Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

A New Initial Alignment Algorithm for Strapdown Inertial Navigation System Using Sensor Output

Kwangjin Kim* and Chan Gook Park**

**Doosan Infracore, Gyeounggi-Do, 448795, Korea*
(Tel: 82-31-270-1545; e-mail: kwangjin1.kim@doosan.com).
***Seoul National University, Seoul, 151744, Korea*
(Tel: 82-2-880-1675; e-mail:chanpark@snu.ac.kr)

Abstract: In this paper, a new alignment algorithm that uses simultaneously both open-loop and closed-loop scheme is designed to increase the convergence rate of the Kalman filter in the fine alignment stage. Generally, the initial alignment is divided into coarse and fine alignment. The fine alignment stage with the 10-state Kalman filter refines the initial estimate of the transformation matrix given by the coarse alignment algorithm. This paper derives a convergence theorem of the Kalman filter for analyzing a problem of the 10-state Kalman filter in the fine alignment. In order to resolve the problem, the new alignment algorithm calculates the attitude angles with the open-loop scheme and estimates the accelerometer and gyro biases with the closed-loop scheme at once. The estimated bias errors are used to correct the sensor errors that are utilized to calculate the attitude angles in the open-loop scheme. The computer simulation results illustrate the efficiency of this new alignment algorithm.

1. INTRODUCTION

The strapdown inertial navigation system (SDINS) provides the position, velocity, and attitude of any moving vehicle with a known start position and is now being used more widely for the navigation of aeroplanes, ships, vehicles, and rockets, etc. Alignment is the process whereby the orientation of the axes of a SDINS is determined with respect to the reference axis system (Titterton et al., 1997). In many applications, It is essential to achieve an accurate alignment for the precision navigation over long periods of time without any form of adding. There are two fundamental types of alignment process: initial alignment and in-flight alignment.

The initial alignment process is of vital importance to SDINS (Jiang, 1998). The purpose of initial alignment of the SDINS is to get a coordinate transformation matrix from body frame to navigation frame, and drive the misalignment angle to zero. Normally, alignment process is divided into two phases, i.e., coarse and fine alignment. The purpose of the coarse alignment is to provide a fairly good initial condition for the fine alignment. Generally, the coarse alignment stage would use the analytic alignment scheme, which utilizes the measurement of the gravity and earth rotation vectors to directly compute the transformation matrix (Britting, 1971). This is defined as open-loop alignment.

The fine alignment stage refines the initial estimate of the transformation matrix calculated by the coarse alignment algorithm. To do this, the 10-state Kalman filter is generally used in the fine alignment for SDINS (Lee et al., 1992). The Kalman filter provides estimates of the attitude errors, the north and east velocity errors, the accelerometer biases, and the gyro biases. The estimated values are used to correct the pure navigation algorithm. This is defined as closed-loop alignment. However, since the observability of SDINS is weak, the alignment performance is degraded (Fang et al., 1995).

In this paper, a new alignment algorithm that uses both open-loop and closed-loop scheme is designed to increase the convergence rate of the Kalman filter in the fine alignment stage. In order to analyze the problem of the 10-state Kalman filter, a convergence theorem of the Kalman filter is proposed and applied to the alignment model. The new alignment algorithm calculates the attitude angles with the open-loop scheme and estimates the accelerometer and gyro biases with the closed-loop scheme. The estimated bias errors are used to correct the sensor errors that are utilized to calculate the attitude angles in the open-loop scheme.

This paper is organized as follows. Section 2 provides a brief introduction of the coarse alignment and fine alignment algorithms. In Section 3, a convergence theorem is derived and applied to the 10 state Kalman filter. In Section 4, a new alignment algorithm is proposed. Section 5 shows the simulation results. Finally, a concluding remark is provided in Section 6.

2. INITIAL ALIGNMENT FOR SDINS

The requirement of the initial alignment is related to the necessity for the transformation of the sensor output into a best estimate of the attitude, velocity and position of a vehicle with respect to the reference navigation frame. A two-stage alignment scheme appears promising in this regard, i.e., the coarse alignment and the fine alignment. Since SDINS is entirely self-contained, it can align itself by using the measurements of local gravity and earth rate.

2.1 Coarse Alignment

The coarse alignment utilizes directly the body mounted sensors with accelerometers and gyros for attitude estimation. The accelerometer outputs are used for solving the levelling problem while the gyro outputs are required for azimuth estimation. The accelerometers and gyros will measure components of the specific force needed to overcome gravity and components of earth rate, denoted by the vector quantities f^b and ω^b respectively. These vectors are related to the gravity and Earth's rate vectors specified in the local geographic frame, f^n and ω^n respectively, in accordance with the following equations:

$$f^b = C_n^b f^n = C_n^b \begin{bmatrix} 0 & 0 & -g \end{bmatrix}^T \tag{1}$$

$$\omega^b = C_n^b \omega^n = C_n^b \begin{bmatrix} \Omega_{ie}\cos L & 0 & -\Omega_{ie}\sin L \end{bmatrix}^T \tag{2}$$

where g and Ω_{ie} represent the magnitude of gravity and Earth rate, respectively, and L is the local geographical latitude.

Inserting the transformation matrix C_n^b into (1) yields the following equations

$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} g\sin\theta \\ -g\sin\phi\cos\theta \\ -g\cos\phi\cos\theta \end{bmatrix} \tag{3}$$

where ϕ and θ represent roll and pitch angle, respectively.

From (3), the roll and pitch angles are calculated by

$$\phi = \tan^{-1}\left(\frac{-g\sin\phi\cos\theta}{-g\cos\phi\cos\theta}\right) = \tan^{-1}\left(\frac{f_y}{f_z}\right) \tag{4}$$

$$\theta = \tan^{-1}\left(\frac{g\sin\theta}{g\cos\theta}\right) = \tan^{-1}\left(\frac{f_x}{\sqrt{f_y^2 + f_z^2}}\right) \tag{5}$$

Eqs. (4) and (5) show that the roll and pitch angles are decided by using only accelerometer outputs.

To calculating yaw angle, the transformation matrix C_n^b is divided into the matrix C_1 containing roll and pitch angle and C_2 containing yaw angle. Hence, C_1 and C_2 are given by

$$C_1 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\phi\sin\theta & \cos\phi & \sin\phi\cos\theta \\ \cos\phi\sin\theta & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \tag{6}$$

$$C_2 = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Inserting (6) and (7) into (2) and rearranging yields the following equation

$$\begin{bmatrix} \omega_x\cos\theta + \omega_y\sin\phi\sin\theta + \omega_z\cos\phi\sin\theta \\ \omega_y\cos\phi - \omega_z\sin\phi \\ -\omega_x\sin\theta + \omega_y\sin\phi\cos\theta + \omega_z\cos\phi\cos\theta \end{bmatrix} = \begin{bmatrix} \omega_N\cos\psi \\ -\omega_N\sin\psi \\ \omega_D \end{bmatrix} \tag{8}$$

From (8), the yaw angle is calculated by

$$\psi = \tan^{-1}\left(\frac{\omega_z\sin\phi - \omega_y\cos\phi}{\omega_x\cos\theta + \omega_y\sin\phi\sin\theta + \omega_z\cos\phi\sin\theta}\right) \tag{9}$$

From (4), (5) and (9), it is known that the initial attitudes are roughly calculated with an open-loop scheme.

2.2 Fine Alignment

In the fine alignment stage, a Kalman filter is used to estimate the small misalignment angles between reference frame and true frame. Here we have modified Bar-Itzhack and Berman's inertial navigation system error model. The SDINS stationary error model augmented with sensor errors can be written

$$\begin{bmatrix} \dot{x}_f(t) \\ \dot{x}_a(t) \end{bmatrix} = \begin{bmatrix} F & T_i \\ 0_{5\times5} & 0_{5\times5} \end{bmatrix}\begin{bmatrix} x_f(t) \\ x_a(t) \end{bmatrix} + \begin{bmatrix} w_f(t) \\ 0_{5\times1} \end{bmatrix}$$

$$\equiv A_i x(t) + w(t) \quad w(t) \sim N(0, Q) \tag{10}$$

where

$$F = \left[\begin{array}{cc|ccc} 0 & 2\Omega_D & 0 & g & 0 \\ -2\Omega_D & 0 & -g & 0 & 0 \\ \hline 0 & 0 & 0 & \Omega_D & 0 \\ 0 & 0 & -\Omega_D & 0 & \Omega_N \\ 0 & 0 & 0 & -\Omega_N & 0 \end{array}\right]$$

$$T_i = \left[\begin{array}{cc|ccc} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{21} & C_{22} & 0 & 0 & 0 \\ \hline 0 & 0 & C_{11} & C_{12} & C_{13} \\ 0 & 0 & C_{21} & C_{22} & C_{23} \\ 0 & 0 & C_{31} & C_{32} & C_{33} \end{array}\right]$$

$$x_f = \begin{bmatrix} v_N & v_E & \psi_N & \psi_E & \psi_D \end{bmatrix}^T$$

$$x_a = \begin{bmatrix} \nabla_x & \nabla_y & \varepsilon_x & \varepsilon_y & \varepsilon_z \end{bmatrix}^T$$

where the subscript N, E, and D denote the north, east, and down component, respectively.

The measured signals during the stationary alignment are the horizontal velocity errors. Therefore the observation model can be written

$$z(t) = \begin{bmatrix} I_{2\times2} & 0_{2\times8} \end{bmatrix} \begin{bmatrix} x_f(t) \\ \hline x_a(t) \end{bmatrix} + \begin{bmatrix} v_1(t) \\ \hline v_2(t) \end{bmatrix}$$

$$\equiv Hx(t) + v(t) \qquad v(t) \sim N(0, \ R) \tag{11}$$

In order to estimate the state vectors of error model by Kalman filter, the observability analysis of error model must be performed. Due to

$$Rank\begin{bmatrix} H & HA_i & \cdots & HA_i^9 \end{bmatrix}^T = 7$$

the system is not completely observable. Therefore, only 7 states are observable (the estimation value of state is convergent by Kalman filter); the other 3 states are not observable.

3. Convergence Analysis

For the fast initial alignment, the convergence rate of the Kalman filter in the fine alignment is important. In this Section, a convergence theorem is proposed to analyze the problem of the 10-state Kalman filter in Section 2.

3.1 Convergence Theorem

Theorem 1. The system model and measurement model are given by

$$\dot{x} = Fx, \quad z = Hx \tag{12}$$

where, $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$ and their initial values are all zero.

$$F = \begin{bmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, P_0 = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix}, H = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T.$$

If $(a\cdot\sigma_2)^2 >> (b\cdot\sigma_3)^2$, the estimated state $\hat{x}_3$ of the Kalman filter is converged into zero □

Proof. From the discrete Kalman filter, the propagation equations of the state and the covariance are given by

$$\hat{x}_{t_0}(-) = \Phi_{t_0}\hat{x}_{t_0}(+) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

$$P(-) = \begin{bmatrix} \sigma_1^2 + a^2\cdot\sigma_2^2 + b^2\cdot\sigma_3^2 & a\cdot\sigma_2^2 & b\cdot\sigma_3^2 \\ a\cdot\sigma_2^2 & \sigma_2^2 & 0 \\ b\cdot\sigma_3^2 & 0 & \sigma_3^2 \end{bmatrix} \tag{14}$$

The Kalman gain is calculated as follows

$$K_{t_1} = \begin{bmatrix} 1 \\ a\cdot\sigma_2^2/S \\ b\cdot\sigma_3^2/S \end{bmatrix} \tag{15}$$

where $S = \sigma_1^2 + a^2\cdot\sigma_2^2 + b^2\cdot\sigma_3^2$.

From equations (13), (14) and (15), the time update equation for the covariance and state are given by

$$P_{t_1}(+) = \begin{bmatrix} 1 & a\cdot\sigma_2^2 & b\cdot\sigma_3^2 \\ 0 & \sigma_2^2 - a^2\cdot\sigma_2^4/S & -ab\cdot(\sigma_2\sigma_3)^2/S \\ 0 & -ab\cdot(\sigma_2\sigma_3)^2/S & \sigma_3^2 - b^2\cdot\sigma_3^4/S \end{bmatrix} \tag{16}$$

$$\hat{x}_{t_1}(+) = \begin{bmatrix} x_1 \\ (a\cdot\sigma_2^2/S)\cdot x_1 \\ (b\cdot\sigma_3^2/S)\cdot x_1 \end{bmatrix} \tag{17}$$

When $(a\cdot\sigma_2)^2 >> (b\cdot\sigma_3)^2$, the equations (15), (16) and (17) could be approximated as follows:

$$K_{t_1} = \begin{bmatrix} 1 \\ a\cdot\sigma_2^2/S \\ 0 \end{bmatrix} \tag{18}$$

$$P_{t_1}(+) = \begin{bmatrix} 1 & a\cdot\sigma_2^2 & b\cdot\sigma_3^2 \\ 0 & \sigma_2^2 - a^2\cdot\sigma_2^4/S & -ab\cdot(\sigma_2\sigma_3)^2/S \\ 0 & -ab\cdot(\sigma_2\sigma_3)^2/S & \sigma_3^2 \end{bmatrix} \tag{19}$$

$$\hat{x}_{t_1}(+) = \begin{bmatrix} x_1 \\ (a\cdot\sigma_2^2/S)\cdot x_1 \\ 0 \end{bmatrix} \tag{20}$$

■

3.2 Analysis of Fine Alignment

In order to analyze the convergence of the fine alignment, it is assumed that the body frame coincides with the navigation frame. And, the earth rate components, Ω_N and Ω_D, are very small value. Hence, it is possible to approximate to zero. When $C_b^n = \mathrm{I}$ and the earth rate components are zero, the system model F and T_i of (10) are represented by

$$F \approx \left[\begin{array}{cc|ccc} 0 & 0 & 0 & g & 0 \\ 0 & 0 & -g & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right], \ T_i = \left[\begin{array}{cc|ccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right] \tag{21}$$

From (21), the north velocity error and related state equations could be rewritten by

$$\delta\dot{V}_N \approx g\cdot\phi_E + \nabla_x \tag{22}$$

$$\dot{\phi}_E \approx 0 \tag{23}$$

$$\dot{\nabla}_x = 0 \tag{24}$$

The above equations have the same form of the equation (12). Therefore, it is possible to apply the theorem 1 to analyze the relationship between attitude error and gyro bias in (22).

The velocity error equation could be rearranged into the attitude error equation as follows

$$\phi_E = \frac{1}{g}\left(\delta\dot{V}_N - 2\Omega_D \delta V_E\right) - \frac{\nabla_x}{g} \tag{25}$$

From the theorem 1, the accelerometer bias ∇_x of (25) is converged into zero by the Kalman filter. And the velocity error terms of the above equation are known as the filter measurement of the fine alignment. This means that the velocity errors are always observable. Therefore, the Kalman filter could estimate the attitude error ϕ_E as

$$\hat{\phi}_E = \frac{1}{g}\left(\delta\dot{V}_N - 2\Omega_D \delta V_E\right) \tag{26}$$

From (25) and (26), the estimation error of the east attitude is given by

$$\delta\phi_E = -\frac{\nabla_x}{g} \tag{27}$$

Similarly, the east velocity error and related state equations could be rewritten by

$$\delta\dot{V}_E \approx -g \cdot \phi_N + \nabla_y \tag{28}$$

$$\dot{\phi}_N \approx 0 \tag{29}$$

$$\dot{\nabla}_y = 0 \tag{30}$$

From the theorem 1, the accelerometer bias ∇_y is converged into zero. The velocity error equation could be rearranged into the attitude error equation as follows

$$\phi_N = -\frac{1}{g}\left(\delta\dot{V}_E 2\Omega_D \delta V_N\right) + \frac{\nabla_y}{g} \tag{31}$$

The Kalman filter could estimate the attitude error ϕ_N as

$$\hat{\phi}_N = -\frac{1}{g}\left(\delta\dot{V}_E + 2\Omega_D \delta V_N\right) \tag{32}$$

From (31) and (32), the estimation error of the north attitude is given by

$$\delta\phi_N = \frac{\nabla_y}{g} \tag{33}$$

From (26) and (32), the north and east attitude errors are estimable since the x and y accelerometer biases are converged into zero by the Kalman filter. However, the Kalman filter needs some times to make the accelerometer biases converging into zero.

4. NEW ALIGNMENT ALGORITHM

In order to overcome the flaw of fine alignment with the 10-state in Section 3, the new alignment algorithm that uses simultaneously both open-loop and closed-loop scheme is proposed in this section. The new alignment algorithm calculates the attitude angles with the open-loop scheme and estimates the accelerometer and gyro biases with the closed-loop scheme.

The open-loop algorithm was given by (4), (5), and (9).

For the closed-loop scheme, the sensor biases for the system equation of the Kalman filter are modelled as random constants

$$\begin{bmatrix} \dot{\nabla}_x \\ \dot{\nabla}_y \\ \dot{\nabla}_z \\ \hline \dot{\varepsilon}_x \\ \dot{\varepsilon}_y \\ \dot{\varepsilon}_z \end{bmatrix} = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} \nabla_x \\ \nabla_y \\ \nabla_z \\ \hline \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} + \begin{bmatrix} w_{\nabla_x} \\ w_{\nabla_y} \\ w_{\nabla_z} \\ \hline w_{\varepsilon_x} \\ w_{\varepsilon_y} \\ w_{\varepsilon_z} \end{bmatrix} \tag{34}$$

where w_∇ and w_ε represent random white noises.

In the new alignment scheme, the accelerometer and gyro outputs are directly used to the filter measurements since the filter states of (34) are only sensor errors. From the accelerometer outputs (3), the accelerometer measurements are represented as

$$\begin{bmatrix} z_{f_x} \\ z_{f_y} \\ z_{f_z} \end{bmatrix} = \begin{bmatrix} \tilde{f}_x - g\sin\tilde{\theta} \\ \tilde{f}_y + g\sin\tilde{\phi}\cos\tilde{\theta} \\ \tilde{f}_z + g\cos\tilde{\phi}\cos\tilde{\theta} \end{bmatrix} \tag{35}$$

The equation (35) could be calculated by utilizing the perturbation method as follows:

$$\begin{bmatrix} z_{f_x} \\ z_{f_y} \\ z_{f_z} \end{bmatrix} = \begin{bmatrix} \nabla_x - g\cos\theta \cdot \delta\theta \\ \nabla_y + g\cos\phi \cdot \cos\theta \cdot \delta\phi - g\sin\phi \cdot \sin\theta \cdot \delta\theta \\ \nabla_z - g\sin\phi \cdot \cos\theta \cdot \delta\phi - g\cos\phi \cdot \sin\theta \cdot \delta\theta \end{bmatrix} \tag{36}$$

The gyro outputs are given by

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \Omega_N \cdot c\theta \cdot c\psi - \Omega_D \cdot s\theta \\ \Omega_N (s\phi \cdot s\theta \cdot c\psi - c\phi \cdot s\psi) + \Omega_D \cdot s\phi \cdot c\theta \\ \Omega_N (c\phi \cdot s\theta \cdot c\psi + s\phi \cdot s\psi) + \Omega_D \cdot c\phi \cdot c\theta \end{bmatrix} \tag{37}$$

where $s\phi = \sin\phi$, $c\phi = \cos\phi$, $s\theta = \sin\theta$, $c\theta = \cos\theta$, $s\psi = \sin\psi$, and $c\psi = \cos\psi$.

The gyro measurements are represented as

$$\begin{bmatrix} z_{\omega_x} \\ z_{\omega_y} \\ z_{\omega_z} \end{bmatrix} = \begin{bmatrix} \tilde{\omega}_x - \Omega_N \cdot c\tilde{\theta} \cdot c\tilde{\psi} - \Omega_D \cdot s\tilde{\theta} \\ \tilde{\omega}_y - \Omega_N (s\tilde{\phi} \cdot s\tilde{\theta} \cdot c\tilde{\psi} - c\tilde{\phi} \cdot s\tilde{\psi}) - \Omega_D \cdot s\tilde{\phi} \cdot c\tilde{\theta} \\ \tilde{\omega}_z - \Omega_N (c\tilde{\phi} \cdot s\tilde{\theta} \cdot c\tilde{\psi} + s\tilde{\phi} \cdot s\tilde{\psi}) - \Omega_D \cdot c\tilde{\phi} \cdot c\tilde{\theta} \end{bmatrix} \tag{38}$$

The equation (38) could be calculated by utilizing the perturbation method as follows

$$\begin{bmatrix} z_{\omega_x} \\ z_{\omega_y} \\ z_{\omega_z} \end{bmatrix} = \begin{bmatrix} h_1 \cdot \delta\theta + h_2 \cdot \delta\psi + \varepsilon_x \\ h_3 \cdot \delta\phi + h_4 \cdot \delta\theta + h_5 \cdot \delta\psi + \varepsilon_y \\ h_6 \cdot \delta\phi + h_7 \cdot \delta\theta + h_8 \cdot \delta\psi + \varepsilon_z \end{bmatrix} \tag{39}$$

where $h_1 = \Omega_N \cdot s\theta \cdot c\psi + \Omega_D \cdot c\theta$, $h_2 = \Omega_N \cdot c\theta \cdot s\psi$,

$h_3 = -(\Omega_N \, c\phi \cdot s\theta \cdot c\psi + \Omega_D \cdot c\phi \cdot c\theta + \Omega_N \cdot s\phi \cdot s\psi)$,

$h_4 = -\Omega_N \cdot s\phi \cdot c\theta \cdot s\psi + \Omega_D \cdot s\phi \cdot s\theta$,

$h_5 = \Omega_N \cdot s\phi \cdot s\theta \cdot s\psi + \Omega_N \cdot c\phi \cdot c\psi$,

$h_6 = \Omega_N \cdot s\phi \cdot c\theta \cdot c\psi + \Omega_D \cdot s\phi \cdot c\theta - \Omega_N \cdot c\phi \cdot s\psi$,

$h_7 = -\Omega_N \cdot c\phi \cdot c\theta \cdot c\psi + \Omega_D \cdot c\phi \cdot s\theta$,

$h_8 = \Omega_N \cdot c\phi \cdot s\theta \cdot s\psi - \Omega_N \cdot s\phi \cdot c\psi$.

The measurement equations must be constituted with system states that are accelerometer biases and gyro biases. However, the attitude error terms are contained in the measurement equations, (36) and (39). The following attitude error equations are used to get rid of these error terms (Park et al., 1998).

$$\delta\phi = \frac{\nabla_z \sin\phi - \nabla_y \cos\phi}{g \cos\theta} \tag{40}$$

$$\delta\theta = \frac{\nabla_x \cos\theta + \sin\theta(\nabla_y \sin\phi + \nabla_z \cos\phi)}{g} \tag{41}$$

$$\delta\psi = \frac{h_9 \cdot \varepsilon_x + h_{10} \cdot \varepsilon_y + h_{11} \cdot \varepsilon_z}{\Omega_N} \tag{42}$$

where $h_9 = c\theta \cdot s\psi$, $h_{10} = s\phi \cdot s\theta \cdot s\psi + c\phi \cdot c\psi$

$h_{11} = c\phi \cdot c\theta \cdot s\psi - s\phi \cdot c\psi$.

For the Kalman filter, inserting (40), (41) and (42) into (36) and (39) yields the following measurement matrix H.

$$H = \begin{bmatrix} H_{11} & 0_{3\times3} \\ H_{21} & H_{22} \end{bmatrix} \tag{43}$$

where $H_{11} = \begin{bmatrix} s^2\theta & -s\phi \cdot s\theta \cdot c\theta & -c\phi \cdot s\theta \cdot c\theta \\ -s\phi \cdot s\theta \cdot c\theta & s^2\phi \cdot c^2\theta & s\phi \cdot c\phi \cdot c^2\theta \\ -c\phi \cdot s\theta \cdot c\theta & s\phi \cdot c\phi \cdot c^2\theta & c^2\phi \cdot c^2\theta \end{bmatrix}$,

$$H_{21} = \begin{bmatrix} \frac{c\theta \cdot h_1}{g} & \frac{s\phi \cdot s\theta \cdot h_1}{g} & \frac{c\phi \cdot s\theta \cdot h_1}{g} \\ \frac{c\theta \cdot h_4}{g} & \frac{s\phi \cdot s\theta \cdot h_4}{g} - \frac{c\phi \cdot h_3}{g \cdot c\theta} & \frac{c\phi \cdot s\theta \cdot h_4}{g} + \frac{s\phi \cdot h_3}{g \cdot c\theta} \\ \frac{c\theta \cdot h_7}{g} & \frac{s\phi \cdot s\theta \cdot h_7}{g} - \frac{c\phi \cdot h_6}{g \cdot c\theta} & \frac{c\phi \cdot s\theta \cdot h_7}{g} + \frac{s\phi \cdot h_6}{g \cdot c\theta} \end{bmatrix},$$

$$H_{21} = \begin{bmatrix} \frac{h_2 \cdot h_9}{\Omega_N} & \frac{h_2 \cdot h_{10}}{\Omega_N} & \frac{h_2 \cdot h_{11}}{\Omega_N} \\ \frac{h_5 \cdot h_9}{\Omega_N} & \frac{h_5 \cdot h_{10}}{\Omega_N} & \frac{h_5 \cdot h_{11}}{\Omega_N} \\ \frac{h_8 \cdot h_9}{\Omega_N} & \frac{h_8 \cdot h_{10}}{\Omega_N} & \frac{h_8 \cdot h_{11}}{\Omega_N} \end{bmatrix}.$$

5. SIMULATIONS

To evaluate the proposed alignment algorithm, simulations are performed by using the medium grade IMU that has the accelerometer bias with 100 [μg] and the gyro bias with 0.1 [deg/hr].

Fig. 1 and 2 show the simulation results of the 10-state Kalman filter. From the Fig. 1, the roll and pitch angles are fluctuated with time since the estimation of the attitude errors in the 10-state Kalman filter are affected by the characteristic of the accelerometer convergence.

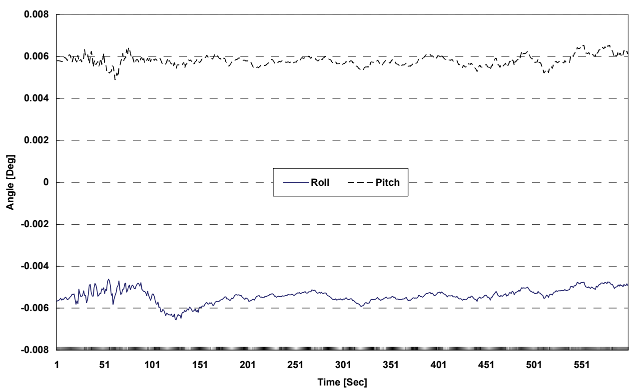

Fig. 1. Roll and pitch angle of the 10-state Kalman filter

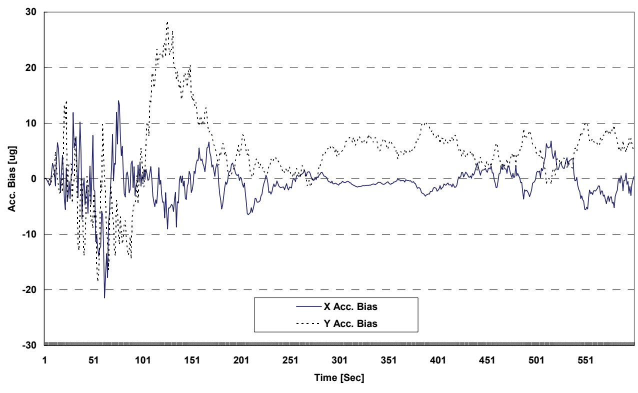

Fig. 2. X and Y Acc. biases of the 10-state Kalman filter

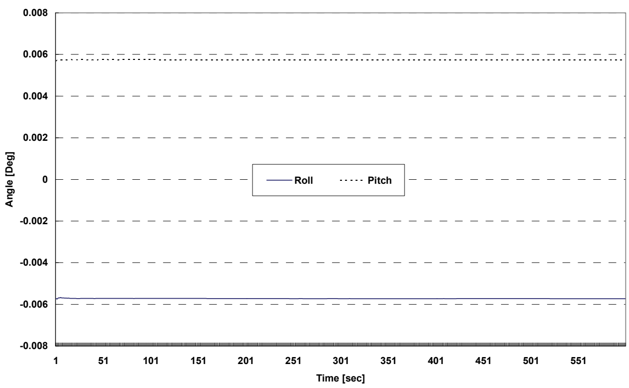

Fig. 3. Roll and pitch angle of the new alignment algorithm

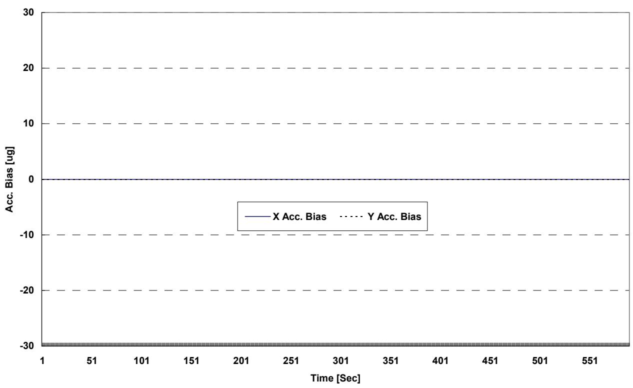

Fig. 4. X and Y Acc. Biases of the new alignment algorithm

Fig. 3 and 4 show the simulation results of the new alignment algorithm. From the Fig. 3, the roll and pitch angles are converged with very fast rate because the proposed algorithm estimates the x and y accelerometer biases independently with the attitude error components. The Fig. 4 shows that the estimated accelerometer biases have some stable value compared with the Fig. 2.

6. CONCLUSIONS

In this paper, a new alignment algorithm is proposed to increase the convergence rate of the Kalman filter in the fine alignment stage. The proposed algorithm uses simultaneously both open-loop and closed-loop scheme, which calculates the attitude angles with the open-loop scheme and estimates the accelerometer and gyro biases with the closed-loop scheme. The convergence theorem of the Kalman filter is derived and applied to analyze the problem of the 10-state Kalman filter in the fine alignment. The simulation results show that the proposed algorithm has superior performance in comparison with the 10-state Kalman filter.

REFERENCES

Titterton, D. H and Weston, J. L (1997). *Strapdwon inertial navigation technology*, IEE Radar, Sonar, Navigation and Avionics series 5.

Jiang, Y. F (1998). Error Analysis of Analytic Coarse Alignment Methods, *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 34, No. 1.

Britting, K. R (1971), *Inertial Navigation Systems Analysis*, John Wiley & Sons, Inc.

Lee, J. G, Park, C. G and Park, H. W (1992). Multiposition Alignment of Strapdown Inertial Navigation Systems, *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 29, No. 4.

Fang, J. C and Wan, D. J (1995). A Fast Initial Alignment Method for Strapdown Inertial Navigation System on Stationary Base, *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 32, No. 4.

Park, C. G, Kim, K. J, Park, H. W and Lee, J. G (1998). Development of an Initial Coarse Alignment Algorithm for Strapdown Inertial Navigation System, *Journal of Control, Automation, and System Engineering*, Vol 4, No. 5.