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A switched MPC approach to hierarchical control ⋆

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Abstract: This paper deals with the problem of designing stable hierarchical control schemes within a Model Predictive Control (MPC) framework. Specifically, the considered control structure is composed by two layers. At the upper level, a switched robust MPC regulator is designed to define, in a slow time scale, the subset of actuators to be activated in the next sampling interval as well as to compute the required control actions they must provide. At the lower level, the selected actuators are controlled through an MPC strategy in order to account for hard control and state constraints. The discrepancy between the control actions required by the system at the upper level and those effectively provided by the systems at the lower level is tolerated in view of the robustness properties guaranteed by the MPC law adopted at the upper level.

Keywords: Model predictive control, hierarchical control, stabilization.

1. INTRODUCTION

Research in hierarchical control is motivated by a large number of real applications, where the system can be viewed as composed by a number of subsystems placed at different layers, see e.g. Bahnasawi et al. [1990], Ng and Stephanopoulos [1996], Tarraf and Asada [2002], Abdelwahed et al. [2005], Katebi and Johnson [1997], Sanders et al. [1998], Roberts and Becerra [2001]. In this framework, the present paper deals with the design of a hierarchical control structure where the system at the upper level (S_u) coordinates a set of independent subsystems (S_{li}, $i = 1, ..., m$) working at the lower level and operating at a faster time scale. System S_u must guarantee some high-level, primary functions of the plant, while the $S'_{li}s$ are actuators (agents, teams) which must provide the required control actions at the upper level. With the aim to achieve global stabilization, at any slow sampling time, S_u chooses the subset of the S'_{li} to be used in the next (long) time interval and computes the control actions u_i they should ideally produce. These values are the reference signals for the S'_{li} s, which must be able to provide the effective control actions $\tilde{u}_i$ with adequate accuracy. However, in view of dynamic and intrinsic limitations of the subsystems, it is likely to happen that $u_i \neq \tilde{u}_i$, so that a robustness problem can arise.

In order to tackle this robustness problem, it is proposed to resort to the Model Predictive Control (MPC) approach. Specifically, at the upper level a robust MPC algorithm, see e.g. Magni and Scattolini [2005], Magni et al. [2003], Magni et al. [2006], is used to design a stabilizing feedback control law in the face of a norm-bounded uncertainty w representing the total effect of the discrepancy between the desired $u'_i s$ and the real $\tilde{u}'_i s$. At the same time, the robust MPC algorithm is extended to cope with switched control systems (see also Parisini and Sacone [1999], Colaneri and Scattolini [2007]), so as to select the active subset of actuators. In this phase, only feasible configurations can be considered, i.e. those which ensure that the overall effect of the equivalent disturbances $w_i = u_i - \tilde{u}_i$ associated with the active $S'_{li}s$ satisfies the robustness condition. Then, among the feasible subsets and with respect to a suitably defined cost function, the optimal one is selected and the corresponding values u_i are definitely computed and transmitted to the systems at the lower level. Finally, at the lower level a number of standard MPC problems is solved at any short time instant to optimize the performance of the selected actuators, to consider state and control constraints and to maintain feasibility at the upper level. A sketch of the considered hierarchical control structure is shown in Fig. 1.

Looking at the discrepancy between the requests of the system at the upper level and what can be effectively provided by the subsystems at the lower level as a disturbance term allows one to largely decouple the design problem while still guaranteeing stability properties of the overall system. This approach has already been considered in Scattolini and Colaneri [2007], where however, only one system at any level of the hierarchy was assumed.

The paper is organized as follows. In Section 2 the control problem is formally stated while Section 3 presents the overall control algorithm and the related stability results. Some concluding remarks close the paper.

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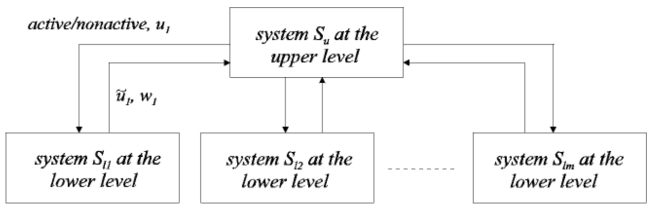

Fig. 1. The considered hierarchical control structure.

2. PROBLEM FORMULATION

In order to cope with a multirate implementation typical of hierarchical control structures where the upper layer acts at a slower rate than the lower layer, in the following different time scales will be considered. Specifically, the fast discrete time index will be denoted by the symbol h, while the slow discrete time index will be represented by the symbol k. Then, given a signal $\phi^f(h)$ in the fast time scale, its sampling in the slow time scale is $\phi(k) = \phi^f(\nu k)$ where ν is a positive integer.

2.1 System at the upper level

In the fast discrete time index, the system at the upper level is described by

$$x^f(h+1) = A^f x^f(h) + \sum_{i=1}^{m} \alpha_i^f b_i^f \left(u_i^f(h) + w_i^f(h)\right) \quad (1)$$

where $x^f \in R^{n_x}$ is the measurable state, $u_i^f \in R^{n_i}$ is the control variable provided by the $i-th$ actuator, $i = 1, ..., m$, such that

$$u_i^f \in U_i \quad (2)$$

U_i being a subset of R^{n_i} containing the origin as an interior point, and w_i^f is a matched disturbance. Then, the total number of control variables is $n_u = \sum_{i=1}^{m} n_i$. As for the parameters α_i^f, they are additional control variables defined as follows

$$\alpha_i^f(h) = \begin{cases} 1 & \text{the } i-th \text{ actuator is used at time } h \\ 0 & \text{otherwise} \end{cases}$$

Concerning system (1), the following assumption will be made.

Assumption 1. The pairs (A^f, b_i^f), $i = 1, ..., m$, are reachable.

The control signals u_i^f and the parameters α_i^f are assumed to be constant over the slow sampling period, i.e.

$$u_i^f(k\nu + j) = u_i^f(k\nu), \quad j = 0, ..., \nu - 1$$
$$\alpha_i^f(k\nu + j) = \alpha_i^f(k\nu), \quad j = 0, ..., \nu - 1$$

Letting

$$x(k) = x^f(\nu k), \;\; u_i(k) = u_i^f(\nu k), \;\; \alpha_i(k) = \alpha_i^f(\nu k)$$

$$A = \left(A^f\right)^{\nu}, \quad b_i = \sum_{j=0}^{\nu-1} \left(A^f\right)^{\nu-j-1} b_i^f$$

and

$$w_i(k) = \sum_{j=0}^{\nu-1} \left(A^f\right)^{\nu-j-1} b_i^f w_i^f(\nu k + j) \quad (3)$$

system (1) can be written in the slow sampling rate as

$$x(k+1) = Ax(k) + \sum_{i=1}^{m} \alpha_i(k) \left(b_i u_i(k) + w_i(k)\right) \quad (4)$$

At any long sampling time k, the vector

$$\alpha(k) = [\, \alpha_1(k) \;\; \alpha_2(k) \;\; \cdots \;\; \alpha_m(k) \,]$$

can assume only $T = 2^m$ configurations; correspondingly define a new integer variable $\sigma(k) = \{1, 2, ..., T\}$ which selects the active set of actuators at the time instant k, that is whose value is associated with a given configuration of $\alpha(k)$. For $\sigma(k) = 1$ none of the actuators is active at time k, while for $\sigma(k) = T$ all of them are in use.

Now, defining

$$u(k) = \begin{bmatrix} u_1(k) \\ \vdots \\ u_m(k) \end{bmatrix}, \quad w(k) = \begin{bmatrix} w_1(k) \\ \vdots \\ w_m(k) \end{bmatrix}$$

and

$$B_{1\sigma(k)} = \begin{bmatrix} \alpha_1(k) I_{n_x} & \alpha_2(k) I_{n_x} & \dots & \alpha_m(k) I_{n_x} \end{bmatrix} \quad (5a)$$
$$B_{2\sigma(k)} = \begin{bmatrix} \alpha_1(k) b_1 & \alpha_2(k) b_2 & \dots & \alpha_m(k) b_m \end{bmatrix} \quad (5b)$$

system (4) can be written as

$$x(k+1) = Ax(k) + B_{1\sigma(k)} w(k) + B_{2\sigma(k)} u(k) \quad (6)$$

Associated with this system, define the output transformation

$$z(k) = C_{\sigma(k)} x(k) + D_{2\sigma(k)} u(k) \quad (7)$$

where the following standard assumptions are made.

Assumption 2. $C_i' D_{2i} = 0$ and $D_{2i}' D_{2i} > 0, \forall i$

2.2 Systems at the lower level (actuators)

The control variables effectively used at the upper level coincide with the outputs $\tilde{u}_i$ of the $i-th$ system at the lower level described in the short sampling time by

$$\zeta_i(h+1) = F_i \zeta_i(h) + G_i v_i(h) \quad (8a)$$
$$\tilde{u}_i(h) = H_i \zeta_i(h) \quad (8b)$$

where $\zeta_i \in R^{n_{\zeta_i}}$ is the measurable state and $v_i \in R^{n_{v_i}}$ is the manipulated input. Moreover, the following state and input constraints must be considered

$$v_i \in V_i, \quad \zeta_i \in Z_i \quad (9)$$

where V_i and Z_i are subsets containing the origin as an interior point.

Concerning the lower level systems (8) the following assumptions are introduced.

Assumption 3.

(i) The pairs (F_i, G_i), $i = 1, ..., m$, are reachable and the pairs (F_i, H_i), $i = 1, ..., m$, are observable.
(ii) The number of control variables of the $i-th$ actuator is not less than the one of its output variables, i.e. $n_{\nu_i} \geq n_i$.
(iii) Systems (8) have no invariant zeros equal to 1, i.e.

$$rank \begin{bmatrix} I - F_i & -G_i \\ H_i & 0 \end{bmatrix} = n_{\zeta_i} + n_i$$

(iv) For any $\tilde{u}_i \in U_i$ there exists a pair $(\bar{v}_i, \bar{\zeta}_i)$ with $\bar{v}_i \in V_i$, $\bar{\zeta}_i \in Z_i$ such that

$$\begin{bmatrix} I - F_i & -G_i \\ H_i & 0 \end{bmatrix} \begin{bmatrix} \bar{\zeta}_i \\ \bar{v}_i \end{bmatrix} = \begin{bmatrix} 0 \\ \tilde{u}_i \end{bmatrix}$$

(v) For any initial state $\zeta_{i0} \in Z_i$ there exists $\varpi > 0$ such that the state $\zeta_i \in Z_i$ is reachable in ϖ steps with bounded control $v_i \in V_i$.

2.3 Hierarchical control problem

In a hierarchical implementation, at any long sampling time k, the upper level decides the value of the integer $\sigma(k)$, denoting the active subsystems in the next ν short sampling times, and the desired value $u(k)$ of the control vector for system (6). However, due to the constraints (9) and to the initial conditions (at $h = k\nu$) of the systems (8), at the lower level, the real control variables $\tilde{u}_i(k\nu + j)$, $j = 0, ..., \nu - 1$, (outputs of (8)) are in general different from the corresponding components $u_i(k)$ of $u(k)$. These variables u_i can be at all effects considered as the piecewise constant reference signals for (8).

Due to the matched nature of the disturbance appearing in (1) and recalling (3), letting

$$\varsigma_h = \left\lceil \frac{h}{\nu} \right\rceil$$

where $\lceil \phi \rceil$ denotes the integer part of ϕ, the difference

$$w_i^f(h) = -u_i(\varsigma_h) + \tilde{u}_i(h) \tag{10}$$

between the desired value of the control variable at the upper level and the one provided by the selected active set of subsystems, can be viewed as the disturbance acting on system (6). Then, the problem consists in deriving suitable control algorithms for the upper and the lower levels guaranteeing that the overall control system has some robust stability properties. The problem lends itself to be solved via an MPC technique, in that it is characterized by hard constraints on the state and control variables to be fulfilled both at the upper and at the lower levels. Moreover, it also has a combinatorial nature, in that one has to select the value of $\sigma(k)$ that specifies the proper subset of actuators to be used at any long sampling time. As a result, the next section presents a solution of the problem based on a switched robust MPC technique.

3. HIERARCHICAL CONTROL ALGORITHM

In this Section, a switched robust MPC algorithm for the upper level is first developed assuming that the term w in (6), due to the lower level actuators' dynamics, can be viewed as a state dependent disturbance. Then, the design of MPC algorithms for the lower level is discussed. Finally, an overall stability result is provided.

3.1 Switched robust MPC for the upper level

For system (6) consider the problem of determining an MPC state-feedback law and a switching policy

$$u(k) = \bar{\kappa}^o(x(k)), \quad \sigma(k) = \bar{\xi}^o(x(k))$$

such that the closed loop system with input w and output z given by (7) has a finite L_2 gain, bounded by a positive attenuation level γ. Internal stability is then achieved for any disturbance w satisfying

$$\|w(k)\|^2 \leq \gamma_d^2 \|z(k)\|^2 \tag{11}$$

with $\gamma\gamma_d < 1$. The set of admissible signals w satisfying (11) is denoted by $\mathcal{W}$.

Auxiliary control law The development of a robust MPC algorithm for the upper level calls for the knowledge of a robustly stabilizing auxiliary control law with guaranteed attenuation level γ. To this regard, the following result can be stated. Its proof can be recovered from Colaneri and Scattolini [2007].

Theorem 1. Suppose that there exist a set of positive definite matrices X_i, a set of matrices Υ_i and a positive scalar $\beta >> 0$ such that, for each $i = 1, ..., T$ and $j = 1, ..., T$, the matrices M_{ij} are positive definite, where

$$M_{ij} = \begin{bmatrix} X_i(1+\beta) & X_i A' + \Upsilon_i' B_{2i}' & X_i C_i' + \Upsilon_i' D_{2i} & \sqrt{\beta} X_i \\ \star & X_i - \dfrac{B_{1i} B_{1i}'}{\gamma^2} & 0 & 0 \\ \star & \star & 0 & 0 \\ \star & \star & \star & X_j \end{bmatrix}$$

Let

$$K_i = \Upsilon_i X_i^{-1} \tag{12}$$

consider the control law

$$u(k) = K_{\sigma(k)} x(k)$$

along with the switching rule

$$\sigma(k) = \xi(x(k)) = \arg\min_i x(k)' P_i x(k) \tag{13}$$

with $P_i = X_i^{-1}$, define the function

$$V_F(x) = \min_i x' P_i x$$

and the set

$$\Omega(K_\sigma, \xi, \gamma, \gamma_d) = \{x : \ V_F(x) \leq \overline{\beta}\}$$

Then the closed loop system (6), (12), (13) is asymptotically stable in $\Omega(K_\sigma, \xi, \gamma, \gamma_d)$ and such that

$$\begin{aligned} \Delta V_F = & V_F(x(k+1)) - V_F(x(k)) \\ & < -\|z(k)\|^2 + \gamma^2 \|w(k)\|^2 \end{aligned} \tag{14a}$$

$$\forall x \in \Omega(K_\sigma, \xi, \gamma, \gamma_d), \quad \forall w \in \mathcal{W} \tag{14b}$$

□

Model predictive switching control In a model predictive control context, we will consider a finite time-interval $[k, k+N]$, where N is a positive integer defining the prediction horizon for the upper level. At a given time k, the designer has to chose a vector of switching strategies

$$\chi(k, N) = \left[\, \xi^0(x(k)) \ \cdots \ \xi^{N-1}(x(k+N-1)) \,\right]$$

and a vector of control laws

$$\mathcal{K}(k, N) = \left[\, \kappa^0(x(k)) \ \cdots \ \kappa^{N-1}(x(k+N-1)) \,\right]$$

where

$$\xi^i : R^n \to \{1, 2, \cdots T\}, \kappa^i : R^{n_x} \to R^{n_u}$$

are the so-called *policies*. The sequence of disturbances chosen by "nature" is denoted as

$$\mathcal{Q}(k, N) = [\, w(k) \ w(k+1) \ \cdots \ w(k+N-1) \,]$$

In the classical Receding Horizon (RH) approach, only open loop strategies are considered, while here, in order to take into account the variation of the state variable due to the unpredictable behavior of the nature, we are well advised to consider closed-loop strategies and, consequently, minimize with respect to the sequence of policies. In general this problem is particular demanding since the policies in $\mathcal{K}(k,N)$ belong to an infinite-dimensional space. Concerning $\chi(k,N)$, there is a finite number of policies since $\xi^i(x)$ may assume only T values.

Now, assume that there exists an auxiliary law $\sigma^{aus}(k) = \xi(x(k))$, $u^{aus}(k) = K_{\sigma(k)}x(k)$, a domain of attraction $\Omega(K_\sigma,\xi,\gamma,\gamma_d)$ whose boundary is a level line of a positive function $V_F(x)$, with $V_F(0) = 0$ such that, $\forall x \in \Omega(K_\sigma,\xi,\gamma,\gamma_d)$ the constraints (2) are satisfied and, $\forall w \in \mathcal{W}$, it results

$$V_F(x(k+1)) - V_F(x(k)) < -\|z(k)\|^2 + \gamma^2\|w(k)\|^2$$

Notably, this auxiliary control law can be the one previously developed. The problem now consists in minimizing with respect to $(\chi(k,N),\mathcal{K}(k,N))$ and maximize with respect to $\mathcal{Q}(k,N)$ the cost function

$$J(\bar{x},\mathcal{K},\chi,\mathcal{Q},N) = V_F(x(k+N)) + \sum_{i=k}^{k+N-1}\left(\|z(k)\|^2 - \gamma^2\|w(k)\|^2\right) \tag{15}$$

subject to system (6), (7) with $x(k) = \bar{x}$ and $x(k+N) \in \Omega(K_\sigma,\xi,\gamma,\gamma_d) \subset R^{n_x}$. If $(\bar{\sigma}(k,N),\bar{\mathcal{K}}(k,N),\bar{\mathcal{Q}}(k,N))$ is the optimal solution of this min-max problem, according to the receding horizon principle, set

$$\sigma(k) = \bar{\xi}^0(x(k)), \quad u(k) = \bar{\kappa}^0(x(k)) \tag{16}$$

The control law (16) turns out to be the required MPC control law, for which the following result holds, see again Colaneri and Scattolini [2007] for a proof.

Theorem 2. Let X^{MPC} the set of all states $\bar{x}$ such that the above min-max problem admits a solution. Then,

(i) X^{MPC} is a positively invariant set for the closed loop system (6), (16).

(ii) $\Omega(K_\sigma,\xi,\gamma,\gamma_d) \subseteq X^{MPC}$, $\forall N$.

(iii) The origin is asymptotically attractive for the closed loop system (6), (16) in X^{MPC}.

□

Remark 1. In view of the particular nature of w, see the previous Section and in particular equations (3) and (10), at the upper level it is not possible to guarantee a-priori that the disturbance is an admissible one, since it depends on the output of the actuators effectively used at the lower level. This means that only for some values of $\sigma(k)$ the disturbance term is admissible, that is $w \in W$. In the development of the overall control scheme, this condition will allow to select the proper configuration of the actuators to be used at any slow sampling time.

3.2 MPC for the lower level

At any fast sampling time h, $k\nu \le h < k\nu + \nu$, the required output $\bar{u}_i(h)$ and the corresponding equilibrium state and input vectors $\bar{\zeta}_i(h)$, $\bar{v}_i(h)$ can be computed for any actuator by setting $\bar{u}_i(h) = \alpha_i(\varsigma_h)u_i(\varsigma_h)$ and by solving the set of equations

$$\begin{bmatrix} I - F_i & -G_i \\ H_i & 0 \end{bmatrix}\begin{bmatrix} \bar{\zeta}_i(h) \\ \bar{v}_i(h) \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{u}_i(h) \end{bmatrix}$$

Note that the dynamics of these variables are governed by the following equations

$$\begin{aligned} \bar{\zeta}_i(h+1) &= \bar{\zeta}_i(h) + \\ &\quad + \left(\bar{\zeta}_i(\varsigma_{h+1}) - \bar{\zeta}_i(\varsigma_h)\right)\delta\left(h+1-\varsigma_{h+1}\right) \\ \bar{v}_i(h+1) &= \bar{v}_i(h) + \\ &\quad + \left(\bar{v}_i(\varsigma_{h+1}) - \bar{v}_i(\varsigma_h)\right)\delta\left(h+1-\varsigma_{h+1}\right) \\ \bar{u}_i(h+1) &= \bar{u}_i(h) + \\ &\quad + \left(\bar{u}_i(\varsigma_{h+1}) - \bar{u}_i(\varsigma_h)\right)\delta\left(h+1-\varsigma_{h+1}\right) \end{aligned}$$

where δ is the Kroenecker function.

By defining

$$\delta\zeta_i(h) = \zeta_i(h) - \bar{\zeta}_i(h) \tag{17a}$$

$$\delta v_i(h) = v_i(h) - \bar{v}_i(h) \tag{17b}$$

$$\delta u_i(h) = \tilde{u}_i(h) - \bar{u}_i(h) \tag{17c}$$

the models (8) can be written as

$$\begin{aligned} \delta\zeta_i(h+1) &= F_i\delta\zeta_i(h) + G_i\delta v_i(h) + \\ &\quad - \left(\bar{\zeta}_i(\varsigma_{h+1}) - \bar{\zeta}_i(\varsigma_h)\right)\delta\left(h+1-\varsigma_{h+1}\right) \end{aligned} \tag{18}$$

$$\delta u_i(h) = H_i\delta\zeta_i(h) \tag{19}$$

For these systems, at any time instant h and by assuming

$$\varsigma_{h+j} = \varsigma_h, \quad j > 0 \tag{20}$$

it is possible to minimize with respect to the future sequence of control variables

$$\Theta_i(h,\nu) = [\,\delta v_i(h) \;\cdots\; \delta v_i(h+\nu-1)\,]$$

the performance indices

$$J_{li}(\delta\zeta_i(h),\Theta_i(h,\nu),\nu) = \sum_{j=0}^{\nu-1}\|\delta u_i(h+j)\|^2 \tag{21}$$

subject to (18), (19), (20), to the input and state constraints (9) and to the additional state terminal constraint

$$\delta\zeta_i(h+\nu) = 0 \tag{22}$$

In view of Assumption 3, point (v), this problem is feasible at any time instant.

Letting $\Theta_i^o(h,\nu) = \left[\,\delta v_{i,h}^o(h) \;\cdots\; \delta v_{i,h}^o(h+\nu-1)\,\right]$ be the optimal future control sequence, and according to the Receding Horizon principle, only the first value $\delta v_{i,h}^o(h)$ of $\Theta_i^o(h,\nu)$ is applied and the overall procedure is repeated at any short time instant. This implicitly defines the state-feedback control law

$$\delta v_i(h) = \eta_i(\delta\zeta_i(h)) \tag{23}$$

Theorem 3. For any $\nu \ge \varpi$, the origin of the closed-loop system (18), (19), (20), (23) is asymptotically attractive with region of attraction Z_i.

□

Associated to the optimal future control sequence $\Theta_i^o(h,\nu)$ it is also possible to compute the value function

$$\tilde{J}_{li}^o(\delta\zeta_i(h),\Theta_i^o(h,\nu),\nu)=\sum_{j=0}^{\nu-1}\left\|\left(A^f\right)^{\nu-j-1}b_i^f\delta u_i(h+j)\right\|^2 \tag{24}$$

(for short $\tilde{J}_{li}^o(h)$) which at any long sampling time ($h=\nu k$) represents the equivalent disturbance term for the system at the upper level due to the mismatch between the required control value ($\bar{u}_i(h)$) and the one ($\tilde{u}_i(h)$) provided by the $i-th$ subsystem, recall (3), (10) and (17c).

3.3 The overall hierarchical control algorithm

The overall hierarchical control algorithm can now be defined as follows:

1. *Negotiation and optimization at the upper level*

At any long sampling time k, solve at the upper level the MPC optimization problem with performance index (15). In this phase, in order to check if a given strategy $\chi(k,N)$, $\mathcal{K}(k,N)$ is feasible, i.e. $w(k)\in\mathcal{W}$, for any value of $\sigma(k)=1,...,T$, compute the corresponding $\alpha(k)$ and $u(k)=\kappa^0(x(k))$, send to the systems at the lower level these values and solve at the lower level the related MPC optimization problems with performance indices (21). Associated to the optimal solution $\Theta_i^o(h,\nu)$, compute $\tilde{J}_{li}^o(k)$ as defined in (24). Then, if the condition

$$\sum_{i=1}^{m}\alpha_i(k)\tilde{J}_{li}^o(k)\le\gamma_d^2\|z(k)\|^2$$

is verified, the corresponding configuration of actuators at the lower level is feasible.

2. *Optimization at the lower level*

Let $\bar{\chi}(k,N)$, $\bar{\mathcal{K}}(k,N)$ be the optimal strategies at time k and $\alpha(k)$, $u(k)$ the corresponding optimal configurations and control variables computed at the upper level. Note that, during step 1, also the optimal values $v_{i,k}^o(h)$ of the control variables $v_i(h)$, $h=\nu k+j$, $j=0,...,\nu-1$, $i=1,...m$, have been computed and could be applied along the whole interval $[\nu k,\nu(k+1))$. However, this is an open-loop solution for the systems at the lower level between two successive long time instants. Then, in order to compute a closed-loop solution preserving the overall robust stability property, at any short sampling time $h\neq\nu k$, at the lower level it is possible to solve the optimization problem defined by the performance indices (21) subject to (18), (19), (20), (9), (22) and to the additional constraint

$$\sum_{j=h}^{\varsigma_h+\nu-1}\left\|\left(A^f\right)^{\nu+h-j-1}b_i^f\delta u_i(j)\right\|^2\le \sum_{j=h}^{\varsigma_h+\nu-1}\left\|\left(A^f\right)^{\nu+h-j-1}b_i^f\delta u_{i,k}^o(j)\right\|^2$$

where $\delta u_{i,k}^o$ is the value of δu_i computed with the sequence $v_{i,k}^o(h)$. Note that this optimization problem is always feasible in view of the feasibility at time $h=\nu k$.

Concerning the overall closed-loop system, the following result can be stated.

Theorem 4. If, at any long sampling time k, step 1 of the above procedure admits a feasible solution, i.e. there is at least a feasible configuration of the actuators, then the origin of the overall closed-loop system is asymptotically attractive with region of attraction $X^{MPC}\bigoplus Z_1...\bigoplus Z_m$.

□

The overall control scheme previously described is sketched in Fig. 2, where the thick arrows denote the slow time scale variables while the thin arrows stand for the fast time scale ones.

4. CONCLUSIONS

A robust switched MPC approach has been devised in order to solve a hierarchical control problem characterized by two layers. The upper one, working at a slow time scale, decides the ideal control inputs and negotiates the set of possible active actuators at the lower level which compute the actual control law. The discrepancy between the ideal and actual control laws justifies the use of the robust paradigm, whereas the presence of hard bounds on the control and state variables enforces the use of the MPC approach.

Future work will be devoted to the analysis and synthesis of decentralized networking control problems.

5. APPENDIX - SKETCH OF THE PROOFS

Proof of Theorem 3

The proposed method is a classical *zero terminal constraint* algorithm, see e.g. Clarke and Scattolini [1991], Mayne and Michalska [1990]. To prove stability, note that in view of its definition and Assumption A3, (i) , $J_{li}>0$ for any $\delta u_i(h+j)\neq 0$, $j=0,...,\nu-1$.

Now define

$$\Theta_i^*(h+1,\nu)=\left[\,\delta v_{i,h}^o(h+1)\ \cdots\ \delta v_{i,h}^o(h+\nu-1)\ 0\,\right]$$

and note that this is a feasible control sequence at time $h+1$. Correspondingly, let $J_{li}^*(\delta\zeta_i(h+1),\Theta_i^*(h+1,\nu),\nu)$ (for short $J_{li}^*(h+1)$) be the associated cost function at time $h+1$. Then, it is easy to verify that

$$J_{li}^*(h+1)-J_{li}^o(h)\le 0$$

Finally, since $J_{li}^o(h+1)\le J_{li}^*(h+1)$ one has

$$J_{li}^o(h+1)-J_{li}^o(h)\le-\varepsilon\|\delta\zeta_i(h+j)\|^2$$

Therefore, J_{li}^o is a decreasing positive definite function for the closed-loop system and the result follows.

Proof of Theorem 4

In view of the theorem assumption and Theorems 1, 2, the small gain property (11) is satisfied for system (1), so that its state asymptotically converges to the origin. In turn, this means that $\bar{u}_i$, $\bar{\zeta}_i$, $\bar{v}_i$ tend to zero and in view of Theorem 2 the result follows.

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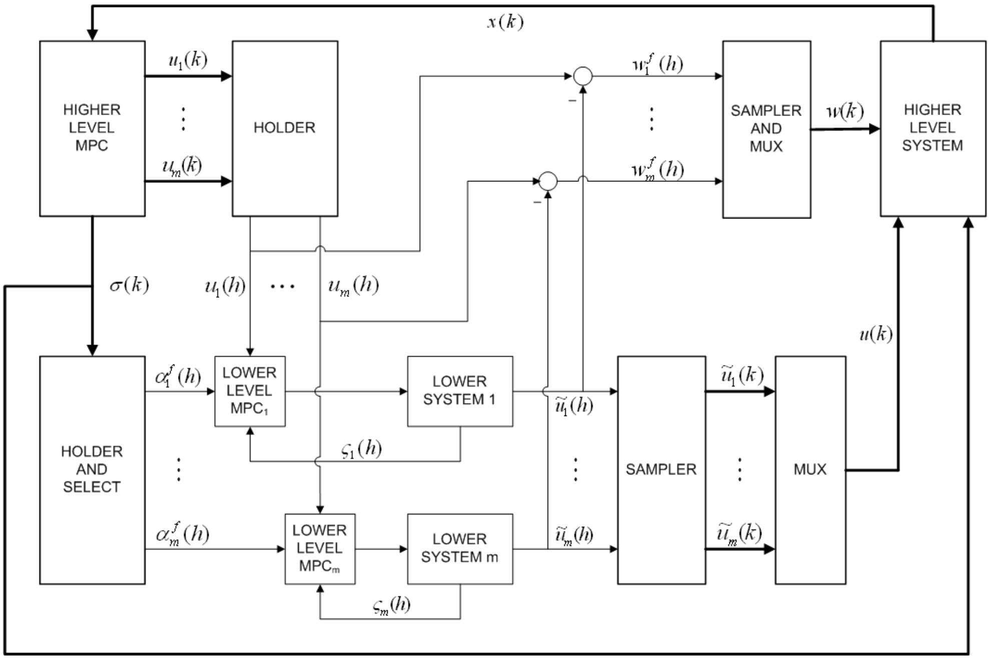

Fig. 2. The overall control scheme.

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