Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Distributed Formation Algorithm for Multi-agent Systems with a Relaxed Connectivity Condition ⋆

Xiaoli Li * **Yugeng Xi** *

* *Shanghai Jiao Tong University, Shanghai, 200240 China (Tel: +86-021-34204297; e-mail: skill-li@ sjtu.edu.cn, ygxi@sjtu.edu.cn.*

Abstract: This paper proposes a distributed formation algorithm for multi-agent systems with a relaxed connectivity condition. In our study, velocity information exchange among agents depends on the group communication topology, and the available position information flows among agents are determined by a special subgraph of the communication topology. Our distributed formation algorithm guarantees that once this subgraph is connected at some time instant, the topology will keep connected thereafter, and the formation objective of a multi-agent system is proved to achieve in a completely distributed style.

1. INTRODUCTION

The multi-agent formation problem aims at designing proper distributed control strategies to achieve and maintain a pre-specified configuration via local interactions among agents, where the coupling of agents depends on the group interaction topology [Caughman et al., 2005]. The existing representative works were provided in [Caughman et al., 2005, Fax et al., 2004] and [Leonard et al., 2001, Do, 2006, Gennaro et al., 2006], etc., where the averaging feedback control algorithms and the nonlinear control algorithms deduced from artificial potential functions are adopted, respectively. Almost all of these results rely on the assumption that the topology is connected for all time. Similar preconditions are also provided for other distributed cooperative control problems of multi-agent systems, such as consensus [Moreau, 2005, Ren and Beard, 2005], flocking [Olfati-Saber, 2006, Tanner et al., 2007].

However, the group topology may evolve over time due to the motions of agents, and its connectivity relies on the actual distances between agents. Since almost all of the algorithms proposed above do not ensure the maintenance of connectivity, system stabilization can only be established for the cases that the connected topology is satisfied throughout the group evolution. Therefore, efforts should be made to preserve connectivity of the topology while achieving the desired collective objective. The past research on this subject include the geometric connectivity robustness and its applications [Spanos and Murray, 2004], transforming connectivity condition into the constraint on the motion range of agents to solve rendezvous problems of single integrator dynamic systems [Ando et al., 1999, Corts et al., 2006], the admissible set that allows the double integrator to remain inside disks, and the corresponding double-integrator disk graph [Notarstefano et al., 2006]. Ji et al. [2005, 2006] have investigated the rendezvous and formation problems of multi-agent systems with single integrator dynamics while preserving the group connectivity.

In this paper, the formation problem of multi-agent systems with second order dynamic models will be solved with a relaxed connectivity condition. Firstly, an artificial potential function is designed by considering both the formation objective and the connectivity requirement of the topology, which is similar to [Zavlanos and Pappas, 2007]. Furthermore, the collective potential function is constructed on a special subgraph of the topology. From this, a distributed formation algorithm is presented for each agent in the group. It is proved that the multi-agent system steered by our distributed algorithm asymptotically converges to the goal formation, provided this special subgraph of the topology is connected at some time instant.

2. PRELIMINARIES

Consider a multi-agent system with N homogeneous mobile agents, of which each has the dynamics as

$$\begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = u_i \end{cases} \quad (1)$$

where $q_i \in \mathbb{R}^2$ and $p_i \in \mathbb{R}^2$ denote the position and velocity of the i-th agent, respectively. For clarity, the group state is described by $q = col\,(q_1, \cdots q_N)$ and $p = col\,(p_1, \cdots p_N)$. In order to achieve a pre-specified formation, the control law u_i is composed by

$$u_i = f_i^q + f_i^p \quad (2)$$

where f_i^q acts as the induced term for achieving the desired configuration, f_i^p is the velocity consensus term and is used to promote the velocities of agents to a common value.

Assume the communication radius of agents is Δ, then the communication topology of the multi-agent system can be described by an undirected graph $G_p = (\mathcal{V}, \mathcal{E}_p)$, where the set of vertices $\mathcal{V} = \{1, 2..., N\}$ corresponds to the N agents, and the edge set is defined by

$$\mathcal{E}_p = \{(i,j) \mid \|q_j(t) - q_i(t)\| < \Delta, i, j \in \mathcal{V}, j \neq i\} \quad (3)$$

⋆ This work is supported by the National Science Foundation of China (Grant No. 60474002, 60674041) and the National High Tech. Project (Grant No. 2006AA04Z173).

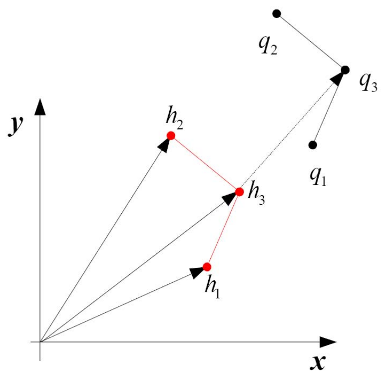


Fig. 1. A group with relative positions determined by h

Thus, the neighbor set of the i-th agent can be described as

$$N_i(G_p) = \{j \in \mathcal{V} | \, (i,j) \in \mathcal{E}_p\} \tag{4}$$

2.1 Definition of the formation objective

As is well known, the collective objective of formation problem can be described by the relative positions and the relative velocities of agents. In our study, the desired relative positions are uniquely determined by utilizing a formation vector $h = col(h_1, h_2, \cdots h_N) \in \mathbb{R}^{nN}$, $h_i \in \mathbb{R}^n$, which is illustrated by Fig. 1.

Define the goal topology of a multi-agent system as $G_g = (\mathcal{V}, \mathcal{E}_g)$, where the set of vertices satisfies $\mathcal{V}(G_g) = \mathcal{V}(G_p)$, the edge set is given by

$$\mathcal{E}_g = \{(i,j) \,|\, \|h_j - h_i\| < \Delta, i, j \in \mathcal{V}, j \neq i\} \tag{5}$$

Thus, the formation objective of a multi-agent system can be compactly described as follows by considering both the relative positions and the relative velocities of agents.

Definition 1. Given a formation vector $h = col(h_1, h_2, ..., h_N)$ satisfying $G_g = (\mathcal{V}, \mathcal{E}_g)$ is connected. Then a multi-agent system is in the formation h, if the agent states satisfy

$$\begin{cases} q_j - q_i = h_j - h_i \\ p_j = p_i \end{cases} \tag{6}$$

for any $(i,j) \in \mathcal{E}_g$.

2.2 Some results on graph theory

In this section, we briefly introduce some significant graph theoretic results used later.

Consider an undirected graph $G = (\mathcal{V}, \mathcal{E})$. a_{ij} is a positive coefficient associated with the edge $(i,j) \in \mathcal{E}$. The adjacency matrix of G is denoted by $A = [a_{ij}]$, which can be used to describe the diagonal degree matrix of G by $D = [d_{ii}]$, where $d_{ii} = \sum\limits_{j \in N_i} a_{ij}$. The Laplacian matrix associated with G is defined by

$$L(G) = L = D - A \tag{7}$$

It is known that $L(G)$ is always symmetric and positive semidefinite for undirected graph G and satisfies

$$z^T L z = \frac{1}{2} \sum_{i=1}^{N} \sum_{j \in N_i} a_{ij} (z_j - z_i)^2 \tag{8}$$

where $z = col(z_1, z_2, \cdots, z_N)$.

3. FORMATION CONTROL

3.1 Background

In most past research related to our work, such as [Olfati-Saber, 2006, Tanner et al., 2007], the distributed control law u_i with form of (2) is often designed by utilizing the state information from agent i and its neighbour agents, and is given by

$$u_i = \underbrace{- \sum_{j \in N_i(G_p)} \nabla_{q_i} V_{ij}}_{f_i^q} + \underbrace{\sum_{j \in N_i(G_p)} a_{ij}(q) \cdot (p_j - p_i)}_{f_i^p} \tag{9}$$

where $N_i(G_p)$ is given by (4), $a_{ij}(q)$ is a positive coefficient which indicates how any one of two agents i and j contributes to the other's velocity. To simplify the following analysis, we assume that $a_{ij}(q)$ in (9) is uniform and smoothly varies from 1 to 0 when the distance between agents i and j increases from 0 to Δ. V_{ij} is a continuous potential function and has its unique minimum when agents i and j are located at a desired distance. Thus, under the control law u_i, the state of agent i changes in the direction of approaching the desired distances from itself to its neighbor agents and decreasing the velocity difference between itself and its neighbor agents. The desired stable group motion has been proved to achieve under the assumption that the topology is connected for all time in above literatures. In the following, a simple proof outline is proposed towards understanding how connectivity of the topology plays the crucial role in the multi-agent cooperative controls, which is also the starting point of our work.

Design that the continuous potential function V_{ij} always equals to $V_{ij}(\Delta)$ when $\|q_i - q_j\| \geq \Delta$. Then the collective potential function is always continuous and can be given by

$$V = \frac{1}{2} \sum_{i=1}^{N} \left(\sum_{j \in N_i(G_p)} V_{ij} + \sum_{j \notin N_i(G_p)} V_{ij}(\Delta) \right) \tag{10}$$

Important results are established by considering the following Hamiltonian function [Olfati-Saber, 2006]

$$H = V + K \tag{11}$$

where V is given by (10), $K = \frac{1}{2} \sum\limits_{i=1}^{N} \|p_i\|^2$ is the group kinetic energy. It has been shown that

$$\dot{H} = \dot{V} + \dot{K} = -p^T \left(L_{G_p} \otimes I_n \right) p \tag{12}$$

where L_{G_p} is the Laplacian associated with the topology G_p, and $\otimes$ denotes the Kronecker production. By utilizing (8), we have

$$\dot{H} \leq 0 \tag{13}$$

Denote $q_j - q_i$ as q_{ji}. If the topology G_p is connected for all time, it is ensured that $\|q_{ji}\|$ is always bounded. Besides, if H_0 is finite, the nonincreasing characteristic of H makes $\|p_i\|^2 \leq 2K \leq 2H \leq 2H_0$, and further $\|p_i\| \leq$

$\sqrt{2H_0}$. Thus, the set $\Omega = \{(q_{ji}, p_i) \,|H \le H_0 \ll \infty, t \ge 0\}$ is a compact invariant set of the multi-agent system. By utilizing LaSalle's invariant principle, system state starting from Ω will converge to the largest invariant set in

$$E = \left\{(q_{ji}, p_i) \in \Omega | \dot{H} = 0\right\}$$

According to (12), $\dot{H} = 0$ implies that

$$p^T \left(L_{G_p} \otimes I_n\right) p = \frac{1}{2} \sum_{i=1}^{N} \sum_{j \in N_i(G_p)} a_{ij} \cdot (p_j - p_i)^2 = 0 \quad (14)$$

where a_{ij} is always positive. In order to achieve a stable goal configuration, all agent velocities in the largest invariant set in E must be synchronized, i.e. $p_1 = p_2 = \cdots = p_N$. It may be guaranteed by assuming that the topology $G_p(t)$ is connected according to equation (14).

From the above analysis, it is obvious that the stabilization of such multi-agent system depends on the connectivity of the topology. However, the above control laws to steer the motions of the agents do not guarantee the maintenance of the connections among the agents, and further can not preserve the group connectivity. For this reason, we will reconstruct the potential function V_{ij} in (9) so that the corresponding connection (i, j) is maintained during maneuvers.

3.2 A new potential function for preserving connectivity

In this section, we will define a special formation potential function instead of the general potential functions used in (9) to ensure that if the topology is connected at some time instant, its connectivity can be preserved thereafter and the desired formation objective can be achieved as time grows. The new potential function is designed for the edges in $\mathcal{E}_p(t) \cap \mathcal{E}_g$ based on the following principles:

i) V_{ij} is always nonnegative and differentiable;

ii) $V_{ij}(q_i, q_j)$ has a single minimal value zero at $q_j - q_i = h_j - h_i$;

iii) $V_{ij} \to +\infty$, if $\|q_j - q_i\| \to \Delta$.

One example of such a potential function is given by

$$V_{ij} = \frac{\|(q_j - q_i) - (h_j - h_i)\|_\sigma^b}{\left(\|\Delta\|_\sigma - \|q_j - q_i\|_\sigma\right)^c}, \quad (i, j) \in (\mathcal{E}_p(t) \cap \mathcal{E}_g) \quad (15)$$

where b, c are both positive constant, and $b \ge 1$, $\|\cdot\|_\sigma = \sqrt{\left(1 + \|\cdot\|_2^2\right)} - 1$ denotes a nonnegative σ-norm [Olfati-Saber, 2006]. An example of V_{ij} is shown in Fig. 2 for $n = 1$, $b = c = 1$.

Except for $(i, j) \in (\mathcal{E}_p(t) \cap \mathcal{E}_g)$, the potential function V_{ij} is designed as $V_{ij} = 0$ for any i and j. Thus, the collective potential function is essentially constructed on $\mathcal{E}_p \cap \mathcal{E}_g$. We now prove that the edges in $\mathcal{E}_g \cap \mathcal{E}_p$ can be maintained by adopting the new potential function (15).

Theorem 1. Consider a multi-agent system steered by the distributed formation algorithm (9), where the potential function is given by (15). If the initial Hamiltonian H_0 is finite and no new edges are added into $\mathcal{E}_p \cap \mathcal{E}_g$ during maneuvers, then all the edges in $\mathcal{E}_p \cap \mathcal{E}_g$ are preserved all the time.

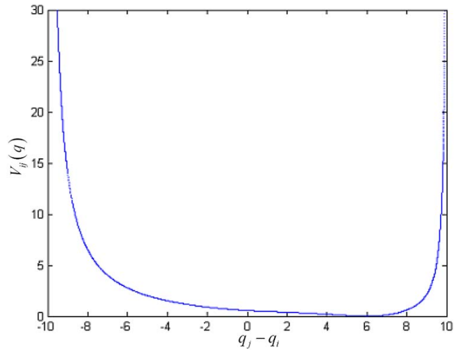


Fig. 2. The formation potential function V_{ij} with $h_j - h_i = 6$, $\Delta = 10$

Proof. Consider a multi-agent system steered by (9), where the potential function is given by (15). It is assumed that there exists an arbitrarily time interval $[0, t_k)$, during which the edge set $\mathcal{E}_p \cap \mathcal{E}_g$ keeps fixed. Then the Hamiltonian function of the continuous system (1) is differentiable and nonincreasing during $[0, t_k)$ based on (13). Thus, the potential function V is bounded by H_0:

$$V \le H \le H_0 \ll \infty$$

However, based on (15) V will increases to infinity whenever $\|q_j - q_i\| \to \Delta$ for any $(i, j) \in (\mathcal{E}_g \cap \mathcal{E}_p)$. Thus, the finite V implies $\|q_j - q_i\|$ will never converge to Δ for any $(i, j) \in (\mathcal{E}_g \cap \mathcal{E}_p)$, i.e. no edges in $\mathcal{E}_p \cap \mathcal{E}_g$ are lost during $[0, t_k)$. If no new edges are added into $\mathcal{E}_p \cap \mathcal{E}_g$, the corresponding Hamiltonian function is always differentiable and nonincreasing, which further guarantees that all the edges in $\mathcal{E}_p \cap \mathcal{E}_g$ are preserved for all time. □

3.3 A new control law based on a subset topology

In Theorem 1, it is assumed that no new edges are added into $\mathcal{E}_p \cap \mathcal{E}_g$, so that the continuity and differentiability of the system Hamiltonian function are ensured. In practice, it is possible that G_p may be switched due to the motions of the agents. To solve this problem, we will define an edge set $\mathcal{E}_q$ instead of $\mathcal{E}_p \cap \mathcal{E}_g$ to model the available position information flows among agents. Based on this edge set $\mathcal{E}_q$, the collective potential function of a multi-agent system is constructed. We define the initial edge set $\mathcal{E}_q(0)$ as

$$\mathcal{E}_q(0) = \{(i, j) | (i, j) \in (\mathcal{E}_p(0) \cap \mathcal{E}_g), i, j \in \mathcal{V}, j \ne i\} \quad (16)$$

When $t > 0$, the new formed edges in $\mathcal{E}_p \cap \mathcal{E}_g$ are also expected to join into set $\mathcal{E}_q$ so that more position information can be used to pursue the group formation objective, which however may destroy the differentiability of the system Hamiltonian function. Therefore, we define that an edge $(i, j) \in (\mathcal{E}_p \cap \mathcal{E}_g)$ can be added into $\mathcal{E}_q$ only when

$$q_j(t) - q_i(t) = h_j - h_i \quad (17)$$

With (17), at the switching time, edge (i, j) has a zero potential energy and a zero potential energy gradient according to (15). Besides, all the edges in $\mathcal{E}_q$ will never be lost according to Theorem 1, that is

$$\mathcal{E}_q(t_1) \subseteq \mathcal{E}_q(t_2), t_1 \le t_2 \quad (18)$$

Thus, even if the edge set $\mathcal{E}_p \cap \mathcal{E}_g$ is arbitrarily changed, the corresponding collective potential function constructed on $\mathcal{E}_q$ can be always smooth and differentiable. Thus the distributed control law of agent i is reconstructed as follows for the multi-agent formation problem

$$u_i = \underbrace{-\sum_{(i,j)\in\mathcal{E}_q} \nabla_{q_i} V_{ij}}_{f_i^q} + \underbrace{\sum_{j\in N_i(G_p)} a_{ij}(q)\cdot(p_j - p_i)}_{f_i^p} \quad (19)$$

For clarity, we define the subgraph set of a graph G as

$$S(G) = \{G_{sub} | \mathcal{V}(G_{sub}) = \mathcal{V}(G)\ ,\ \mathcal{E}(G_{sub}) \subseteq \mathcal{E}(G)\} \quad (20)$$

Denote the graph $(\mathcal{V}, \mathcal{E}_q)$ as G_q. Since when $t > 0$ just part of the new formed edges in $\mathcal{E}_p \cap \mathcal{E}_g$ is added into $\mathcal{E}_q$, it can be easily find that G_q is a subgraph of $G_p(t) \cap G_g$, i.e.

$$G_q \in S(G_p(t) \cap G_g) \quad (21)$$

4. ANALYSIS

In the following, we first introduce a simpler condition of multi-agent formation problems as follows.

Theorem 2. Consider any $G_{sub} \in S(G_g)$ for a connected multi-agent system. If G_{sub} is connected and the agent states satisfy $\begin{cases} q_j - q_i = h_j - h_i \\ p_j = p_i \end{cases}$ for all $(i,j) \in \mathcal{E}(G_{sub})$, then the multi-agent system is in the formation h.

Proof. To prove by contradiction, we assume that the multi-agent system is not in the formation h. That is, at least two agents in G_g, for example a and b, satisfy $q_a - q_b \neq h_a - h_b$ or $p_a \neq p_b$.

Since G_{sub} is connected, the relative position of agent a to b can be expressed as

$$q_a - q_b = (q_a - q_{k_1}) + (q_{k_1} - q_{k_2}) + \cdots + (q_{k_m} - q_b)$$

where all $(a, k_1), (k_1, k_2), \cdots (k_m, b) \in \mathcal{E}(G_{sub})$. Since $\begin{cases} q_j - q_i = h_j - h_i \\ p_j = p_i \end{cases}$ is always satisfied for all $(i,j) \in \mathcal{E}(G_{sub})$, then we have

$$\begin{cases} (q_a - q_{k_1}) + \cdots + (q_{k_m} - q_b) = h_a - h_b \\ p_a = p_{k_1} = p_{k_2} = \cdots = p_{k_m} = p_b \end{cases}$$

This contradicts the assumption of $q_a - q_b \neq h_a - h_b$ or $p_a \neq p_b$. □

By using Theorem 2 and LaSalle's invariance principle, we have the following result.

Theorem 3. Consider a multi-agent system with dynamics (1) and each agent steered by the distributed control law (19). Given an initial finite group Hamiltonian H_0. Assume the subgraph G_q is connected at some time instant t_s, $t_s < \infty$, then the multi-agent system asymptotically converges to the pre-specified formation.

Proof. According to the fact that $\mathcal{E}_q(t_1) \subseteq \mathcal{E}_q(t_2)$, $t_1 \leq t_2$ given by (18) and the assumption that the subgraph $G_q(t_s)$ is connected, it can be deduced that $G_q(t)$ is always connected if $t \geq t_s$ according to Theorem 1. Since $G_q \in S(G_p(t))$ based on equation(21), $G_p(t)$ is also connected when $t \geq t_s$.

From the assumption that H_0 is finite and the fact of $\dot{H} \leq 0$, the group Hamiltonian H keeps finite for all time. So we have $V_{ij} \leq V \leq H_0 \ll \infty$ and $K \leq H_0 \ll \infty$. The distance between any two neighbors in $G_q(t)$ is restricted by $\|q_j - q_i\| \leq V_{ij}^{-1}(H_0) < \Delta$. For a connected topology $G_q(t)$, the path which connects agent i and j has length of no grater than $(N-1)\left(V_{ij}^{-1}(H_0)\right)$. On the other hand, $K = \frac{1}{2}\sum_{i=1}^{N} \|p_i\|^2 \leq H_0$ guarantees $\|p_i\| \leq \sqrt{2H_0}$. Then the set $\Omega = \{(q_{ji}, p_i) | H \leq H_0, t \geq 0\}$ is a compact invariant set of the multi-agent system. From LaSalle's invariance principle, system state starting from Ω will converge to the largest invariant set in $E = \left\{(q_{ji}, p_i) \in \Omega | \dot{H} = 0\right\}$.

According to (12), the group velocity vector satisfies

$$p^T\left(L_{G_p} \otimes I_n\right) p = \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i(G_p)} a_{ij}\cdot(p_j - p_i)^2 = 0 \quad (22)$$

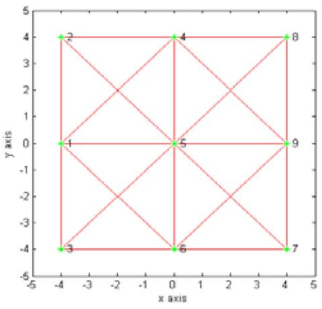
Then for the connected topology $G_p(t)$, $t \geq t_s$, equation (22) is valid only when $p_1 = p_2 = \cdots = p_N = p*$.

The common velocity of agents ensures that q_{ji} keeps constant for any agent i and j in the largest invariant set in E. Thus, the corresponding potential energy is invariant, which makes the dynamic energy of the multi-agent system invariant because of $\dot{H} = 0$. Therefore, in the largest invariant set in E, the velocities of the agents keep constant, which follows that

$$\begin{aligned} \dot{p}_i &= -\sum_{j\in N_i(G_q)} \nabla_{q_i} V_{ij}(q_{ji}) \\ &= -\sum_{j\in N_i(G_q)} \left(-\frac{b\cdot\left(\sqrt{1+(q_j - q_i - h_{ji})^2} - 1\right)^{b-1}}{\left(\sqrt{1+\Delta^2} - \sqrt{1+(q_j - q_i)^2}\right)^{c}} \cdot \frac{(q_j - q_i - h_{ji})}{\sqrt{1+(q_j - q_i - h_{ji})^2}} - \frac{(q_j - q_i)}{\sqrt{1+(q_j - q_i)^2}} \cdot \frac{c\cdot\left(\sqrt{1+(q_j - q_i - h_{ji})^2} - 1\right)^{b}}{\left(\sqrt{1+\Delta^2} - \sqrt{1+(q_j - q_i)^2}\right)^{c+1}} \right) \\ &= 0, i = 1, \cdots, N \end{aligned} \quad (23)$$

based on (15), where $h_{ji} = h_j - h_i$. Equation (23) is satisfied for all $i = 1, \cdots N$ when $q_j - q_i - (h_j - h_i) = 0$ is achieved for each $(i,j) \in G_q$. Therefore, the group states in the largest invariant set in E satisfy $\begin{cases} q_j - q_i = h_j - h_i \\ p_j = p_i \end{cases}$ for all $(i,j) \in G_q$. From the facts that G_q is connected and belongs to $S(G_g)$ deduced by (21), the multi-gent system asymptotically converges to the pre-specified formation h by utilizing Theorem 2. □

Remark 2. From Theorem 3, the formation objective of a multi-agent system can be achieved under the control law (19) as long as the initial subgraph $G_q(0)$ is connected.



(a) (b)

Fig. 3. The initial and goal configurations of the multi-agent system

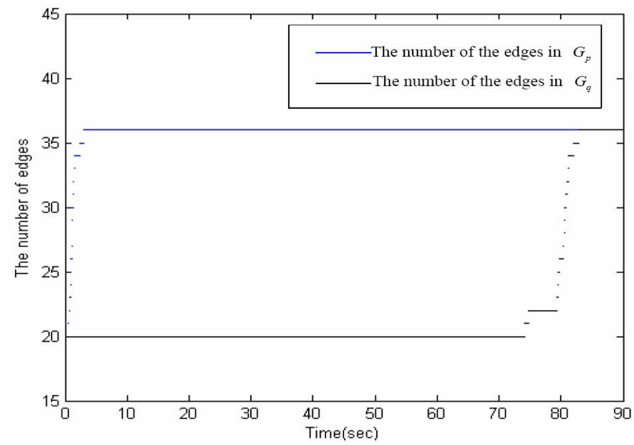


Fig. 4. The numbers of the edges in G_p and $G_q\left(0\right)$ during maneuvers

5. SIMULATION

In this section, we will provide a simulation of applying the proposed distributed formation algorithm on a multi-agent system. We assume the communication radius of agents is 7.5. The parameter b and c in the potential function (15) are both 1. The initial and final configurations of the multi-agent system are described in Fig. 3.a and Fig. 3.b, respectively. One can easily find that the initial subgraph $G_q\left(0\right)$ is connected.

We illustrate the formation course of the multi-agent system by Fig. 4 and Fig. 5. From the initial connected $G_q\left(0\right)$, the topology G_p of the driven multi-agent system keeps connected for all time, which is consistent with the result proved in previous sections. The numbers of the edges in G_p and G_q during maneuvers are shown in Fig. 4, respectively. With the connected topology G_p, the desired relative states of the agents in the goal formation are achieved with a consensus agent velocity, which means that the multi-agent system asymptotically converges to the pre-specified formation according to Fig. 5.

6. CONCLUSIONS

This paper aims at solving the formation problem of multi-agent systems. A potential function is designed to steer the corresponding neighbor agents to the desired relative

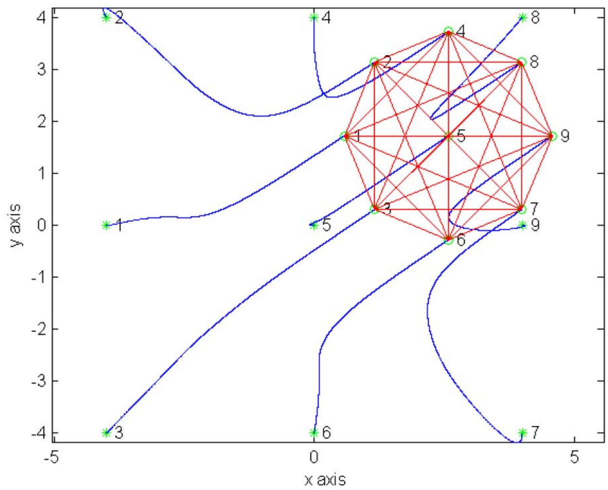


Fig. 5. The multi-agent system asymptotically achieves the desired collective objective

positions while preserving the connection among them. The velocity and position information flows among agents are described by the group communication topology and its special subgraph, respectively. Based on this potential function and the information networks, a distributed formation algorithm is presented. It is guaranteed that the multi-agent system asymptotically converges to the pre-specified formation as long as this subgraph is connected at some time instant.

REFERENCES

H. Ando,Y. Osuzuki, and M. Yamashita. Distributed memoryless point convergengce algorithm for mobile robots with limited visibility. *IEEE Transactions on Robotics and Automation*, 15(5):818–828,1999.

J. S. Caughman, G. Lafferriere, J. J. P. Veerman, and A. Williams. Decentralized control of vehicle formations. *Systems and Control Letters* 54:899–910, 2005.

J. Corts, S. Martnez, and F. Bullo. Robust rendezvous for mobile autonomous agent via proximity graphs in arbitrary dimensions. *IEEE Transactions on Robotics and Automation* 51(8):1289–1298, 2006.

M. C. De Gennaro, and A.Jadbabaie. Formation control for a cooperative multi-agent system using decentralized navigation functions. In *Proc. American Control Conf.*, pages 1346–1351, 2006.

K. D. Do. Formation control of mobile agents using local potential functions. In *Proc. American Control Conf.*, pages 2148–2153, Minneapolios, Minnesota, USA, 2006.

J. A. Fax, and R. M. Murray. Information flow and cooperative control of vehicle formations. *IEEE Transactions on Automatic Control* 49(9):1465–1476, 2004.

M. Ji, and M. Egerstedt. Connectedness preserving distributed coordination control over dynamic graphs. In *Proc. American Control Conf.*, pages 93–98, USA, 2005.

M. Ji, and M. Egerstedt. Distributed formation control while preserving connectedness. In *Proc. IEEE Conf. Decision Control*, pages 5962–5967, San Diego, CA, USA, 2006.

N. E. Leonard, and E. Fiorelli Virtual leaders, artificial potentials, and coordinated control of groups. In *Proc. IEEE Conf. Decision Control*, pages 2968–2973, Orlando, Florida, USA, 2001.

L. Moreau. Stability of multiagent systems with time-dependent communication links. *IEEE Transactions on Automatic Control* 50(2):169–182, 2005.

G. Notarstefano, K. Savla, F. Bullo, and A. Jadbabaie. Maintaining limited-range connectivity among second-order agents. In *Proc. American Control Conf.*, pages 2124–2129, 2006.

R. Olfati-Saber. Flocking for multi-agent dynamic systems: algorithms and theory. *IEEE Transactions on Automatic Control* 51(3):401–420, 2006.

W. Ren and R. W. Beard. Consensus seeking in multi-agent systems under dynamically changing interaction topologies. *IEEE Transactions on Automatic Control* 50(5):655–661, 2005.

D.P. Spanos and R. M. Murray. Robust connectivity of networked vehicles. In *Proc. IEEE Conf. Decision Control*, pages 2893–2898, Pasadena, CA, USA, 2006.

H. G. Tanner, A. Jadbabaie, and G. J. Pappas. Flocking in fixed and switching networks. *IEEE Transactions on Automatic Control* 52(5):863–868, 2007.

M. M. Zavlanos and G. J. Pappas. Flocking while Preserving Network Connectivity. In *Proc. IEEE Conf. Decision Control*, pages 2919–2924, 2007.