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Fault Detection and Isolation for Nonlinear Systems with Full State Information ⋆

Wei Wang * **Yang Bo** * **Kemin Zhou** ** **Zhang Ren** *

* *School of Automation Science and Electrical Engineering, Beihang University, Beijing, China (Email: wwinney@126.com, yangbo02@gmail.com, renzhang@buaa.edu.cn).*
** *Department of Electrical and Computer Engineering, Louisiana State University, Baton Rouge, LA 70803, USA (Email: kemin@ece.lsu.edu)*

Abstract: This paper considers robust fault detection for nonlinear systems with full state information. We propose and solve a multi-objective fault detection criterion by maximizing the smallest singular value of the transformation from faults to fault detection residuals while decoupling/or minimizing the largest singular value of the transformation from disturbance to the fault detection residuals.

1. INTRODUCTION

Research and development of fault diagnosis technology for modern control systems have received considerable attention in the recent years in view of the ever-increasing complexity of modern control systems and the desirability for improving the reliability and safety of the control systems. The model-based method is one of the most important and widely used analytic approaches for fault detection in automation processes (Patton [1997]). It is relatively easy to implement and effective in detecting sensors, actuators and system components faults when good models of the system can be obtained. However, the requirement of good system models is also the weakest point of most model-based fault detection methods. An accurate model of any practical system is almost impossible to obtain due to disturbances, measurement noises and modeling errors. Hence any model-based fault detection method must take into consideration of these effects, i.e. the detection mechanism must be robust to these effects.

A truly robust fault detection design involves a difficult multiple objective design task (Chen et al. [1999]). The fault detection mechanism must reject disturbance, noise, and be insensitive to modeling uncertainties and at the same time be as sensitive as possible to faults. An ideal fault detection mechanism would be able to completely decouple the disturbance, noises, and modeling error from the faults and be able to recover or isolate the faults exactly. However, this is in general impossible and a trade-off between decoupling the disturbance, noises, and uncertainties and recovering or isolating the faults must be made. Several methods have been proposed to tackle these multiple objective trade-off problems. For example, the unknown input observer, eigenstructure assignment, parity relation approach, H_∞ optimization method, H_2/H_∞ problem, H_∞/H_∞ problem, and H_2/H_2 problem (Chen et al. [1999]; Ding et al. [2000]; Frank et al. [1997]; Hou et al. [1996]; Jaimoukha et al. [2006]; Liu et al. [2005]; Liu et al. [2007]; Zhang et al. [2006]; Zhong et al. [2003]).

Most of the existing work on robust fault detection is limited to linear systems. Robust fault detection for general nonlinear systems has so far been very difficult (De Persis et al. [2001]). Motivated from a recent work for some over-instrumented systems where full states are assumed to be available (Aravena et al. [2006]), we propose in this paper an approach for robust fault detection and isolation of a class of nonlinear uncertain systems as a step towards the solution to the general robust nonlinear fault detection problem.

2. MAIN RESULTS

Consider a nonlinear time varying system given by

$$\dot{x} = F(t,x,u) + g_d(t,x,u)\,d(t) + g_f(t,x,u)\,f(t) \qquad (1)$$

$$y = h(t,x,u) \qquad (2)$$

Where $x \in \Re^n$ is the state, $u \in \Re^m$ is the input, $d \in \Re^{n_d}$ is the disturbance, and $f \in \Re^{n_f}$ is the possible fault. Assume that F, g_d, g_f, h are known functions.

Assumption: *g_d and g_f have full column normal rank, i.e.* $\max rank(g_d) = n_d$ *and* $\max rank(g_f) = n_f$.

The above assumption can be made without loss of generality since otherwise disturbance and fault signals can be regrouped to form new signals with such properties.

Our objective is to design a signal processing strategy that can reliably detect the possible fault f under the influence of the disturbance d. This problem is extremely hard in general. Here we shall take one step towards the direction of solving this general problem by assuming that the state of the dynamical system is actually measurable, i.e.

$$h(t,x,u) = x \qquad (3)$$

In some cases, this assumption is not too restrictive, for example, some over-instrumented flight control problems where all state variables are directly or indirectly measured. Hence, theoretically $\dot{y}$ and $F(t,x,u)$ can be computed from measurement.

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Now define

$$e(t) = \dot{y}(t) - F(t,x,u) \tag{4}$$

Then

$$e(t) = g_d(t,x,u)\,d(t) + g_f(t,x,u)\,f(t)$$

Define a residual function as

$$\begin{aligned} r = W(t,x,u)\,e = W(t,x,u)\,g_d(t,x,u)\,d(t) \\ + W(t,x,u)\,g_f(t,x,u)\,f(t) \end{aligned} \tag{5}$$

Where $W \in \Re^{n_f \times n}$ is to be designed to satisfy some predefined performance criteria.

Remark: *In general, we can let $W \in \Re^{p\times n}$ for any p. However, it can be shown that this extra freedom does not offer any advantage. Hence it is sufficient to consider $p = n_f$.*

It is desirable to decouple the residual r from the disturbance as much as possible. Hence it is highly desirable to choose $W(t,x,u)$ so that $W(t,x,u)\,g_d(t,x,u)$ is small in some way. At the same time, it is desirable to choose $W(t,x,u)$ so that all components of the faulty signal f can be recovered as much as possible. Mathematically, we can formulate the problem as follows.

Problem: *Find a $W \in \Re^{n_f\times n}$ so that*

$$\max\{\underline{\sigma}(Wg_f) : \ \overline{\sigma}(Wg_d) \le \gamma, \ \|W\| \le 1\} \tag{6}$$

Where $\gamma \ge 0$ is predefined. In the case of perfect decoupling, $\gamma = 0$ and $Wg_d = 0$.

This optimization problem is in general quite difficult. We shall consider first the case $\gamma = 0$.

Theorem 1. Let g_d have the following singular value decomposition:

$$g_d = U_d \begin{bmatrix} \Sigma_d \\ O_{(n-n_d)\times n_d} \end{bmatrix} V_d^H \tag{7}$$

where $U_d \in \Re^{n\times n}$, $0 < \Sigma_d \in \Re^{n_d\times n_d}$, $V_d \in \Re^{n_d\times n_d}$. Define

$$\tilde{g}_f := \begin{bmatrix} O_{(n-n_d)\times n_d} & I_{n-n_d} \end{bmatrix} U_d^H g_f \tag{8}$$

Let $\tilde{g}_f$ have the following singular value decomposition:

$$\tilde{g}_f = \tilde{U}_f \begin{bmatrix} \tilde{\Sigma}_f \\ O_{(n-n_d-n_f)\times n_f} \end{bmatrix} \tilde{V}_f^H \tag{9}$$

where $\tilde{U}_f \in \Re^{(n-n_d)\times(n-n_d)}$, $0 < \tilde{\Sigma}_f \in \Re^{n_f\times n_f}$, $\tilde{V}_f \in \Re^{n_f\times n_f}$.

Then

$$\max\{\underline{\sigma}(Wg_f) : \ Wg_d = 0, \ \|W\| \le 1\} = \underline{\sigma}(\tilde{g}_f) \tag{10}$$

And an optimal W is given by

$$W_{opt} = \begin{bmatrix} I_{n_f} & O_{n_f\times(n-n_f-n_d)} \end{bmatrix} \tilde{U}_f^H \begin{bmatrix} O_{(n-n_d)\times n_d} & I_{n-n_d} \end{bmatrix} U_d^H \tag{11}$$

In particular, if $rank[g_d,\, g_f] = n_d + n_f$, then $\underline{\sigma}(\tilde{g}_f) > 0$. Otherwise, $\underline{\sigma}(\tilde{g}_f) = 0$.

Proof: Without loss of generality, we can assume that W is partitioned in the following form

$$W = [W_1 \quad W_2]\,U_d^H$$

with $W_1 \in \Re^{n_f\times n_d}$, $W_2 \in \Re^{n_f\times(n-n_d)}$. Then from

$$0 = Wg_d = W_1\Sigma_d V_d^H$$

We get $W_1 = 0$ and

$$Wg_f = [0 \quad W_2]\,U_d^H g_f = W_2\tilde{g}_f$$

Now the problem becomes

$$\begin{aligned} &\max\{\underline{\sigma}(Wg_f) : \ Wg_d = 0, \ \|W\| \le 1\} \\ &= \max\{\underline{\sigma}(W_2\tilde{g}_f) : \ \|W_2\| \le 1\} \end{aligned}$$

Since

$$\begin{aligned} \underline{\sigma}^2(W_2\tilde{g}_f) = \underline{\lambda}(\tilde{g}_f^H W_2^H W_2 \tilde{g}_f) \le \overline{\sigma}^2(W_2)\underline{\lambda}(\tilde{g}_f^H\tilde{g}_f) \\ \le \underline{\lambda}(\tilde{g}_f^H\tilde{g}_f) = \underline{\sigma}^2(\tilde{g}_f) \end{aligned}$$

It is clear that an optimal solution is

$$W_2 = [I_{n_f} \ O_{n_f\times(n-n_f-n_d)}]\tilde{U}_f^H$$

which gives

$$\underline{\sigma}^2(W_2\tilde{g}_f) = \underline{\sigma}^2(\tilde{g}_f)$$

Note that

$$\begin{aligned} U_d^H[g_d,\, g_f] &= [U_d^H g_d,\, U_d^H g_f] \\ &= \left[\begin{bmatrix} \Sigma_d \\ O_{(n-n_d)\times n_d} \end{bmatrix} V_d^H,\, U_d^H g_f\right] \\ &= \left[\begin{bmatrix} \Sigma_d V_d^H \\ O_{(n-n_d)\times n_d} \end{bmatrix}, \begin{bmatrix} I_{n_d} & O \\ O & I_{n-n_d} \end{bmatrix} U_d^H g_f\right] \\ &= \begin{bmatrix} \Sigma_d V_d^H & [I_{n_d} \ O_{n_d\times(n-n_d)}]U_d^H g_f \\ O_{(n-n_d)\times n_d} & \tilde{g}_f \end{bmatrix} \end{aligned}$$

Then

$$rank[g_d, g_f] = rank[\Sigma_d] + rank[\tilde{g}_f] = n_d + rank[\tilde{g}_f]$$

Let $q = rank[g_d, g_f] \le n_d + n_f$, then it is clear that

$$rank\,\tilde{g}_f = q - n_d$$

Hence if $q < n_d + n_f$, then

$$rank\,\tilde{g}_f = q - n_d < n_f \text{ and } \underline{\sigma}(\tilde{g}_f) = 0$$

Otherwise,

$$rank\,\tilde{g}_f = q - n_d = n_f \text{ and } \underline{\sigma}(\tilde{g}_f) > 0$$

□

Corollary 2. For fault detection and isolation, we define

$$r_f = W_f(t,x,u)\,e(t) \tag{12}$$

If $q = n_d + n_f$, let $W_f \in \Re^{n_f\times n}$ be given by

$$\begin{aligned} W_f =& \tilde{V}_f\tilde{\Sigma}_f^{-1}\,[I_{n_f} \ O_{n_f\times(n-n_f-n_d)}]\,\tilde{U}_f^H \\ &\cdot [O_{(n-n_d)\times n_d} \ I_{n-n_d}]\,U_d^H \end{aligned} \tag{13}$$

Then

$$W_f g_d = 0,\, W_f g_f = I_{n_f} \tag{14}$$

and faulty signals can be completely isolated and decoupled from the disturbance.

If $q < n_d + n_f$, a complete fault isolation is impossible. However, let $W_f \in \Re^{(q-n_d)\times n}$ be given by

$$\begin{aligned} W_f =& \tilde{V}_f\tilde{\Sigma}_f^{+}\,[I_{n_f} \ O_{n_f\times(n-n_f-n_d)}]\,\tilde{U}_f^H \\ &\cdot [O_{(n-n_d)\times n_d} \ I_{n-n_d}]\,U_d^H \end{aligned} \tag{15}$$

Then $W_f g_d = 0$ and

$$W_f g_f = \tilde{V}_f\,\tilde{\Sigma}_f^{+}\,\tilde{\Sigma}_f\,\tilde{V}_f^H \tag{16}$$

and those combinations of faulty signals that are in the range of the disturbance space cannot be isolated.

Fault detection with completely disturbance coupling may not be possible. This is the case if

$$rank[g_d, g_f] < n_d + n_f$$

In this case, the residual function generated using the W in Theorem 1 will always be zero for some combinations of faults. Hence, detection of faults in that combination is impossible if a perfect disturbance decoupling is desired (because this combination of faults is also decoupled from the residual in this case). Therefore, it is desirable to design a residual so that it rejects the disturbance as much as possible but still be able to detect the faults. This will require solution for the general case where $\gamma > 0$.

However, the general problem $\gamma > 0$ seems to be much hard to solve. Nevertheless, the following theorem will produce a reasonably good approximation.

Theorem 3. Let Σ_d have the following form

$$\Sigma_d = diag(\sigma_1, \cdots, \sigma_r, \sigma_{r+1}, \cdots, \sigma_{n_d})$$

and

$$\sigma_1 \geq \cdots \geq \sigma_r \geq \gamma \geq \sigma_{r+1} \geq \cdots \geq \sigma_{n_d}$$

Define

$$\tilde{\tilde{g}}_f := [O_{(n-r)\times r}\ I_{n-r}]\, U_d^H\, g_f \tag{17}$$

Let $\tilde{\tilde{g}}_f$ have the following singular value decomposition:

$$\tilde{\tilde{g}}_f = \tilde{\tilde{U}}_f \begin{bmatrix} \tilde{\tilde{\Sigma}}_f \\ O_{(n-r-n_f)\times n_f} \end{bmatrix} \tilde{\tilde{V}}_f^H \tag{18}$$

where $\tilde{\tilde{U}}_f \in \Re^{(n-r)\times(n-r)}$, $0 < \tilde{\tilde{\Sigma}}_f \in \Re^{n_f\times n_f}$, $\tilde{\tilde{V}}_f \in \Re^{n_f\times n_f}$.

Let

$$W_{sub} = \left[I_{n_f}\ O_{n_f\times(n-n_f-r)}\right] \tilde{\tilde{U}}_f^H [O_{(n-r)\times r}\ I_{n-r}] U_d^H \tag{19}$$

Then

$$\overline{\sigma}(W_{sub}g_d) \leq \sigma_{r+1} \leq \gamma, \quad \underline{\sigma}(W_{sub}g_f) = \underline{\sigma}(\tilde{\tilde{g}}_f) \tag{20}$$

Proof: Note that

$$\begin{aligned} W_{sub}g_d &= [I_{n_f}\ O_{n_f\times(n-n_f-r)}]\tilde{\tilde{U}}_f^H[O_{(n-r)\times r}\ I_{n-r}]U_d^H g_d \\ &= [I_{n_f}\ O_{n_f\times(n-n_f-r)}]\tilde{\tilde{U}}_f^H[O_{(n-r)\times r}\ I_{n-r}] \\ &\quad \cdot U_d^H U_d \begin{bmatrix} \Sigma_d \\ O_{(n-n_d)\times n_d} \end{bmatrix} V_d^H \\ &= [I_{n_f}\ O_{n_f\times(n-n_f-r)}] \\ &\quad \cdot \tilde{\tilde{U}}_f^H \begin{bmatrix} & \sigma_{r+1} & & \\ O_{(n_d-r)\times r} & & \ddots & \\ & & & \sigma_{n_d} \\ O_{(n-n_d)\times r} & O_{(n-n_d)\times(n_d-r)} & & \end{bmatrix} V_d^H \end{aligned}$$

and

$$\begin{aligned} W_{sub}g_f &= [I_{n_f}\ O_{n_f\times(n-n_f-r)}]\tilde{\tilde{U}}_f^H[O_{(n-r)\times r}\ I_{n-r}]U_d^H g_f \\ &= [I_{n_f}\ O_{n_f\times(n-n_f-r)}]\tilde{\tilde{U}}_f^H \tilde{\tilde{g}}_f \\ &= [I_{n_f}\ O_{n_f\times(n-n_f-r)}]\tilde{\tilde{U}}_f^H \tilde{\tilde{U}}_f \begin{bmatrix} \tilde{\tilde{\Sigma}}_f \\ O \end{bmatrix} \tilde{\tilde{V}}_f^H \\ &= [I_{n_f}\ O_{n_f\times(n-n_f-r)}] \begin{bmatrix} \tilde{\tilde{\Sigma}}_f \\ O \end{bmatrix} \tilde{\tilde{V}}_f^H = \tilde{\tilde{\Sigma}}_f \tilde{\tilde{V}}_f^H \end{aligned}$$

Hence

$$\overline{\sigma}(W_{sub}g_d) \leq \sigma_{r+1} \leq \gamma, \qquad \underline{\sigma}(W_{sub}g_f) = \underline{\sigma}(\tilde{\tilde{g}}_f)$$

□

3. SIMULATION EXAMPLE

We will illustrate our optimal fault diagnosis technique proposed above by a three-tank system in Fig. 1 (Li et al. [2005]).

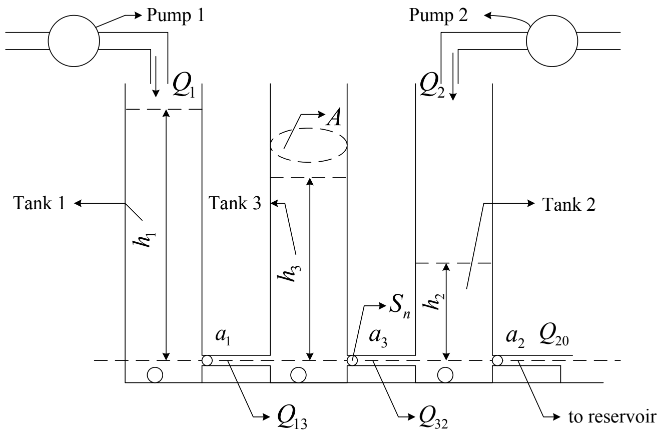

Fig. 1. The layout of the DT200 three-tank system

The mathematic model of the three-tank system is described as follows:

$$\begin{aligned} A\frac{\mathrm{d}h_1}{\mathrm{d}t} &= -\, a_1 S_n sgn(h_1-h_3)\sqrt{2g|h_1-h_3|} + Q_1 \\ A\frac{\mathrm{d}h_2}{\mathrm{d}t} &= a_3 S_n sgn(h_3-h_2)\sqrt{2g|h_3-h_2|} \\ &\quad - a_2 S_n\sqrt{2gh_2} + Q_2 \\ A\frac{\mathrm{d}h_3}{\mathrm{d}t} &= a_1 S_n sgn(h_1-h_3)\sqrt{2g|h_1-h_3|} \\ &\quad - a_3 S_n sgn(h_3-h_2)\sqrt{2g|h_3-h_2|} \end{aligned} \tag{21}$$

The states are the levels of tanks: $x = [h_1\ h_2\ h_3]^T$; the input is the controlled pump flow $u = [Q_1\ Q_2]^T$, and the output is $y = [h_1\ h_2\ h_3]^T$. The actual parameters are $g = 9.81m\cdot s^{-2}$, pipe coefficients $a_1^0 = 0.5$, $a_2^0 = 0.6$, $a_3^0 = 0.45$, tank cross-section area $A = 0.0154m^2$, pipe cross-section area $S_n = 5\times10^{-5}m^2$, $Q_{1\max} = Q_{2\max} = 100ml\cdot s^{-1}$. The levels of T1 and T2 are both controlled by PI controllers, with parameters $K_P = 0.001$ (gain constant) and $T_I = 5s$ (integral time constant).

Consider three types of faults:

(1) Leakage in T1: $Q_{leak}^1 = a_{r_1}\pi r_1^2\sqrt{2gh_1}$, r_1 is the leak hole's radius in T1;
(2) Leakage in T2: $Q_{leak}^2 = a_{r_2}\pi r_2^2\sqrt{2gh_2}$, r_2 is the leak hole's radius in T2;
(3) Clogging between T3 and T2: $a_3 = (1-\delta_3)a_3^0$.

The system white noise is considered as the disturbance. For detecting fault we need to get the system with fault described as (1). Define the right side of (21) as $F(t,x,u)$, we applied our method in two different fault case.

(a) Firs case: Leakage in T1 and Clogging between T3 and T2.

Let $\alpha = -a_{r_1}\pi\sqrt{2gh_1}$, $\beta = -a_3^0 S_n sgn(h_3-h_2)\cdot\sqrt{2g|h_3-h_2|}$, then the whole system with disturbance and faults is described as:

$$A\dot{x} = F(t,x,u) + \begin{bmatrix}1\\1\\1\end{bmatrix} d(t) + \begin{bmatrix}\alpha & 0\\ 0 & \beta\\ 0 & -\beta\end{bmatrix}\begin{bmatrix} r_1^2 \\ \delta_3\end{bmatrix} \tag{22}$$

According to Theorem 1, because $q = rank[g_d\quad g_f] = n_d + n_f$, the optimal W is given by

$$W = \begin{bmatrix} \sqrt{\frac{2}{3}} sgn(\alpha) & -\sqrt{\frac{1}{6}} sgn(\alpha) & -\sqrt{\frac{1}{6}} sgn(\alpha) \\ 0 & \sqrt{\frac{1}{2}} sgn(\beta) & -\sqrt{\frac{1}{2}} sgn(\beta) \end{bmatrix} \tag{23}$$

For fault isolation and estimation, W_f is given by

$$W_f = \begin{bmatrix} \frac{sgn(\alpha)}{\sqrt{\alpha^2}} & -\frac{1}{2}\frac{sgn(\alpha)}{\sqrt{\alpha^2}} & -\frac{1}{2}\frac{sgn(\alpha)}{\sqrt{\alpha^2}} \\ 0 & \frac{1}{2}\frac{sgn(\beta)}{\sqrt{\beta^2}} & -\frac{1}{2}\frac{sgn(\beta)}{\sqrt{\beta^2}} \end{bmatrix} \tag{24}$$

Fault changes with time in the T1 tank leak hole's radius,

$$r_1 = \begin{cases} 0m, & t \le 40s \\ 0.0002(t-30)m, & 40s < t \le 65s \\ 0.007m, & 65s < t < 80s \end{cases} \tag{25}$$

with pipe coeffcient $a_{r_1} = 0.5$. Clogging between T3 and T2: $\delta_3 = 0.3\,(t \ge 50s)$. The disturbance $d(t)$ is $(0,1)$ white noise.

Fig. 2 shows the residual signals $r(1)$, $r(2)$ generated by our method and the actual fault signals r_1^2, δ_3, the $r(1)$ is exactly the same as r_1^2 and the $r(2)$ is exactly the same as δ_3. It can be seen that the fault signals decouple completely from the disturbance.

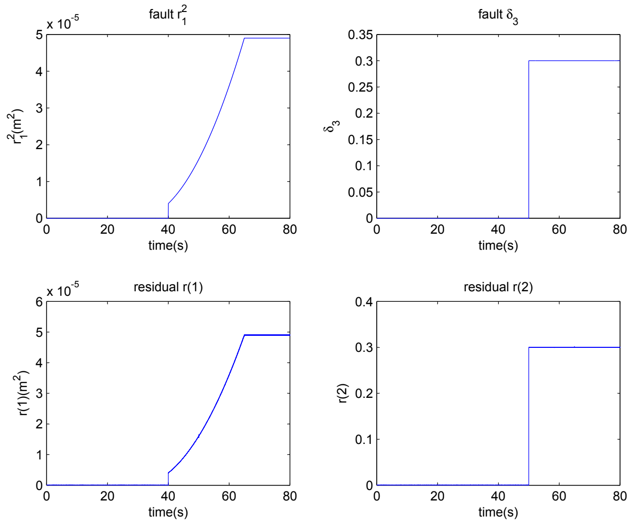

Fig. 2. Fault signals and residual signals

(b) Second case: Leakage both in T1 and T2. Define $\alpha = -a_{r_1}\pi\sqrt{2gh_1}$, $\theta = -a_{r_2}\pi\sqrt{2gh_2}$

$$A\dot{x} = F(t,x,u) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} d(t) + \begin{bmatrix} \alpha & 0 \\ 0 & \theta \\ 0 & 0 \end{bmatrix} \begin{bmatrix} r_1^2 \\ r_3^2 \end{bmatrix} \tag{26}$$

According to Theorem 1, since $q = rank[g_d\ g_f] < n_d + n_f$, the optimal W is given by

$$W = \begin{bmatrix} 0 & sgn(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{27}$$

For fault isolation and estimation, W_f is given by

$$W_f = \begin{bmatrix} 0 & \frac{sgn(\theta)}{\sqrt{\theta^2}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{28}$$

Faults are changes in the T1 and T2 tank leak hole's radius r_1, r_2, with pipe coeffcients $a_{r_1} = 0.5$ and $a_{r_2} = 0.6$. r_1 is the same as described in (25), r_2 changes with time,

$$r_1 = \begin{cases} 0m, & t \le 20s \\ 0.002m, & 20s < t \le 50s \\ 0.004m, & 50s < t < 80s \end{cases} \tag{29}$$

Fig. 3 shows only one fault r_2 which is not in the disturbance space can be detected and decoupled from the disturbance.

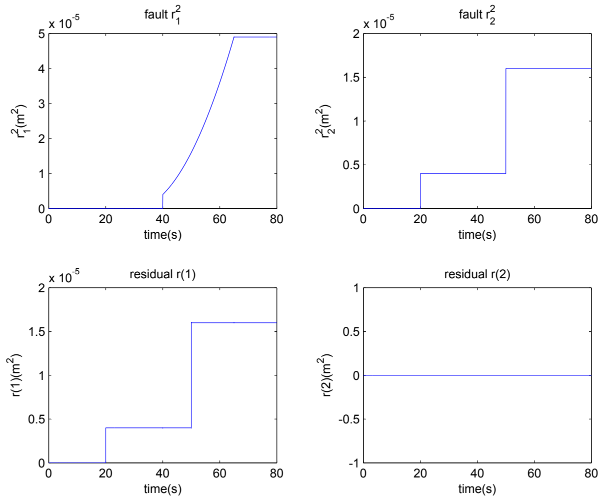

Fig. 3. Fault signals and residual signals

4. CONCLUSIONS

We have taken a step in solving the fault diagnosis problems for nonlinear systems with full state information and assuming that a reasonably accurate derivative can be evaluated. This is by no means practical for many industrial systems and hence much research is needed in extending this small step to more practical setup.

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