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FAULT DETECTION FOR SINGULAR TS FUZZY SYSTEMS WITH TIME-DELAY ⋆

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Abstract: The robust fault detection filter design problem for singular TS fuzzy systems with time-delay is studied. Using an observer-based fuzzy fault detection filter as the residual generator, the fault detection filter design is converted to an H_∞ filtering problem such that the generated residual is the H_∞ estimation of the fault. Sufficient conditions are given, which guarantee the robust H_∞ fault detection filter exists. And by using the cone complementarity linearization iterative algorithm, the fault detection filter design is converted to solving a sequence of convex optimization problems subject to LMIs. The premise variables of the designed fuzzy filter are not demanded to be the same as the premise variables of the TS fuzzy model of the plant.

1. INTRODUCTION

Over the past few decades, the problem of fault detection and isolation (FDI) in dynamic systems has attracted considerable attention of many researchers (Chen *et al.*, 1999; Chen *et al.*, 2000; Ding *et al.*, 2000; Jiang *et al.*, 2003; Zhong *et al.*, 2005). Among the approaches for FDI, the model-based approach has been extensively studied. As is well known, the presence of time delays must be taken into account in a realistic FDI filter design. However, it seems that there are very few previous results on the FDI problem for time-delay nonlinear systems (B. Castillo-Toledo *et al.*, 2005; Chen *et al.*, 2006; E. Alcorta *et al.*, 2003; Magdy *et al.*, 2006; Sing, Peng and Steven, 2006; Sing, Ping and Steven, 2006). In (B. Castillo-Toledo *et al.*, 2005; Chen *et al.*, 2006; E. Alcorta *et al.*, 2003; Magdy *et al.*, 2006; Sing, Ping and Steven, 2006), fault detection problem for nonlinear systems was studied without considering time delays. In (Sing, Peng and Steven, 2006), fault estimation problem for time-delay nonlinear systems described by TS fuzzy models was studied and assumed that the premise variables of the residual generator were the same as the premise variables of the TS model of the plant. To the best of authors' knowledge, however, there is a lack to the research on FDI for singular TS fuzzy systems with time-delay, which motivates the present study.

In this paper, we will deal with the problem of fault detection for a class of singular time-delay nonlinear systems described by TS fuzzy models. Using an observer-based fuzzy fault detection filter as a residual generator, the design of fault detection filter (FDF) will be formulated as an H_∞-filtering problem firstly. Then sufficient conditions on the existence of a robust H_∞-FDF for singular time-delay fuzzy systems will be derived in terms of linear matrix inequalities (LMIs) by using the cone complementarity linearization iterative algorithm and a solution to the robust H_∞-FDF can be obtained.

2. PROBLEM FORMULATION

Consider a class of singular time-delay system described by the following TS fuzzy model
Rule i: IF $z_1(t)$ is Θ_{i1} and $\cdots$ and $z_q(t)$ is Θ_{iq}, THEN

$$\begin{cases} E\dot{x}(t) = A_i x(t) + A_{\tau i}x(t-\tau) + B_d d(t) + B_f f(t) \\ y(t) = C_i x(t) + D_d d(t) + D_f f(t) \\ x(\theta) = \phi(\theta), \forall\theta \in [-\tau, 0] \end{cases} \quad (1)$$

where $i = 1, 2, \cdots, r$, Θ_{ij} $(j = 1, 2, \cdots, q)$ are fuzzy sets; $z(t) = \begin{bmatrix} z_1(t) & \cdots & z_q(t) \end{bmatrix}^T$ is the premise variable which may be a measurable variable or the state of the system; $x(t) \in \mathbf{R}^n$, $y(t) \in \mathbf{R}^{n_y}$, $f(t) \in \mathbf{R}^{n_f}$, $d(t) \in \mathbf{R}^{n_d}$ are the state, measurement output, fault and unknown input, respectively; f and d are assumed to be $\mathcal{L}_2$ -norm bounded; $rankE = p$, $0 < p < n$. E, A_i, $A_{\tau i}$, B_d, B_f, C_i, D_d and D_f are known matrices with appropriate dimensions; τ is an unknown constant delay satisfying

$$0 \le \tau_m \le \tau \le \tau_M \quad (2)$$

where τ_m and τ_M are known constants; $\phi(\cdot)$ is a continuous vector valued initial function; r is the number of IF-THEN rules. In this paper, it is supposed that system (1) with $f(t) = 0$ and $d(t) = 0$ is asymptotically stable and $E = diag(I, 0)$.

The resulting fuzzy system model is inferred as the weighted average of the local models of the form

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$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t))[A_i x(t) + A_{\tau i} x(t-\tau) \\ \qquad\qquad + B_d d(t) + B_f f(t)] \\ y(t) = \sum_{i=1}^{r} \mu_i(z(t))[C_i x(t) + D_d d(t) + D_f f(t)] \\ x(\theta) = \phi(\theta), \forall \theta \in [-\tau, 0] \end{cases} \tag{3}$$

where $\mu_i(z(t)) = \frac{\prod_{j=1}^{q} \Theta_{ij}(z_j(t))}{\sum_{i=1}^{r}\prod_{j=1}^{q} \Theta_{ij}(z_j(t))} \geq 0$, $\sum_{i=1}^{r} \mu_i(z(t)) = 1$. $\Theta_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in Θ_{ij}. Note that the premise variable $z(t)$ may be a measurable variable or the state of the system, so system (3) is a nonlinear system. For the convenience of notations, we let $\mu_i = \mu_i(z(t))$.

Defining $\tau_{av} = \frac{1}{2}(\tau_M + \tau_m)$, using $x(t-\tau) - x(t-\tau_{av}) = \int_{t-\tau_{av}}^{t-\tau} \dot{x}(\theta)d\theta$, we can rewrite system (3) as

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{r} \mu_i[A_i x(t) + A_{\tau i} \int_{t-\tau_{av}}^{t-\tau} \dot{x}(\theta)d\theta \\ \qquad\qquad + A_{\tau i} x(t-\tau_{av}) + B_d d(t) + B_f f(t)] \\ y(t) = \sum_{i=1}^{r} \mu_i[C_i x(t) + D_d d(t) + D_f f(t)] \\ x(\theta) = \phi(\theta), \forall \theta \in [-\tau, 0] \end{cases} \tag{4}$$

The main objective of this paper is to design an asymptotically stable FDF such that the generated residual signal r satisfies the H_∞ performance

$$||r - W_f(s)f||_2 \leq \gamma ||w||_2 \tag{5}$$

for a prescribed $\gamma > 0$, where $W_f(s)$ is a given stable weighting matrix and $w(t) = \left[\, d^T(t) \;\; f^T(t) \,\right]^T$.

Remark 1. The introducing of a suitable weighting matrix $W_f(s)$ was used to limit the frequency interval, in which the fault should be identified, and the system performance could be improved.

Without loss of generality, we suppose that one minimal realization of $W_f(s)$ is

$$\begin{cases} \dot{x}_f(t) = A_W x_f(t) + B_W f(t), x_f(0) = 0 \\ r_f(t) = C_W x_f(t) \end{cases} \tag{6}$$

where $x_f(t) \in \mathbf{R}^{n_W}$, $r_f(t) \in \mathbf{R}^{n_f}$, A_W, B_W and C_W are known constant matrices. Augmenting (4) and (6) yields

$$\begin{cases} E_s\dot{x}_s(t) = \sum_{i=1}^{r} \mu_i[A_{si} x_s(t) + A_{\tau si} x_s(t-\tau_{av}) \\ \qquad\qquad + A_{\tau si} \int_{t-\tau_{av}}^{t-\tau} \dot{x}_s(\theta)d\theta + B_s w(t)] \\ y(t) = \sum_{i=1}^{r} \mu_i[C_{si} x_s(t) + D_s w(t)] \\ r_f(t) = C_{sf} x_s(t) \\ x_s(\theta) = \phi_s(\theta), \forall \theta \in [-\tau, 0] \end{cases} \tag{7}$$

where

$x_s(t) = \begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}$, $E_s = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}$, $A_{si} = \begin{bmatrix} A_i & 0 \\ 0 & A_W \end{bmatrix}$,

$A_{\tau si} = \begin{bmatrix} A_{\tau i} & 0 \\ 0 & 0 \end{bmatrix}$, $B_s = \begin{bmatrix} B_d & B_f \\ 0 & B_W \end{bmatrix}$, $C_{si} = [\, C_i \;\; 0 \,]$,

$\phi_s(\theta) = \begin{bmatrix} \phi(\theta) \\ 0 \end{bmatrix}$, $\forall \theta \in [-\tau, 0]$, $D_s = [\, D_d \;\; D_f \,]$,

$C_{sf} = [\, 0 \;\; C_W \,]$.

Consider the following fuzzy FDF of observer-type
Rule i: IF $\hat{z}_1(t)$ is Θ_{i1} and $\cdots$ and $\hat{z}_q(t)$ is Θ_{iq}, THEN

$$\begin{cases} E_s\dot{\hat{x}}_s(t) = A_{si}\hat{x}_s(t) + A_{\tau si}\hat{x}_s(t-\tau_{av}) \\ \qquad\qquad + H_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C_{si}\hat{x}_s(t) \\ r(t) = C_{ri}\hat{x}_s(t) \\ \hat{x}_s(\theta) = \phi_s(\theta), \forall \theta \in [-\tau, 0] \end{cases} \tag{8}$$

where $i = 1, 2, \cdots, r$, $\hat{z}_i(t)$ is the estimation of $z_i(t)$, $\hat{x}_s(t) \in \mathbf{R}^{n+n_W}$ is the estimation of $x_s(t)$ and $r(t) \in \mathbf{R}^{n_f}$ is the residual of FDF. H_i and C_{ri} are the matrices to be designed.

The final fuzzy FDF is inferred as the weighted average of the local models of the following form

$$\begin{cases} E_s\dot{\hat{x}}_s(t) = \sum_{i=1}^{r} \hat{\mu}_i[A_{si}\hat{x}_s(t) + A_{\tau si}\hat{x}_s(t-\tau_{av}) \\ \qquad\qquad + H_i(y(t) - \hat{y}(t))] \\ \hat{y}(t) = \sum_{i=1}^{r} \hat{\mu}_i C_{si}\hat{x}_s(t) \\ r(t) = \sum_{i=1}^{r} \hat{\mu}_i C_{ri}\hat{x}_s(t) \\ \hat{x}_s(\theta) = \phi_s(\theta), \forall \theta \in [-\tau, 0] \end{cases} \tag{9}$$

Remark 2. In (B. Castillo-Toledo et al., 2005; E. Alcorta et al., 2003; Magdy et al., 2006; Sing, Peng and Steven, 2006; Sing, Ping and Steven, 2006), the premise variables of the residual generator are assumed to be the same as the premise variables of the fuzzy systems model. This actually means that the premise variables of the fuzzy systems model are assumed to be measurable. However, in general, it is extremely difficult to derive an accurate fuzzy systems model by imposing that all the premise variables are measurable. In this paper, we do not impose that condition, we choose the premise variables of the residual generator to be the estimated premise variables of the plant.

Defining $e(t) = x_s(t) - \hat{x}_s(t)$, $r_e(t) = r(t) - r_f(t)$, we have

$$\begin{cases} E_s\dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \hat{\mu}_i\hat{\mu}_j(A_{si} - H_i C_{sj})e(t) \\ \qquad + \sum_{i=1}^{r} \hat{\mu}_i A_{\tau si} e(t-\tau_{av}) \\ \qquad + \sum_{i=1}^{r} (\mu_i - \hat{\mu}_i) A_{si} x_s(t) \\ \qquad - \sum_{i=1}^{r}\sum_{j=1}^{r} \hat{\mu}_i(\mu_j - \hat{\mu}_j) H_i C_{sj} x_s(t) \\ \qquad + \sum_{i=1}^{r} (\mu_i - \hat{\mu}_i) A_{\tau si} x_s(t-\tau_{av}) \\ \qquad + \sum_{i=1}^{r} (\mu_i - \hat{\mu}_i) A_{\tau si} \int_{t-\tau_{av}}^{t-\tau} \dot{x}_s(\theta)d\theta \end{cases} \tag{10a}$$

$$\begin{cases} \quad + \sum_{i=1}^{r} \hat{\mu}_i A_{\tau si} \int_{t-\tau_{av}}^{t-\tau} \dot{x}_s(\theta) d\theta \\ \quad + \sum_{i=1}^{r} \hat{\mu}_i (B_s - H_i D_s) w(t) \\ r_e(t) = \sum_{i=1}^{r} \hat{\mu}_i (C_{ri} - C_{sf}) x_s(t) - \sum_{i=1}^{r} \hat{\mu}_i C_{ri} e(t) \\ e(\theta) = 0, \forall \theta \in [-\tau, 0] \end{cases} \tag{10b}$$

Defining $\tilde{x}(t) = \begin{bmatrix} x_s^T(t) & e^T(t) \end{bmatrix}^T$, using $\tilde{x}(t-\tau) - \tilde{x}(t-\tau_{av}) = \int_{t-\tau_{av}}^{t-\tau} \dot{\tilde{x}}(\theta) d\theta$, one obtains

$$\begin{cases} \tilde{E}\dot{\tilde{x}}(t) = G_{ij} + \sum_{i=1}^{r} \hat{\mu}_i \Delta\tilde{H}_i \tilde{x}(t) + \Delta\tilde{A}\tilde{x}(t) \\ \quad +\Delta\tilde{A}_\tau \tilde{x}(t-\tau) \\ r_e(t) = \sum_{i=1}^{r} \hat{\mu}_i \tilde{C}_{ri} \tilde{x}(t) \\ \tilde{x}(\theta) = \tilde{\phi}(\theta), \forall \theta \in [-\tau, 0] \end{cases} \tag{11}$$

where

$$G_{ij} = \sum_{i=1}^{r}\sum_{j=1}^{r} \hat{\mu}_i \hat{\mu}_j [(\tilde{A}_{ij} + \tilde{A}_{\tau i,2})\tilde{x}(t) + \tilde{B}_{wi} w(t) - \tilde{A}_{\tau i,2} \int_{t-\tau_{av}}^{t} \dot{\tilde{x}}(\theta) d\theta + \tilde{A}_{\tau i,1} \tilde{x}(t-\tau)],$$

$\tilde{E} = \begin{bmatrix} E_s & 0 \\ 0 & E_s \end{bmatrix}$, $\tilde{A}_{ij} = \begin{bmatrix} A_{si} & 0 \\ 0 & A_{si} - H_i C_{sj} \end{bmatrix}$,

$\tilde{A}_{\tau i,2} = \begin{bmatrix} 0 & 0 \\ -A_{\tau si} & A_{\tau si} \end{bmatrix}$, $\tilde{A}_{\tau i,1} = \begin{bmatrix} A_{\tau si} & 0 \\ A_{\tau si} & 0 \end{bmatrix}$,

$\Delta\tilde{A} = \begin{bmatrix} 0 & 0 \\ \Delta A & 0 \end{bmatrix}$, $\Delta\tilde{A}_\tau = \begin{bmatrix} 0 & 0 \\ \Delta A_\tau & 0 \end{bmatrix}$,

$\tilde{B}_{wi} = \begin{bmatrix} B_s \\ B_s - H_i D_s \end{bmatrix}$, $\Delta\tilde{H}_i = \begin{bmatrix} 0 & 0 \\ -H_i \Delta C & 0 \end{bmatrix}$,

$\tilde{\phi}(\theta) = \begin{bmatrix} \phi_s(\theta) \\ 0 \end{bmatrix}$, $\forall \theta \in [-\tau, 0]$,

$\tilde{C}_{ri} = [\, C_{ri} - C_{sf} \;\; -C_{ri} \,]$, $\Delta A = \sum_{i=1}^{r} (\mu_i - \hat{\mu}_i) A_{si}$,

$\Delta A_\tau = \sum_{i=1}^{r} (\mu_i - \hat{\mu}_i) A_{\tau si}$, $\Delta C = \sum_{j=1}^{r} (\mu_j - \hat{\mu}_j) C_{sj}$.

Based on the above discussion, the FDF problem to be addressed is stated as follows.

The FDF Problem: Design an FDF of observer-type in form (8) such that it is a robust H_∞-FDF of system (1) if (i) system (11) with $w(t) = 0$ is asymptotically stable; (ii) the H_∞ performance $\|r_e\|_2 < \gamma \|w\|_2$ is guaranteed for all nonzero $w(t) \in \mathcal{L}_2[0, \infty)$ and a prescribed $\gamma > 0$ under the condition $\phi(\theta) = 0$, $\forall \theta \in [-\tau, 0]$.

3. MAIN RESULTS

The following theorem is essential for solving the FDF problem formulated in the previous section.

Theorem 1. For given scalars $\gamma > 0$ and τ satisfying (2), system (8) is a robust H_∞-FDF of system (1), if there exist scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, matrices $Q > 0$, $S > 0$, $X \geq 0$, $Z > 0$ and P, Y, such that the following matrix inequalities hold for $i, j = 1, 2, \cdots, r$

$$\tilde{E}P = P^T\tilde{E} \geq 0 \tag{12}$$

$$\begin{bmatrix} X & Y \\ * & \tilde{E}Z\tilde{E} \end{bmatrix} \geq 0 \tag{13}$$

$$\begin{bmatrix} \omega_{11} & P^T\tilde{A}_{\tau i,1} & \omega_{13} & P^T\tilde{B}_{wi} & \omega_{15} & \omega_{16} \\ * & -Q & 0 & 0 & \omega_{25} & \omega_{26} \\ * & * & -S & 0 & 0 & \omega_{36} \\ * & * & * & -\gamma^2 I & 0 & \omega_{46} \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & \omega_{66} \end{bmatrix} < 0 \tag{14}$$

where

$\omega_{11} = P^T\tilde{A}_{ij} + \tilde{A}_{ij}^T P + \tau_{av} X + Y + Y^T + Q + S + \varepsilon_1^2 I$,

$\omega_{13} = P^T\tilde{A}_{\tau i,2} - Y$, $\omega_{16} = [\, \tau_{av}\tilde{A}_{ij}^T Z \;\; 0 \;\; 0 \;\; 0 \,]$,

$\omega_{15} = \begin{bmatrix} \varepsilon_2 P^T & \varepsilon_3 P^T & \frac{\sqrt{2}}{\varepsilon_1} P^T \Omega_i & \frac{\sqrt{2}}{\varepsilon_2} \Phi^T & \tilde{C}_{ri}^T & 0 \end{bmatrix}$,

$\omega_{25} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{\varepsilon_3} \Psi^T \end{bmatrix}$, $\Omega_i = \begin{bmatrix} 0 & 0 \\ 0 & H_i \hat{C} \end{bmatrix}$,

$\omega_{26} = [\, \tau_{av}\tilde{A}_{\tau i,1}^T Z \;\; \tau_{av}\tilde{A}_{\tau i,1}^T Z \;\; 0 \;\; 0 \,]$,

$\omega_{36} = [\, \tau_{av}\tilde{A}_{\tau i,2}^T Z \;\; \tau_{av}\tilde{A}_{\tau i,2}^T Z \;\; \tau_{av}\tilde{A}_{\tau i,2}^T Z \;\; 0 \,]$,

$\omega_{46} = [\, \tau_{av}\tilde{B}_{wi}^T Z \;\; \tau_{av}\tilde{B}_{wi}^T Z \;\; \tau_{av}\tilde{B}_{wi}^T Z \;\; \tau_{av}\tilde{B}_{wi}^T Z \,]$,

$\omega_{66} = diag(-\tau_{av}Z, -\tau_{av}Z, -\tau_{av}Z, -\tau_{av}Z)$,

$\Phi = \begin{bmatrix} \hat{A} & 0 \\ 0 & 0 \end{bmatrix}$, $\Psi = \begin{bmatrix} \hat{A}_\tau & 0 \\ 0 & 0 \end{bmatrix}$, $i = 1, 2, \cdots, r$,

$\hat{A} = \sum_{i=1}^{r} A_{si}$, $\hat{A}_\tau = \sum_{i=1}^{r} A_{\tau si}$, $\hat{C} = \sum_{j=1}^{r} C_{sj}$.

Proof. Choose a Lyapunov function candidate as

$$V(t) = \tilde{x}^T(t)P^T\tilde{E}\tilde{x}(t) + \int_{t-\tau}^{t} \tilde{x}^T(\theta)Q\tilde{x}(\theta)d\theta + \int_{t-\tau_{av}}^{t} \tilde{x}^T(\theta)S\tilde{x}(\theta)d\theta + \int_{-\tau_{av}}^{0} ds \int_{t+s}^{t} \dot{\tilde{x}}^T(\theta)\tilde{E}Z\tilde{E}\dot{\tilde{x}}(\theta)d\theta$$

Taking the derivative of $V(t)$ with respect to t along the trajectory of (11) yields

$$\begin{aligned} \dot{V}(t) = {} & 2\tilde{x}^T(t)P^T[G_{ij} + \sum_{i=1}^{r} \hat{\mu}_i \Delta\tilde{H}_i \tilde{x}(t) + \Delta\tilde{A}\tilde{x}(t) \\ & +\Delta\tilde{A}_\tau \tilde{x}(t-\tau)] - \tilde{x}^T(t-\tau)Q\tilde{x}(t-\tau) \\ & +\tilde{x}^T(t)Q\tilde{x}(t) - \tilde{x}^T(t-\tau_{av})S\tilde{x}(t-\tau_{av}) \\ & +\tilde{x}^T(t)S\tilde{x}(t) + \tau_{av}\dot{\tilde{x}}^T(t)\tilde{E}Z\tilde{E}\dot{\tilde{x}}(t) \\ & - \int_{t-\tau_{av}}^{t} \dot{\tilde{x}}^T(s)\tilde{E}Z\tilde{E}\dot{\tilde{x}}(s)ds \end{aligned}$$

Using (13), we have

$$\begin{aligned} & -2\tilde{x}^T(t)P^T\tilde{A}_{\tau i,2} \int_{t-\tau_{av}}^{t} \dot{\tilde{x}}(\theta)d\theta \\ = {} & \int_{t-\tau_{av}}^{t} \begin{bmatrix} \tilde{x}(t) \\ \dot{\tilde{x}}(\theta) \end{bmatrix}^T \begin{bmatrix} 0 & -P^T\tilde{A}_{\tau i,2} \\ * & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \dot{\tilde{x}}(\theta) \end{bmatrix} d\theta \\ \leq {} & \int_{t-\tau_{av}}^{t} \begin{bmatrix} \tilde{x}(t) \\ \dot{\tilde{x}}(\theta) \end{bmatrix}^T \begin{bmatrix} X & Y - P^T\tilde{A}_{\tau i,2} \\ * & \tilde{E}Z\tilde{E} \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \dot{\tilde{x}}(\theta) \end{bmatrix} d\theta \\ = {} & \tau_{av}\tilde{x}^T(t)X\tilde{x}(t) + 2\tilde{x}^T(t)(Y - P^T\tilde{A}_{\tau i,2})(\tilde{x}(t) \\ & -\tilde{x}(t-\tau_{av})) + \int_{t-\tau_{av}}^{t} \dot{\tilde{x}}^T(\theta)\tilde{E}Z\tilde{E}\dot{\tilde{x}}(\theta)d\theta \end{aligned} \tag{15}$$

Noticing that

$$\begin{aligned} & \tilde{x}^T(t)P^T(\sum_{i=1}^{r} \hat{\mu}_i \Delta\tilde{H}_i \tilde{x}(t)) + (\sum_{i=1}^{r} \hat{\mu}_i \tilde{x}^T(t)\Delta\tilde{H}_i^T)P\tilde{x}(t) \\ \leq {} & \varepsilon_1^2 \tilde{x}^T(t)\tilde{x}(t) + \frac{1}{\varepsilon_1^2}(\sum_{i=1}^{r} \hat{\mu}_i \tilde{x}^T(t)P^T\Delta\tilde{H}_i) \\ & \times(\sum_{i=1}^{r} \hat{\mu}_i \Delta\tilde{H}_i^T P\tilde{x}(t)) \end{aligned}$$

$$\leq \varepsilon_1^2 \tilde{x}^T(t)\tilde{x}(t) + \frac{1}{\varepsilon_1^2}\sum_{i=1}^{r}\hat{\mu}_i \tilde{x}^T(t)P^T\Delta\tilde{H}_i\Delta\tilde{H}_i^T P\tilde{x}(t) \leq \varepsilon_1^2 \tilde{x}^T(t)\tilde{x}(t) + \frac{2}{\varepsilon_1^2}\sum_{i=1}^{r}\hat{\mu}_i \tilde{x}^T(t)P^T\Omega_i\Omega_i^T P\tilde{x}(t) \quad (16)$$

Similarly, we have

$$\tilde{x}^T(t)P^T\Delta\tilde{A}\tilde{x}(t) + \tilde{x}^T(t)\Delta\tilde{A}^T P\tilde{x}(t) \leq \varepsilon_2^2\tilde{x}^T(t)P^TP\tilde{x}(t) + \frac{2}{\varepsilon_2^2}\tilde{x}^T(t)\Phi^T\Phi\tilde{x}(t) \quad (17)$$

$$\tilde{x}^T(t)P^T\Delta\tilde{A}_\tau\tilde{x}(t-\tau) + \tilde{x}^T(t-\tau)\Delta\tilde{A}_\tau^T P\tilde{x}(t) \leq \varepsilon_3^2\tilde{x}^T(t)P^TP\tilde{x}(t) + \frac{2}{\varepsilon_3^2}\tilde{x}^T(t-\tau)\Psi^T\Psi\tilde{x}(t-\tau) \quad (18)$$

Denote $\xi_1(t) = \begin{bmatrix} \tilde{x}^T(t) & \tilde{x}^T(t-\tau) & \tilde{x}^T(t-\tau_{av}) \end{bmatrix}^T$. In the case of $w(t) = 0$, from (15)-(18) we obtain

$$\dot{V}(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r}\hat{\mu}_i\hat{\mu}_j\xi_1^T(t)\Upsilon_1\xi_1(t) \quad (19)$$

where

$$\Upsilon_1 = \begin{bmatrix} \bar{\omega}_{11} & \bar{\omega}_{12} & \bar{\omega}_{13} \\ * & \bar{\omega}_{22} & \tau_{av}\tilde{A}_{\tau i,1}^T Z\tilde{A}_{\tau i,2} \\ * & * & \tau_{av}\tilde{A}_{\tau i,2}^T Z\tilde{A}_{\tau i,2} - S \end{bmatrix},$$

$$\bar{\omega}_{11} = P^T\tilde{A}_{ij} + \tilde{A}_{ij}^T P + \tau_{av}X + Y + Y^T + Q + S + \varepsilon_1^2 I + \tau_{av}\tilde{A}_{ij}^T Z\tilde{A}_{ij} + (\varepsilon_2^2 + \varepsilon_3^2)P^TP + \frac{2}{\varepsilon_1^2}P^T\Omega_i\Omega_i^TP + \frac{2}{\varepsilon_2^2}\Phi^T\Phi,$$

$\bar{\omega}_{12} = P^T\tilde{A}_{\tau i,1} + \tau_{av}\tilde{A}_{ij}^T Z\tilde{A}_{\tau i,1}$,
$\bar{\omega}_{13} = P^T\tilde{A}_{\tau i,2} - Y + \tau_{av}\tilde{A}_{ij}^T Z\tilde{A}_{\tau i,2}$,
$\bar{\omega}_{22} = \frac{2}{\varepsilon_3^2}\Psi^T\Psi - Q + \tau_{av}\tilde{A}_{\tau i,1}^T Z\tilde{A}_{\tau i,1}$.

Noticing that (14) implies $\Upsilon_1 < 0$, there exists a scalar $\delta > 0$ such that

$$\Upsilon_1 + diag(\delta I, 0, 0) < 0 \quad (20)$$

From (19)-(20), we have

$$\dot{V}(t) < -\delta\tilde{x}^T(t)\tilde{x}(t)$$

which means that the system (11) with $w(t) = 0$ is asymptotically stable.
Define

$$J = \int_0^{+\infty}(r_e^T(t)r_e(t) - \gamma^2 w^T(t)w(t))dt$$

under zero initial condition, it can be shown that for any nonzero $w(t) \in \mathcal{L}_2[0,\infty)$ and $t > 0$

$$J \leq \int_0^{+\infty}(r_e^T(t)r_e(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t))dt$$

Observing that

$$(\sum_{i=1}^{r}\hat{\mu}_i\tilde{x}^T(t)\tilde{C}_{ri}^T)(\sum_{i=1}^{r}\hat{\mu}_i\tilde{C}_{ri}\tilde{x}(t)) \leq \sum_{i=1}^{r}\hat{\mu}_i\tilde{x}^T(t)\tilde{C}_{ri}^T\tilde{C}_{ri}\tilde{x}(t)$$

we have

$$r_e^T(t)r_e(t) \leq \sum_{i=1}^{r}\hat{\mu}_i\tilde{x}^T(t)\tilde{C}_{ri}^T\tilde{C}_{ri}\tilde{x}(t) \quad (21)$$

Denote

$$\xi_2(t) = \begin{bmatrix} \tilde{x}^T(t) & \tilde{x}^T(t-\tau) & \tilde{x}^T(t-\tau_{av}) & w^T(t) \end{bmatrix}^T,$$

From (15)-(18) and (21), we have

$$J \leq \sum_{i=1}^{r}\sum_{j=1}^{r}\hat{\mu}_i\hat{\mu}_j \int_0^{+\infty}\xi_2^T(t)\Upsilon_2\xi_2(t)dt \quad (22)$$

where

$$\Upsilon_2 = \begin{bmatrix} \tilde{\omega}_{11} & \tilde{\omega}_{12} & \tilde{\omega}_{13} & P^T\tilde{B}_{wi} + \tau_{av}\tilde{A}_{ij}^T Z\tilde{B}_{wi} \\ * & \tilde{\omega}_{22} & \tilde{\omega}_{23} & \tau_{av}\tilde{A}_{\tau i,1}^T Z\tilde{B}_{wi} \\ * & * & \tilde{\omega}_{33} & \tau_{av}\tilde{A}_{\tau i,2}^T Z\tilde{B}_{wi} \\ * & * & * & -\gamma^2 I + \tau_{av}\tilde{B}_{wi}^T Z\tilde{B}_{wi} \end{bmatrix},$$

$$\tilde{\omega}_{11} = P^T\tilde{A}_{ij} + \tilde{A}_{ij}^T P + \tau_{av}X + Y + Y^T + Q + S + \varepsilon_1^2 I + \tau_{av}\tilde{A}_{ij}^T Z\tilde{A}_{ij} + (\varepsilon_2^2 + \varepsilon_3^2)P^TP + \frac{2}{\varepsilon_1^2}P^T\Omega_i\Omega_i^TP + \frac{2}{\varepsilon_2^2}\Phi^T\Phi + \tilde{C}_{ri}^T\tilde{C}_{ri},$$

$\tilde{\omega}_{12} = P^T\tilde{A}_{\tau i,1} + \tau_{av}\tilde{A}_{ij}^T Z\tilde{A}_{\tau i,1}$,
$\tilde{\omega}_{23} = \tau_{av}\tilde{A}_{\tau i,1}^T Z\tilde{A}_{\tau i,2}$, $\tilde{\omega}_{33} = \tau_{av}\tilde{A}_{\tau i,2}^T Z\tilde{A}_{\tau i,2} - S$,
$\tilde{\omega}_{13} = P^T\tilde{A}_{\tau i,2} - Y + \tau_{av}\tilde{A}_{ij}^T Z\tilde{A}_{\tau i,2}$,
$\tilde{\omega}_{22} = \frac{2}{\varepsilon_3^2}\Psi^T\Psi - Q + \tau_{av}\tilde{A}_{\tau i,1}^T Z\tilde{A}_{\tau i,1}$.

Applying the Schur complement formula to (14), we obtain $\Upsilon_2 < 0$. Thus $J < 0$, i.e. $||r_e||_2 < \gamma||w||_2$, which implies system (8) is a robust H_∞-FDF of system (1). This completes the proof.

Now, we are in the position to solve the robust H_∞-FDF problem.

Theorem 2. For given scalars $\gamma > 0$ and τ satisfying (2), system (8) is a robust H_∞-FDF of system (1), if there exist scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, matrices P_1, P_2, L_i, C_{ri}, X_k, Q_k, S_k, Z_k, U_k and Y_l, $i = 1, 2, \cdots, r$, $1 \leq k \leq 3$, $1 \leq l \leq 4$, where $X_1 \geq 0$, $X_3 \geq 0$, $Q_1 > 0$, $Q_3 > 0$, $S_1 > 0$, $S_3 > 0$, $Z_1 > 0$, $Z_3 > 0$, $U_1 > 0$, $U_3 > 0$, such that the following matrix inequalities hold for $i, j = 1, 2, \cdots, r$

$$E_sP_1 \geq 0, E_s(P_2 - P_1) \geq 0 \quad (23)$$

$$\begin{bmatrix} \Lambda_{11} & X_1 + X_2^T & \Lambda_{13} & Y_1 + Y_3 \\ * & X_1 & Y_1 + Y_2 & Y_1 \\ * & * & \Lambda_{33} & \Lambda_{34} \\ * & * & * & \Lambda_{44} \end{bmatrix} \geq 0 \quad (24)$$

$$\begin{bmatrix} \Xi_{11} & Z_1\hat{M}U_2\hat{M}^T + Z_2\hat{M}U_3\hat{M}^T \\ \Xi_{21} & Z_2^T\hat{M}U_2\hat{M}^T + Z_3\hat{M}U_3\hat{M}^T \end{bmatrix} = I \quad (25)$$

$$\begin{bmatrix} \Gamma_{11} & \tau_{av}X_1 + \tau_{av}X_2^T + Y_1 + Y_1^T + Y_2 + Y_3 \\ * & \tau_{av}X_1 + Y_1 + Y_1^T \end{bmatrix} \geq 0 \quad (26)$$

$$\begin{bmatrix} \eta_{11} & \eta_{12} & \eta_{13} & \eta_{14} & \eta_{15} & \eta_{16} & \eta_{17} & \eta_{18} & \eta_{19} \\ * & \eta_{22} & \eta_{23} & \eta_{24} & \eta_{25} & \eta_{26} & \eta_{27} & \eta_{28} & \eta_{29} \\ * & * & \eta_{33} & \eta_{34} & 0 & 0 & 0 & \eta_{38} & \eta_{39} \\ * & * & * & -Q_1 & 0 & 0 & 0 & \eta_{48} & \eta_{49} \\ * & * & * & * & \eta_{55} & \eta_{56} & 0 & 0 & 0 \\ * & * & * & * & * & -S_1 & 0 & 0 & \eta_{69} \\ * & * & * & * & * & * & -\gamma^2 I & 0 & \eta_{79} \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & \eta_{99} \end{bmatrix} < 0 \quad (27)$$

where

$\Lambda_{11} = X_1 + X_2 + X_2^T + X_3$, $\Lambda_{13} = Y_1 + Y_2 + Y_3 + Y_4$,
$\Lambda_{33} = E_sP_1\hat{M}U_1\hat{M}^TP_1^TE_s$,
$\Lambda_{34} = E_sP_1\hat{M}U_1\hat{M}^TP_2^TE_s + E_sP_1\hat{M}U_2\hat{M}^T(P_1^T - P_2^T)E_s$,

$$\Lambda_{44} = E_s(P_1 - P_2)\hat{M}U_2^T\hat{M}^TP_2^TE_s + E_sP_2\hat{M}U_2 \times\hat{M}^T(P_1^T - P_2^T)E_s + E_sP_2\hat{M}U_1\hat{M}^TP_2^TE_s + E_s(P_1 - P_2)\hat{M}U_3\hat{M}^T(P_1^T - P_2^T)E_s,$$

$\hat{M} = \begin{bmatrix} \hat{M}_{ij} \end{bmatrix}_{3\times3}$, $\hat{M}_{11}$, $\hat{M}_{23}$ and $\hat{M}_{32}$ are identity matrices with appropriate dimensions, other block matrices are zero matrices.

$$\Gamma_{11} = \tau_{av}X_1 + \tau_{av}X_2 + \tau_{av}X_2^T + \tau_{av}X_3 + Y_1 + Y_1^T + Y_2 + Y_2^T + Y_3 + Y_3^T + Y_4 + Y_4^T,$$

$$\begin{aligned}
\eta_{11} &= P_1^T A_{si} + A_{si}^T P_1 + 2\varepsilon_1^2 I + Q_1 + Q_2 + Q_2^T \\
&\quad + Q_3 + \tau_{av} X_1 + \tau_{av} X_2 + \tau_{av} X_2^T + \tau_{av} X_3 \\
&\quad + S_1 + S_2 + S_2^T + S_3 + Y_1 + Y_2 + Y_3 + Y_4 \\
&\quad + Y_1^T + Y_2^T + Y_3^T + Y_4^T, \\
\eta_{12} &= P_1^T A_{si} + A_{si}^T P_1 + C_{sj}^T L_i + \varepsilon_1^2 I + Q_1 + Q_2^T \\
&\quad + \tau_{av} X_1 + \tau_{av} X_2^T + S_1 + S_2^T + Y_1 + Y_1^T \\
&\quad + Y_2^T + Y_3, \\
\eta_{13} &= \eta_{14} = \eta_{23} = \eta_{24} = P_1^T A_{\tau si}, \eta_{16} = -Y_1 - Y_3, \\
\eta_{15} &= -Y_1 - Y_2 - Y_3 - Y_4,\ \eta_{17} = P_1^T B_s, \\
\eta_{18} &= \left[\varepsilon_2 P_1^T \ 0 \ \varepsilon_3 P_1^T \ 0 \ 0 \ \frac{\sqrt{2}}{\varepsilon_2} \hat{A}^T \ -C_{sf}^T \ 0 \right], \\
\eta_{19} &= \left[\tau_{av} A_{si}^T P_1 \ \tau_{av}(A_{si}^T P_1 + C_{sj}^T L_i) \ 0 \ 0 \right. \\
&\qquad \left. 0 \ 0 \ 0 \ 0 \right], \\
\eta_{22} &= P_2^T A_{si} + A_{si}^T P_2 + \varepsilon_1^2 I + Q_1 + S_1 + \tau_{av} X_1 \\
&\quad + Y_1 + Y_1^T, \\
\eta_{25} &= -Y_1 - Y_2,\ \eta_{26} = P_2^T A_{\tau si} - P_1^T A_{\tau si} - Y_1, \\
\eta_{27} &= P_1^T B_s + L_i^T D_s, \eta_{33} = -Q_1 - Q_2 - Q_2^T - Q_3, \\
\eta_{28} &= \left[\varepsilon_2 P_2^T \ \varepsilon_2(P_1^T - P_2^T) \ \varepsilon_3 P_2^T \ \varepsilon_3(P_1^T - P_2^T) \right. \\
&\qquad \left. -\frac{\sqrt{2}}{\varepsilon_1} L_i^T \hat{C} \ \frac{\sqrt{2}}{\varepsilon_2} \hat{A}^T \ C_{ri}^T - C_{sf}^T \ 0 \right], \\
\eta_{29} &= \left[\tau_{av} A_{si}^T P_1 \ \tau_{av} A_{si}^T P_2 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \right], \\
\eta_{38} &= \eta_{48} = \left[0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \frac{\sqrt{2}}{\varepsilon_3} \hat{A}_\tau^T \right], \\
\eta_{39} &= \eta_{49} = \left[\tau_{av} A_{\tau si}^T P_1 \ \tau_{av} A_{\tau si}^T P_1 \ \tau_{av} A_{\tau si}^T P_1 \right. \\
&\qquad \left. \tau_{av} A_{\tau si}^T P_1 \ 0 \ 0 \ 0 \ 0 \right], \\
\eta_{55} &= -S_1 - S_2 - S_2^T - S_3,\ \eta_{56} = -S_1 - S_2^T, \\
\eta_{69} &= \left[0 \ \tau_{av} A_{\tau si}^T (P_2 - P_1) \ 0 \ \tau_{av} A_{\tau si}^T (P_2 - P_1) \right. \\
&\qquad \left. 0 \ \tau_{av} A_{\tau si}^T (P_2 - P_1) \ 0 \ 0 \right], \\
\eta_{79} &= \left[\tau_{av} B_s^T P_1 \ \tau_{av} T_1 \ \tau_{av} B_s^T P_1 \ \tau_{av} T_1 \right. \\
&\qquad \left. \tau_{av} B_s^T P_1 \ \tau_{av} T_1 \ \tau_{av} B_s^T P_1 \ \tau_{av} T_1 \right], \\
T_1 &= B_s^T P_1 + D_s^T L_i,\ \eta_{34} = -Q_1 - Q_2^T, \\
\eta_{99} &= diag(T_2, T_2, T_2, T_2), \\
T_2 &= \begin{bmatrix} -\tau_{av}(Z_1 + Z_2 + Z_2^T + Z_3) & -\tau_{av}(Z_1 + Z_2^T) \\ * & -\tau_{av} Z_1 \end{bmatrix}.
\end{aligned}$$

In this case, a desired robust H_∞-FDF is given in the form of (8) with parameters as follows

$$H_i = (P_2 - P_1)^{-T} L_i^T, i = 1, 2, \cdots, r \tag{28}$$

Proof. Suppose (23)-(27) hold and the coefficient matrices of the filter (8) are designed in the form of (28). Now we will prove that there exist matrices $Q > 0$, $S > 0$, $X \geq 0$, $Z > 0$ and P, Y satisfying (12)-(14). Denote $P_1 = \left[\tilde{P}_{ij} \right]_{3\times 3}$. From (23) we have $\tilde{P}_{12} = 0$, $\tilde{P}_{32} = 0$ and $\tilde{P}_{31} = \tilde{P}_{13}^T$. Then we obtain $P_1^T E_s = E_s P_1 \geq 0$, $P_2^T E_s = E_s P_2 \geq 0$.

From (26)-(27), it is easy to prove that P_1 is nonsingular. Without loss of generality, it is assumed that $P_1 - P_2$ is nonsingular (Shengyuan *et al.*, 2003). Denote $P = \Pi_2 \Pi_1^{-1}$, where $\Pi_1 = \begin{bmatrix} P_1^{-1} & I \\ P_1^{-1} & 0 \end{bmatrix}$, $\Pi_2 = \begin{bmatrix} I & P_2 \\ 0 & P_1 - P_2 \end{bmatrix}$. Then using theorem 1 in (Shengyuan *et al.*, 2003), we obtain $P = \begin{bmatrix} P_2 & P_1 - P_2 \\ * & -(P_1 - P_2) \end{bmatrix}$ is nonsingular and satisfies (12).

Denote $Q = \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix}$, $S = \begin{bmatrix} S_1 & S_2 \\ * & S_3 \end{bmatrix}$, $X = \begin{bmatrix} X_1 & X_2 \\ * & X_3 \end{bmatrix}$, $Y = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix}$, $\bar{Z} = \begin{bmatrix} Z_1 & Z_2 \\ * & Z_3 \end{bmatrix}$, $Z^{-1} = P^{-T} \bar{Z} P^{-1}$. Pre- and post-multiplying (27) by $diag(P_1^{-T}, I, P_1^{-T}, I, P_1^{-T}, I, I, I, P_1^{-T}, I, P_1^{-T}, I, P_1^{-T}, I, P_1^{-T}, I)$ and its transpose, it can be shown that the LMIs in (27) are equivalent to the LMIs in (14). Similarly, it can be proved that the matrix inequalities in (24)-(25) is equivalent to the matrix inequality in (13). From theorem 1, we know that system (8) is a robust H_∞-FDF of system (1). This completes the proof.

Remark 3. The introducing of the parameters ε_1, ε_2, ε_3 and the matrices X, Y, Z is used to make the matrix inequalities in theorem 1 less conservative.

Remark 4. It is clear that the nonlinear terms in (24)-(25) make that (24)-(25) are not conformable to LMIs. However, by using the cone complementarity linearization iterative algorithm proposed in (Ghaoui et al., 1997) by minor modification, we can convert (24)-(25) to solving a sequence of convex optimization problems subject to LMIs.

4. CONCLUSION

In this paper, fault estimation is adopted as the residual. The robust residual generator design problem for singular time-delay nonlinear systems is converted to a H_∞ filtering problem. Sufficient conditions are given, which guarantee the robust H_∞ fault detection filter exists. And by using the cone complementarity linearization iterative algorithm, the fault detection filter design is converted to solving a sequence of convex optimization problems subject to LMIs. The premise variables of the residual generator are chosen to be the estimated premise variables of the plant so that the premise variables of the residual generator are not demanded to be measurable.

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