $P = \mathrm{diag}(P_1, P_2)$ with $P_1 \in R^{n\times n}$ and $P_2 \in R^{n_r\times n_r}$, $Q \in R^{(n+n_r)\times(n+n_r)}$, $R \in R^{(n+n_r)\times(n+n_r)}$, $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_m)$, and matrix $Y_\xi^T = \begin{pmatrix} Y^T & 0 \end{pmatrix}$ with appropriate dimensions such that LMI (17) is satisfied. Moreover, the controller gain matrix K is calculated by $K = P_1^{-1} Y$.

Remark 4. Is is seen from Proposition 3 that one can design a feedback controller for Lur'e systems to implement nonlinear H_∞ synchronization by solving LMI. It should be point out that, however, the proposed design result is a little conservative due to the particular choice of $P = \mathrm{diag}(P_1, P_2)$.

4. A NUMERICAL EXAMPLE

Consider the following Chua's circuit

$$\begin{cases} \dot{x} = \alpha(y - h(x)) + w(t), \\ \dot{y} = x - y + z, \\ \dot{z} = -\beta y, \end{cases}$$

with nonlinear characteristic

$$h(x) = m_1 x + \frac{1}{2}(m_0 - m_1)(|x + c| - |x - c|),$$

and parameters $m_0 = -\frac{1}{7}$, $m_1 = -\frac{2}{7}$, $\alpha = 9$, $\beta = 14.28$, and $c = 1$. The system can be represented in Lur'e form with

$$A = \begin{pmatrix} -\alpha m_1 & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix},\ B = \begin{pmatrix} -\alpha(m_0 - m_1) \\ 0 \\ 0 \end{pmatrix},$$
$$C = D = H = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix},\ G = 1,$$

and $\varphi(\theta) = \frac{1}{2}(|\theta + c| - |\theta - c|)$ belonging to sector $[0, k]$ with $k = 1$. For a first order Butterworth filter $\mathcal{R}$ with cut-off frequency 10Hz, applying Proposition 3, choosing $\varepsilon = 0.05$, we calculate the minimum allowed value of γ (i.e. γ_{min}) and the controller gain matrix K for different values of time-delay τ. Table 1 lists the obtained γ_{min} for different time-delay.

Table 1: Minimum allowed γ_{min} for different time-delay τ

τ	0.04	0.06	0.08	0.10	0.12
γ_{min}	1.01	1.02	1.03	1.08	2.79

Set $\tau = 0.08$ and $\gamma = 1.03$, we have

$$K = \begin{pmatrix} -7.6222 & -1.4366 & 7.2900 \end{pmatrix}^T.$$

The initial conditions of the master and slave are chosen as

$$\begin{pmatrix} x(0) & y(0) & z(0) \end{pmatrix}^T = \begin{pmatrix} -0.2 & -0.33 & 0.2 \end{pmatrix}^T,$$
$$\begin{pmatrix} x_s(0) & y_s(0) & z_s(0) \end{pmatrix}^T = \begin{pmatrix} 0.5 & -0.1 & 0.66 \end{pmatrix}^T.$$

For a message signal $v(t) = \mathrm{sign}(\sin(0.3t))$, the master system state and slave system state are shown in Figure 1 and Figure 2, respectively. The message signal is invisible on the transmitted signal in Figure 3. We recover the original message by taking $\mathrm{sign}(\beta^T e(t))$. Figure 4 shows signal $\beta^T e(t)$ (solid line) and the recovered message $\mathrm{sign}(\beta^T e(t))$ (dashed line), respectively.

5. CONCLUSION

The problem of nonlinear H_∞ master-slave synchronization for Lur'e systems with time-delay feedback control has

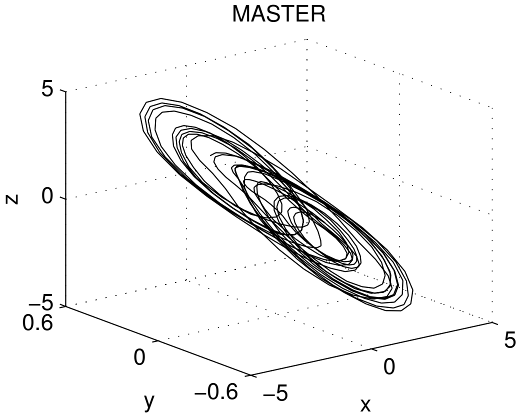

Fig. 1. The master system state

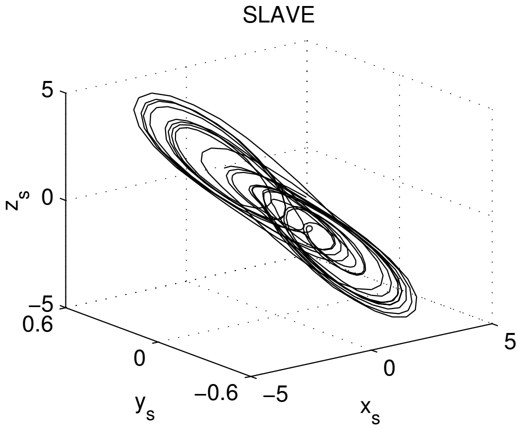

Fig. 2. The slave system state

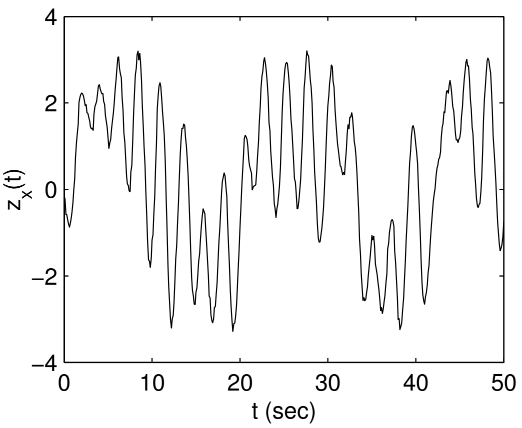

Fig. 3. The transmitted signal $z_x(t)$

been addressed. In this synchronization scheme, the vector field of the master system has been modulated by a filtered binary valued message signal and a static output feedback control with time-delay has been used in the slave system. The master-slave synchronization has been formulated as to minimize a tracking error in the sense of L_2-gain and the message has been recovered from the tracking error. Applying the Lyapunov-Krasovskii functional approach, we have derived a delay-dependent criterion to analyze the error system and, based on the analyzing results, a time-delay feedback control has been obtained by solving LMIs.

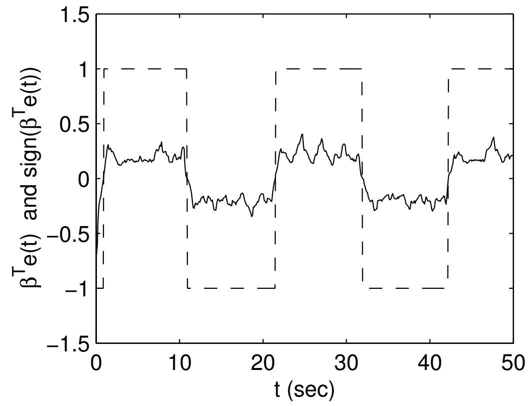

Fig. 4. The signal $\beta^T e(t)$ (solid line) and the recovered message $\mathrm{sign}(\beta^T e(t))$ (dashed line)

The proposed method has been illustrated on Chua's circuit.

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