Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Trajectory Planning for Flatness-based two-degree-of-freedom control of a pumped storage power station

M. Treuer * **T. Weissbach** * **G. Scheffknecht** * **V. Hagenmeyer** **

* *Institute for Process Engineering and Power Plant Technology (IVD), Universität Stuttgart, Pfaffenwaldring 23, 70569 Stuttgart, Germany.*
{treuer,weissbach,scheffknecht}@ivd.uni-stuttgart.de
** *BASF Aktiengesellschaft, Ludwigshafen, Germany.*
veit.hagenmeyer@basf.com

Abstract: A pumped storage plant is one of the fastest reacting types of power generating units within power systems. Typically, the desired value curves for the power output during set point changes are provided in form of non-smooth ramp shaped curves. These cannot be followed exactly by the plant process due to its physical limitations. Resulting oscillations of the water pressure with potentially high pressure peaks can cause severe damage to the plant. Therefore, the gradients of set point changes of the power output have to be carefully limited.
In order to improve the control performance, a flatness-based two-degree-of-freedom control concept is applied: smooth trajectories for set point changes of the generator output are provided, which take the plant dynamics into account. The matter of trajectory planning is thereby addressed in detail. The improved control performance is shown by simulations, also in comparison with the widespread conventional PID control concept.

1. INTRODUCTION

Pumped storage plants are an important element of large power systems. Their ability to react fast allows them to cover a large part of the power system control demand — especially within the framework of secondary control, but also primary control. Since pumped storage plants can generate as well as consume electrical power, they also have the important capability to balance the highly increasing but non-deterministic feed-in of wind power into the grid. Therefore, the operation requirements for pumped storage plants in modern power systems are flexibility and manoeuvrability, which leads to an increasing number of set point changes during plant operation.

Due to the fact that the used medium liquid water has a very low compressibility in comparison to steam used in steam power plants, oscillations of the water pressure with potentially high pressure peaks can occur during set point changes, which can cause severe damage to the plant. This type of oscillations typically are stimulated by desired-value-curves for the generator output that cannot be followed by the plant, as e.g. the edges at the beginning and end of a ramp-like set point change. To apply standard PID control concepts, the gradient of the ramp has to be carefully limited to keep the pressure peaks within tolerated boundaries.

Based on a recently presented approach of applying a flatness-based two-degree-of-freedom control concept to a pumped storage power station [Treuer et al., 2007], this paper presents a novel ansatz for trajectory planning. First, the basic ideas of the control concept are introduced. Then, the dynamic model of a pumped storage plant is presented. Subsequently, a flatness-based two-degree-of-freedom control concept is designed, including the trajectory planning with a novel trajectory type. The achieved improvements are shown via simulation results within the framework of a relevant scenario.

2. FLATNESS-BASED TWO-DEGREE-OF-FREEDOM CONTROL

In order to present the flatness-based two-degree-of-freedom control scheme, the property of differential flatness is briefly recalled in the following. Thereafter, the two-degree-of-freedom control structure is discussed, and it is shown how differential flatness supports a direct design of the dynamic feedforward control part of the two-degree-of-freedom control structure.

2.1 *Differential flatness*

Differential flatness is a structural property of a class of multivariable nonlinear systems, for which, roughly speaking, all system variables can be written in terms of a set of specific variables – the so-called flat outputs – and their derivatives. [1] In this contribution, only SISO flat systems are briefly presented for the sake of simplicity. Given the SISO nonlinear system

$$\Sigma : \begin{aligned} &\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), u(t)), \;\; \mathbf{x}(0) = \mathbf{x}_0 && (1) \\ &y(t) = h(\mathbf{x}(t)) && (2) \end{aligned}$$

where the time $t \in \mathbb{R}$, the state $\mathbf{x}(t) \in \mathbb{R}^n$, the input $u(t) \in \mathbb{R}$ and the controlled output $y \in \mathbb{R}$. The vector field $\mathbf{f} : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ and the function $h : \mathbb{R}^n \to \mathbb{R}$ are smooth. The system (1) is said to be *differentially flat* [Fliess et al., 1995, 1999] if and only if there exists a flat output $z \in \mathbb{R}$ such that

$$z = F(\mathbf{x}) \tag{3}$$

$$\mathbf{x} = \boldsymbol{\phi}(z, \dot{z}, \ldots, z^{(n-1)}) \tag{4}$$

$$u = \psi(z, \dot{z}, \ldots, z^{(n)}) \tag{5}$$

[1] For a text book presentation of differential flatness, cf. Sira-Ramírez and Agrawal [2004].

are smooth at least in an open dense subset of $\mathbb{R}^n$, $\mathbb{R}^n$ and $\mathbb{R}^{n+1}$, respectively. The flat output $z = F(\mathbf{x})$ is a function of the state variables $\mathbf{x}$ and represents in most cases a meaningful physical variable. These equations [2] yield that for every given trajectory of the flat output $t \mapsto z(t)$, the evolution of all other variables of the system $t \mapsto \mathbf{x}(t)$ and $t \mapsto u(t)$ is also given without integration of any system of differential equations. Thus the flat output $z(t)$ and its derivatives parameterize the state $\mathbf{x}(t)$ and the input $u(t)$ via (4) and (5). Thereby it is important to remark that a trajectory $z(t)$ is such that its n-th derivative $z^{(n)}(t)$ admits a left and right limit everywhere. Furthermore $z(t)$ has to be consistent with the initial condition of the system (1), which is given by

$$\mathbf{x}_0 = \boldsymbol{\phi}\left(z(0), \dot{z}(0), \ldots, z^{(n-1)}(0)\right) \tag{6}$$

This relation can also be expressed by

$$\left[z(0), \dot{z}(0), \ldots, z^{(n-1)}(0)\right]^T = \boldsymbol{\phi}^{-1}(\mathbf{x}_0) \tag{7}$$

since the function $\boldsymbol{\phi} : \mathbb{R}^n \mapsto \mathbb{R}^n$ is at least locally bijective.

Moreover, if the controlled output y is a flat output, i.e. $z = y$, then (5) evidently represents the left and right inversion of the system as defined by Respondek [1990]. If the controlled output is not a flat output, i.e. $z \neq y$, then the evolution $t \mapsto y(t)$ of the controlled output is also parameterized by the flat output $z(t)$ and its derivatives, since considering (2), (4) and the results of Hagenmeyer and Zeitz [2004] leads to

$$y = h\left(\boldsymbol{\phi}(z, \dot{z}, \ldots, z^{(n-1)})\right) = \Gamma\left(z, \dot{z}, \ldots, z^{(n-r)}\right) \tag{8}$$

where r is the relative degree of the n-th order flat SISO system (1) with respect to the output (2). Thus (8) represents the parameterization of the controlled output by the flat output and its derivatives up to the order $n - r$.

2.2 *Flatness-based two-degree-of-freedom control*

In many practical applications a model-based feedforward is used in order to enhance the tracking performance of a control loop. Thereby a simple closed-loop control structure consisting of a system Σ and a feedback control Σ_{FB} is extended by an open-loop feedforward control Σ_{FF} as depicted in Fig. 1. The extended structure combining the feedforward control and the feedback control has two degrees of freedom for the independent design of both the tracking performance and the disturbance behaviour, cf. Horowitz [1963]. When using both degrees of freedom for a control with feedforward, it becomes evident that for the design of the feedback control part Σ_{FB} there are many different methods, whereas there are few systematic methods to design dynamic feedforwards Σ_{FF}, which take the desired motion of the controlled variable into account.

In this work a steering between two equilibrium points y_0^* and y_T^* of the controlled output is considered when no desired trajectory $y^*(t)$ is given. The respective boundary points of the flat output $z^*(t)$ can be determined using (8):

$$y_0^* = \Gamma\Big(z^*(0), 0, \ldots, 0\Big), \quad y_T^* = \Gamma\Big(z^*(T), 0, \ldots, 0\Big) \tag{9}$$

[2] The independence of (3) of the input u and the maximal number of derivatives of z in (4) and (5) respectively are due to the results of Jakubczyk and Respondek [1980] and Charlet et al. [1989].

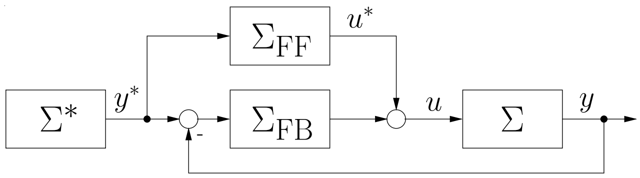

Fig. 1. Two-degree-of-freedom control structure with system Σ, feedback control control Σ_{FB}, feedforward control Σ_{FF} and reference generator Σ^* for a tracking control $y(t) \rightarrow y^*(t)$.

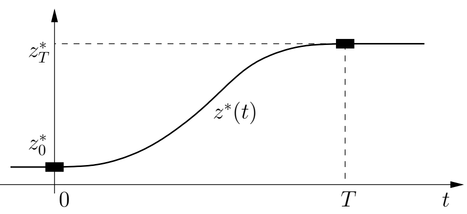

Fig. 2. Polynomial desired trajectory $z^*(t)$ of the flat output for a set point change (11) in the time interval $t \in [0, T]$. At the boundary points, the first n derivatives are zero and therefore continuous, indicated by black boxes.

where T represents the time of the terminal point. These equations have to be solved for $z^*(0)$ and $z^*(T)$, respectively. Thereafter a sufficiently smooth desired trajectory $t \mapsto z^*(t)$ for the flat output can be planned which connects these two points. Given this desired trajectory for the flat output $z^*(t)$, differential flatness yields a direct way to design a two-degree-of-freedom control scheme by system inversion. Equation (5) can be used to design the corresponding feedforward $u^*(t)$ directly:

$$\Sigma_{FF} = \Sigma^{-1}: \qquad u^*(t) = \psi(z^*, \dot{z}^*, \ldots, z^{*(n)}(t)) \tag{10}$$

The related trajectory $y^*(t)$ of the controlled output can be calculated by (8). Then, the flatness-based two-degree-of-freedom control structure can be represented by the blockdiagram shown in Fig. 1, in which the flatness-based inversion feedforward $\Sigma_{FF} = \Sigma^{-1}$ as in (10) holds for a tracking control $y(t) \rightarrow y^*(t)$. Equation (10) clarifies the necessity of the sufficient differentiability of the desired trajectory. For instance, a desired set point change of the flat output $z^*(t)$ $t \in [0, T]$:

$$\begin{aligned} & z^*(0) = z_0^* = F(\mathbf{x}_0^*) \;\rightarrow\; z^*(T) = z_T^* = F(\mathbf{x}_T^*) \\ & \text{with} \quad z^{*(i)}\Big|_{0,T} = 0, \;\; 0 < i \leq n \end{aligned} \tag{11}$$

has to connect both boundary points z_0^* and z_T^* in a sufficiently smooth way; one possible solution is depicted in Fig. 2. The basic idea of flatness-based two-degree-of-freedom control is that the flatness-based feedforward control $\Sigma_{FF} = \Sigma^{-1}$ as in (10) steers the system by inverting its dynamic model Σ, such that the feedback part Σ_{FB} has only to deal with small deviations stemming from parameter uncertainties, exogenous disturbances or modelling errors. This enables the use of linear PID-like structures for the feedback part Σ_{FB} in the blockdiagram shown in Fig. 1.

It is evident, that the pure nominal feedforward (10) does neither guarantee stability nor robustness. Therefore a stabilizing and robustifying feedback Σ_{FB} has to be designed as in Fig. 1. In Hagenmeyer and Delaleau [2003] it has been shown, that the nominal feedforward (10) exactly linearizes the system (1) if the desired trajectory $z^*(t)$ is consistent with the initial condi-

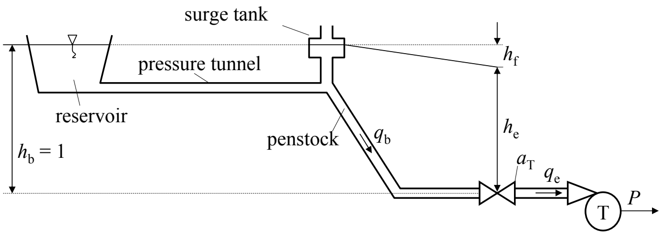

Fig. 3. Schematic outline of a pumped storage hydro power plant.

tion of the system. Thus the feedback can be designed linearly as a PID-like control, see Hagenmeyer and Delaleau [2003] for further details.

In an industrial context, the discussed structure is very useful for tracking control. If a differentially flat system is considered [3], for which there already exists a linear PID feedback control stabilizing the system in the respective vicinities of different operation points, a flatness-based feedforward combined with the existing disturbance rejection optimized PID feedback controller can lead to very good tracking of, for instance, guided set point changes. The latter also represents the main objective of application of the discussed structure on pumped storage plants.

3. DYNAMIC MODEL OF A PUMPED STORAGE PLANT

The classic outline of a pumped storage plant is presented in Fig. 3. All system variables are given in a normalized manner: the initial pressure head is normalized to $h_b = 1$. The total head loss due to friction is taken into account by the head loss h_f. The role of the surge tank is to balance pressure oscillations between pressure tunnel and penstock. The usable pressure head is denoted with h_e.

The configuration of the dynamic model is depicted in Fig. 4a [Treuer et al., 2007]. Neglecting the comparatively slow dynamics of the surge tank, the penstock model consists of two parts: the inertia of the water and the dynamics of the compressible water column. The traveling waves of the compressible water column are substituted by a first order model, representing the fundamental wave of the oscillation of the compressible water column. The mass flow q_e through the turbine depends on the usable pressure head and the turbine valve aperture a_T which is — via the valve characteristic f_1 — a function of the main servo position p_M. The turbine output is $P = q_e h_e$, which is independent of the turbine type but could be corrected by a load dependent efficiency factor.

The main servo position p_M acts as an input to the considered part system, but it has to be positioned by an actuating system consisting of two servos as shown in Fig. 4b [Weber and Zimmermann, 1996]. The main servo is controlled to its desired value u via a permanent droop. Via the characteristic f_2, which realizes precise positioning for small displacements and an over-drive for fast reaction, the main servo is actuated by a pilot servo, usually an electro-hydraulic device. The overall system dynamics are then given by

[3] If the system is not flat, then cf., for instance, the seminal paper of Devasia et al. [1996] and the recent contributions of Graichen et al. [2005], Graichen and Zeitz [2005].

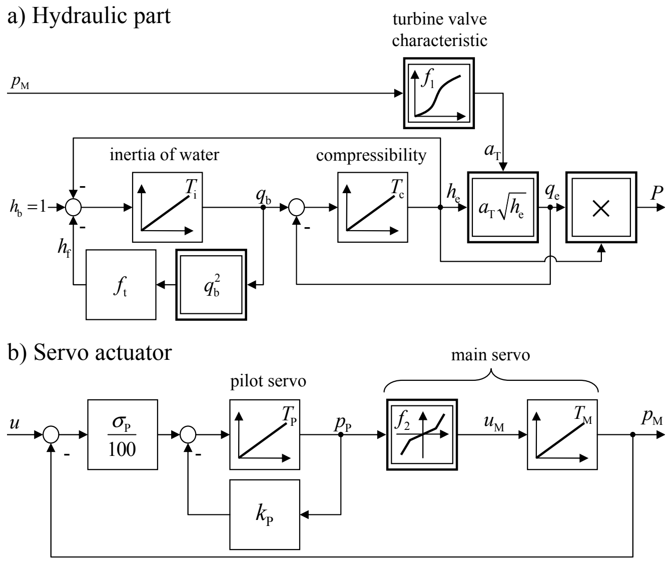

Fig. 4. Dynamic model of a pumped storage plant with the hydraulic power generation part and the electro-hydraulic servo actuator.

$$\frac{\mathrm{d}q_b(t)}{\mathrm{d}t} = \frac{1}{T_i}\left(h_b - h_e(t) - f_t q_b^2(t)\right) \tag{12}$$

$$\frac{\mathrm{d}h_e(t)}{\mathrm{d}t} = \frac{1}{T_c}\left(q_b(t) - \sqrt{h_e(t)}f_1(p_M(t))\right) \tag{13}$$

$$\frac{\mathrm{d}p_M(t)}{\mathrm{d}t} = \frac{1}{T_m} f_2(p_P(t)) \tag{14}$$

$$\frac{\mathrm{d}p_P(t)}{\mathrm{d}t} = \frac{1}{T_p}\left((u(t) - p_M(t))\frac{\sigma_P}{100} - k_p p_P\right) \tag{15}$$

$$P(t) = h_e(t)^{\frac{3}{2}} f_1(p_M(t)) \,. \tag{16}$$

With $\mathbf{x}(t) = [q_b(t),\ h_e(t), p_M(t), p_P(t)]^T$ and $y(t) = P(t)$ the system (12)–(15) and output (16) corresponds to the representation of a nonlinear SISO system in (1) and (2). The system order is $n = 4$ and its relative degree is $r = 2$. Furthermore the system is non-minimum phase [Hoppe, 1981]. The values of the parameters are taken from Treuer et al. [2007].

When connected to a larger power system, the common control task is to maintain a desired power output. This is achieved by a PID-controller together with a static feedforward of the valve position corresponding to the stationary desired output value. A negative transient droop accounts for the non-minimum phase behavior of the hydro-mechanical system [Weber and Zimmermann, 1996].

In Treuer et al. [2007] a validation of the model with measurements from a power plant is given.

4. FLATNESS-BASED TWO-DEGREE-OF-FREEDOM CONTROL FOR A PUMPED STORAGE POWER STATION

Typically the two-degree-of-freedom control structure as shown in Fig. 1 is already implemented in pumped storage hydro power stations. But the open-loop feedforward part Σ_{FF} is usually only static and does therefore not consider the dynamic behaviour of the system during set point changes. Within the framework of differential flatness, a dynamic feedforward control can be designed, which accounts for the dynamic system behaviour. Therefore it avoids the excitation of unwanted pressure oscillations.

4.1 Application of the concept

The water mass flow q_b at the beginning of the penstock is a flat output of the system (12)–(15), which will be shown in the following. With the flat output

$$q_\mathrm{b}(t) = z(t) = \phi_1(z) \tag{17}$$

and (12) the pressure can be expressed as

$$h_\mathrm{e}(t) = h_\mathrm{b} - f_\mathrm{t} z^2(t) - T_\mathrm{i}\dot{z}(t) = \phi_2(z,\dot{z})\,. \tag{18}$$

Further, with (13) the main servo position is given by

$$p_\mathrm{M}(t) = f_1^{-1}\left(\frac{z + 2 f_\mathrm{t} T_\mathrm{c} z\dot{z} + T_\mathrm{c} T_\mathrm{i}\ddot{z}}{\sqrt{h_\mathrm{b} - f_\mathrm{t} z^2 - T_\mathrm{i}\dot{z}}}\right) = \phi_3(z,\dot{z},\ddot{z})\,. \tag{19}$$

In the same way (14) yields a representation

$$p_\mathrm{P}(t) = \phi_4\left(z,\dot{z},\ddot{z},z^{(3)}\right) \tag{20}$$

of the pilot servo position wherein ϕ_4 includes the inverse function $f_2^{-1}(u_\mathrm{M})$ of the main servo characteristic $u_\mathrm{M} = f_2(p_\mathrm{P})$ and the first derivative of the inverse function of the valve characteristics $\frac{\mathrm{d}f_1^{-1}(a_\mathrm{T})}{\mathrm{d}a_\mathrm{T}}$. Finally, together with (15) a representation for the input

$$u(t) = \psi\left(z,\dot{z},\ddot{z},z^{(3)},z^{(4)}\right) \tag{21}$$

can be derived, which additionally includes the first derivative of the inverse function of main servo characteristic $\frac{\mathrm{d}f_2^{-1}(u_\mathrm{M})}{\mathrm{d}u_\mathrm{M}}$ and the second derivative of the inverse function of the valve characteristics $\frac{\mathrm{d}^2 f_1^{-1}(a_\mathrm{T})}{\mathrm{d}a_\mathrm{T}^2}$.

Equations (17)–(20) express the state variables of the system (12)–(15) in terms of the flat output and its $n-1=3$ first time derivatives and therefore correspond to (4). In the same way (21) corresponds to (5) since it states the system input u in terms of the flat output and its $n=4$ first time derivatives.

Finally, the controlled output can be written considering (16) as

$$\begin{aligned} P(t) &= \left(h_\mathrm{b} - f_\mathrm{t} z^2 - T_\mathrm{i}\dot{z}\right)\left(z + 2 f_\mathrm{t} T_\mathrm{c} z\dot{z} + T_\mathrm{c} T_\mathrm{i}\ddot{z}\right) \\ &= \Gamma(z,\dot{z},\ddot{z})\,. \end{aligned} \tag{22}$$

This corresponds to (8) and represents the controlled output in terms of the flat output and its $n-r=2$ first time derivatives.

Due to the existence of the expressions (17)–(21), $z = q_\mathrm{b}$ is proven to be a flat output of the system (12)–(16), as long as the inverse functions mentioned above exist and are differentiable once or twice, respectively. Since the valve characteristic f_1 is smooth and invertible and the servo characteristic f_2 is at least continuous and invertible, these requirements are met. The system is therefore a flat one and methods described above can be applied for designing a feedforward control.

4.2 Trajectory planning

Considering the problem of a set point change, no desired trajectory $y^*(t)$ is given in advance. The target is to change the power output from an initial stationary value P_0 to a final stationary value P_T which is to be reached after a transition time T, i.e. a steering between two equilibrium points $y_0^* = P_0$ and $y_T^* = P_T$, cf. section 2.2.

Using (22), the respective boundary points of $z^*(t)$ can be found by solving

$$y_0^* = \Gamma(z_0^*,0,0) \quad \text{and} \quad y_T^* = \Gamma(z_T^*,0,0) \tag{23}$$

for z_0^* and z_T^*, respectively. Thereafter a sufficiently smooth desired trajectory $z^*(t)$ can be planned which connects these two points, i.e.

$$z^*(0) = z_0^* \quad \text{and} \quad z^*(T) = z_T^*\,. \tag{24}$$

Then (21) is used to get the nominal feedforward $u^*(t)$, and the related trajectory $y^*(t)$ of the controlled output can be calculated by (22).

In addition to connecting the boundary points (24), the trajectory $z^*(t)$ has to satisfy the conditions

$$\begin{aligned} \dot{z}^*(0) &= 0\,, \quad \ddot{z}^*(0) = 0\,, \quad z^{*(3)}(0) = 0 \\ \dot{z}^*(T) &= 0\,, \quad \ddot{z}^*(T) = 0\,, \quad z^{*(3)}(T) = 0 \end{aligned} \tag{25}$$

because the to-be-connected boundary points are equilibrium points of the system. Furthermore $z^*(t)$ has to be four times differentiable on $[0,T]$, since the derivatives up to the forth derivative $z^{*(4)}$ are required to calculate the nominal feedforward $u^*(t)$ in (21). Because the input $u(t)$ is an electrical variable, steps in the evolution of $u^*(t)$ can be allowed and therefore $z^{*(4)}$ does not have to be continuous. A continuity condition for the fourth time derivative — as demanded in (11) — can therefore be waived. The choice of an ansatz function for a trajectory meeting the requirements (24) and (25) is a degree of freedom in the design of the flatness-based feedforward control.

In Treuer et al. [2007] three different ansatz functions are compared, namely a polynomial, a Gevrey function and a mewly introduces spline. The spline ansatz proposed in the current contribution defined by

$$\begin{aligned} \Psi_{T,\kappa,\sigma}(t) = \frac{T^4}{4!}\sum_{i=1}^{4} b_i \Bigg[&\left(\frac{t}{T} - c_i(\kappa,\sigma)\right)^4 h\left(\frac{t}{T} - c_i(\kappa,\sigma)\right) \\ &+ w\left(\frac{t}{T} - 1 + c_i(\kappa,\bar{\sigma})\right)^4 h\left(\frac{t}{T} - 1 + c_i(\kappa,\bar{\sigma})\right)\Bigg] \end{aligned} \tag{26}$$

where $h(\cdot)$ is the Heaviside function. The step sizes $\mathbf{b} = (b_1,\cdots,b_4)$ are given by

$$\mathbf{b} = (1,\,-2,\,2,\,-1) \tag{27}$$

and the step times $\mathbf{c}(\kappa,\sigma) = (c_1(\kappa,\sigma),\cdots,c_4(\kappa,\sigma))$ are

$$\mathbf{c}(\kappa,\sigma) = \left(0,\,\frac{1}{4}\kappa - \sigma,\,\frac{3}{4}\kappa - \sigma,\,\kappa\right)\,. \tag{28}$$

In order to fulfill the consistency conditions

$$\Psi_{T,\kappa,\sigma}^{(i)}(t)\Big|_{0,T} = 0, \quad 0 < i \le 3\,, \tag{29}$$

the variables w and $\bar{\sigma}$ have to be determined appropriately. For the boundary $t=0$ as well as for the third time derivative at the boundary $t=T$ these conditions are met inherently. An evaluation of the second time derivative at the boundary $t=T$ yields

$$\Psi_{T,\kappa,\sigma}^{(2)}(t)\Big|_{T} = T\kappa(\sigma + w\bar{\sigma}) = 0 \tag{30}$$

which provides

$$w = -\frac{\sigma}{\bar{\sigma}} \tag{31}$$

Setting the first time derivative of (26) to zero at the boundary $t=T$ yields the quadratic equation

$$\bar{\sigma}^2 + (\sigma + 2\kappa - 2 - \frac{\kappa^2}{\sigma 16})\bar{\sigma} - \frac{\kappa^2}{16} = 0 \tag{32}$$

that can be solved for $\bar{\sigma}$.

With the choice of $\sigma = \bar{\sigma} = 0$ and $w = 1$ the ansatz (26) is equivalent to the spline defined in Treuer et al. [2007], except for the factor $T/4!$ that is introduced for convenience.

During the main time of the transition, i.e. $t \in [\kappa T, (1-\kappa)T]$, a constant slope is maintained. Only for $t \in [0, \kappa T]$ at the beginning and $t \in [(1-\kappa)T, T]$ at the end of the trajectory a tight curvature is used to obtain a smooth changeover. Due to the symmetry of (26) with $\sigma = 0$ the first, second, and third time derivative is zero at the transition boundaries $t = 0$ and $t = T$, i.e. the consistency conditions (29) are met. Furthermore, the second time derivative is zero for $t \in [\kappa T, (1-\kappa)T]$, which yields the constant slope described above. With the design parameter $\sigma \neq 0$ the second time derivative results in a constant value not equal to zero for $t \in [\kappa T, (1-\kappa)T]$, and therefore (26) has a constant curvature during the main time of the transitions.

In the following $\kappa = 1/8$ is used, i.e. the changeover takes one eighth of the transition time T at the beginning and at the end, respectively, and during three fourth of the transition the curvature is constant. Fig. 5 shows the spline function $\Psi_{T,\kappa,\sigma}(t)$ and its first four time derivatives for $\kappa = 1/8$ and a transition time $T = 10\,\mathrm{s}$. Two choices for the parameter σ are depicted. For $\sigma = 0$ the constant slope of $\Psi_{T,\kappa,\sigma}(t)$ during the main time of the transition becomes obvious by looking at the first time derivative, which is constant. For $\sigma = -3.5e^{-4}\,\mathrm{s}$ the slope of $\Psi_{T,\kappa,\sigma}(t)$ has a curvature, which can be observed in the slope of the first time derivative. The tiny shifts in the step-times of the fourth derivative are hardly noticeable, but the resulting change in the step height at the end of the transition is considerable.

A desired trajectory for the flat output satisfying the conditions (24) and (25) is given by

$$z^*(t) = z_0^* + \frac{z_T^* - z_0^*}{\Psi_{T,\kappa,\sigma}(T)} \Psi_{T,\kappa,\sigma}(t), \quad t \in (0, T). \tag{33}$$

4.3 Simulation results

A set point change output from 90% to 40% of the power output within 10 seconds is considered in the following. Conventionally this as achieved by applying a ramp like desired value curve to a static feedforward and the PID control. The control action is retarded by a negative transient droop.

In comparison a flatness based feedforward is designed. A desired trajectory for the flat output is planed with the spline ansatz (33). A nominal feedforward $u^*(t)$ is determined via (21) and applied directly to the system, i.e. not modified by the transient droop. A reference output trajectory $y^*(t)$ is calculated via (22) and used as the reference trajectory for the existing PID-controller.

In Fig. 6 the simulation results are shown for the conventional ramp as reference and for the flatness based feedforward with two different parametrisations of the spline ansatz. Both use $\kappa = 1/8$ and $T = 10\mathrm{s}$. The design parameter σ is chosen to $\sigma = 0\,\mathrm{s}$ (dotted) and $\sigma = -3.5e^{-4}\,\mathrm{s}$ (solid).

When reducing the power output, the pressure in front of the turbine increases to a higher stationary value, due to reduced friction losses. But during the transition the pressure reaches significantly higher peak values due to the inertia of the water masses in the penstock. This peak values can cause severe damage to the plant. Therefore the considered negative set point

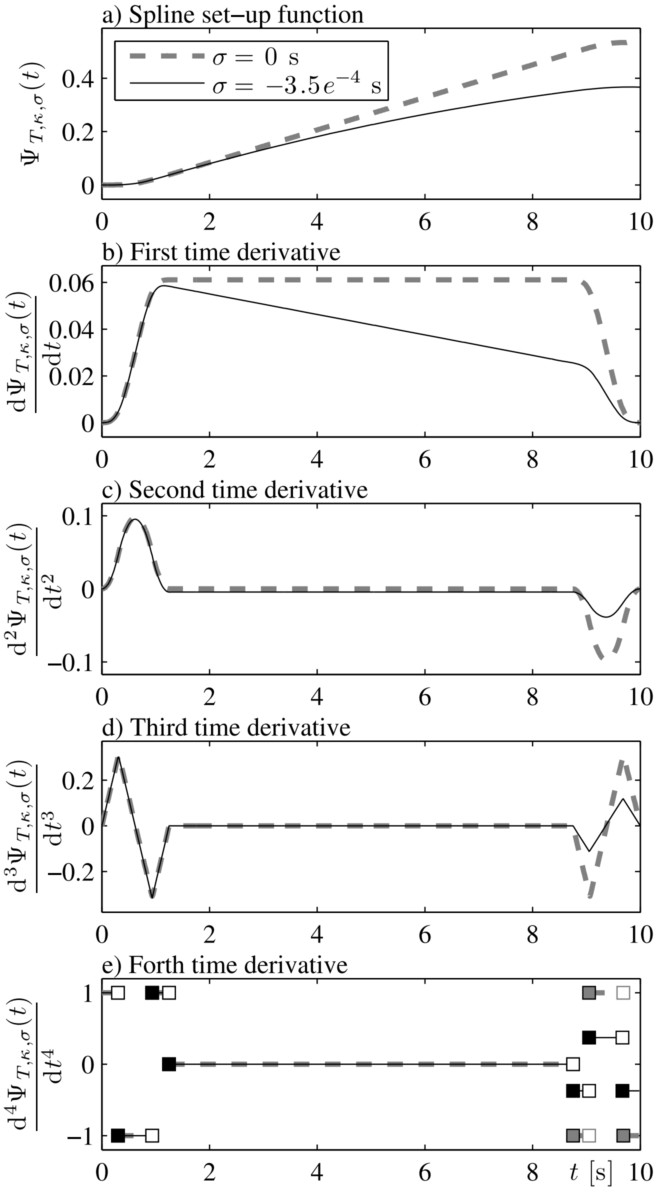

Fig. 5. Spline function $\Psi_{T,\kappa,\sigma}(t)$ and its first four time derivatives for $\kappa = 1/8$, a transition time $T = 10\,\mathrm{s}$ and the parameter $\sigma = 0\,\mathrm{s}$ (gray dashed) and $\sigma = -3.5e^{-4}\,\mathrm{s}$ (solid).

change is in fact a critical case. Furthermore, the non-minimum phase behaviour of the plant due to these pressure dynamics can be observed at the beginning of the transition, where the power output in Fig. 6b increases for a short period of time. During this time the increase in pressure is stronger than the decrease in mass flow and therefore raises the power output.

A static feed forward, like in the reference simulation, does not account for the pressure dynamics, resulting in an undershoot in the power output at the end of the transition, that is corrected slowly. The pressure in front of the turbine, plotted in Fig. 6b, shows significant oscillations.

In case of the flatness based feedforward control the transition is planned in terms of the flat output $z = q_\mathrm{b}$, where the spline can be seen in Fig. 6d. It connects the initial stationary value for the water mass flow corresponding to the output at time $t = 0\,\mathrm{s}$ smoothly with the final stationary value at time $t = T = 10\,\mathrm{s}$.

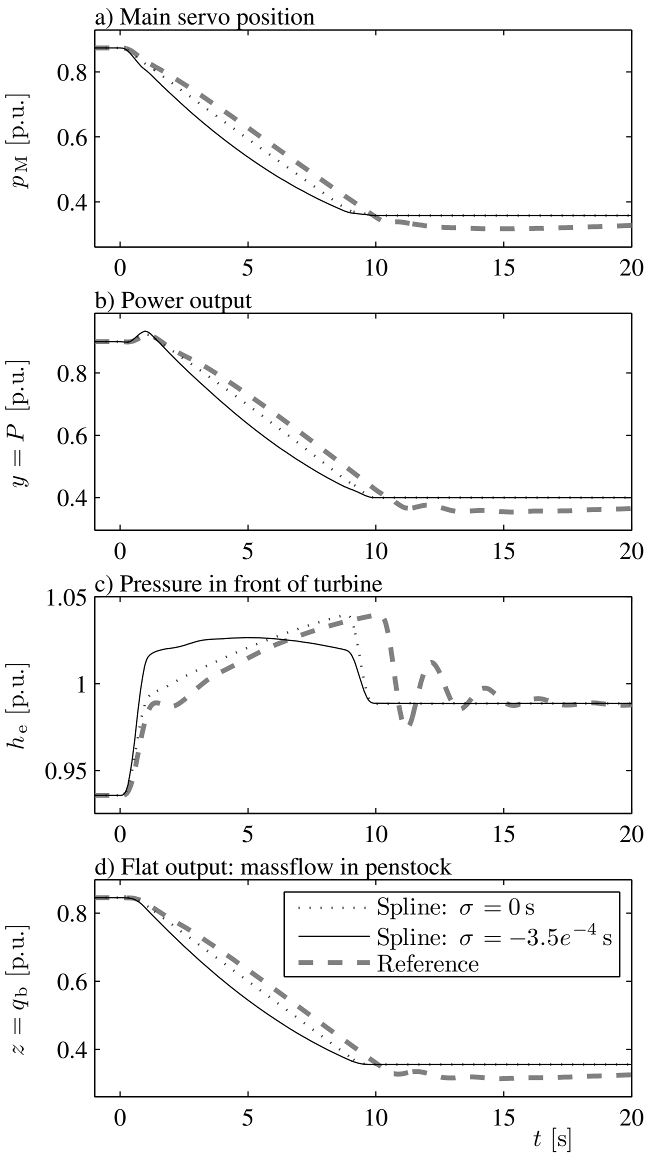

Fig. 6. Set point transition of the pumped storage plant using flatness-based feedforward control with spline trajectories and parameters $\kappa = 1/8, \sigma = 0\,\mathrm{s}$ (dotted) and $\kappa = 1/8$, $\sigma = -3.5e^{-4}\,\mathrm{s}$ (solid). Reference simulation using static feedforward and ramped desired values (dashed gray).

The resulting output, shown in Fig. 6b, follows from (22). The pressure in front of the turbine, plotted in Fig. 6c, does not show the oscillating behaviour of the reference case.

Its overshoot is due to the inertia of the water masses and depends on the slope of the water mass flow as (18) reveals. With a constant slope during the main time of the transition, as in the case of the spline with $\sigma = 0\,\mathrm{s}$, the pressure shows a constant offset to its quasi-stationary value. But since the quasi-stationary value increases during the transition, a higher offset could be tolerated at the beginning and a lower offset at the end of the transition. The offset is related to the slope of the flat output and decreasing slope is realized by a spline parameter $\sigma = -3.5e^{-4}\,\mathrm{s}$. Thereby the peak value of the pressure is significantly reduced.

5. CONCLUSIONS

The applied flatness-based two-degree-of-freedom control concept results in a considerably improved control performance of the considered pumped storage plant. Especially the critical pressure oscillations during set point changes are prevented, which leads to a preservation of the plant fittings. The type of ansatz function used for trajectory planning strongly affects the dynamic behaviour of the plant. The introduced spline ansatz allows for a wider but less pronounced pressure peak during the critical negative set point changes. The peak value is therefore considerably reduced.

REFERENCES

B. Charlet, J. Lévine, and R. Marino. On dynamic feedback linearization. *Syst. Contr. Lett.*, 13:143–151, 1989.

S. Devasia, D. Chen, and B. Paden. Nonlinear inversion-based output tracking. *IEEE Trans. Automatic Control*, 41:930–942, 1996.

M. Fliess, J. Lévine, Ph. Martin, and P. Rouchon. Flatness and defect of nonlinear systems: introductory theory and examples. *Int. J. Control*, 61(6):1327–1361, 1995.

M. Fliess, J. Lévine, Ph. Martin, and P. Rouchon. A Lie-Bäcklund approach to equivalence and flatness of nonlinear systems. *IEEE Trans. Automat. Contr.*, 44(5):922–937, 1999.

K. Graichen and M. Zeitz. Feedforward control design for nonlinear systems under input constraints. In T. Meurer, Graichen K., and Gilles E.D., editors, *Control and Observer Design for Nonlinear Finite- and Infinite-Dimensional Systems*, LNCIS, pages 235–252. Springer, 2005.

K. Graichen, V. Hagenmeyer, and M. Zeitz. A new approach to inversion-based feedforward control design for nonlinear systems. *Automatica*, 41:2033–2041, 2005.

V. Hagenmeyer and E. Delaleau. Exact feedforward linearization based on differential flatness. *Int. J. Control*, 76:537–556, 2003.

V. Hagenmeyer and M. Zeitz. Internal dynamics of flat nonlinear SISO systems with respect to a non-flat output. *Syst. Contr. Lett.*, 52:323–327, 2004.

M. Hoppe. *Die Regelung von Systemen mit Allpaß-Verhalten – dargestellt durch theoretische und experimentelle Untersuchung einer Wasserkraftanlage.* Number 16 in Schriftenreihe des Lehrstuhls für Meß- und Regelungstechnik. Ruhr-Universität Bochum, 1981.

I. M. Horowitz. *Synthesis of Feedback Systems*. Academic Press, New York, 1963.

B. Jakubczyk and W. Respondek. On linearization of control systems. *Bull. Acad. Pol. Sci. Sér. Sci. Math.*, 28:517–522, 1980.

W. Respondek. Right and left invertibility of nonlinear control systems. In H.J. Sussmann, editor, *Nonlinear Controllability and Optimal Control*, pages 133–176. Marcel Dekker, New York, 1990.

H. Sira-Ramírez and S. K. Agrawal. *Differentially flat systems*. Marcel Dekker, New York, 2004.

M. Treuer, T. Weißbach, M. Kurth, and V. Hagenmeyer. Flatness-based two-degree-of-freedom control of a pumped storage power station. In *Proceedings of the European Control Conference, Kos, Greece*, pages 4087–4094, July 2007.

H. Weber and D. Zimmermann. Investigation of the dynamic Behaviour of a High Pressure Hydro Power Plant in the Swiss Alps during the Transition from interconnected to isolated Operation. In *12th Power Systems Computation Conference, Dresden*, August 1996.