

## Modelling and control of the air system of a turbocharged gasoline engine

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**Abstract:** This paper investigates the modelling and the control of a turbocharged air system of a gasoline engine. The purpose of the work described here is to propose a new control strategy based on an original physical model of the system. This first part describes the development of a simple model of the system. Based on a complete representation of the system, some simplifications and assumptions are proposed in order to obtain a model with the adequate level of complexity for an integration in a control law. We describe a model based innovative control strategy. Experimental results are proposed on a 4 cylinder turbocharged gasoline engine. Conclusions stress the possibility of taking into account the model of this system by a simple, yet efficient in practice, control law.

**Keywords:** Engine control, Engine modelling, Model-based control, Turbochargers.

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### 1. INTRODUCTION

In the search for a reduction in the CO<sub>2</sub> automotive engines emissions, the downsizing approach seems to be a promising solution. The purpose is to decrease the volumetric capacity of the engine, and hence improve its efficiency chain via a diminution in friction and pumping losses. This is particularly the case for gasoline engines on which we will concentrate in this paper, even though this technique can also show some advantages for diesel technologies. In downsized engines, the lack of power induced by the reduction of air charge capacity is compensated for by the use of supercharging or, more specifically, turbocharging. Therefore, the turbocharger is a key component of the system, which brings two drawbacks. Indeed, its function is to use some of the engine exhaust energy, otherwise lost, to compress the gases at the engine intake. At low engine speeds, the exhaust energy available is limited, which results in insufficient supercharging capabilities and slow time responses. These issues are partly addressed through an increase in the complexity of the engine architecture (variable valve timing, direct injection). In order to obtain the best performances of the system, a fast and accurate control strategy is also needed.

The standard controllers are composed mostly by linear controller. They exhibit some disadvantages linked to the application of linear control techniques to non linear systems : the robustness/performance compromise is suboptimal on the whole operating range of the system. These issues are partly corrected by the use of gain scheduling techniques and the addition of static feed forward terms leading to a high calibration effort. These are based on

parameters given by maps which depend on the system state.

For diesel technologies, the complexity of the turbocharging systems is also growing, leading to the development of new control strategies. The introduction of model based structures seems to be a promising way to improve the standard linear controllers with feed forward. Some works have already been published around this topic proposing tuning methods for PI controllers Däubler et al. [2006], or new non linear strategies (Schwarzmann et al. [2006], Stefanopoulou et al. [2000]). In a gasoline context, the problem is slightly different : the system is simpler, but less sensors are available. The control solutions investigated must take account of these differences and address the specific issues. A novel strategy was proposed in a diesel engine technology context in Youssef et al. [2007]. This paper proposes to extend this work to a gasoline context with simplifications made on the model which lead to a different strategy.

The plan of this paper is as follows : after a description of the context in terms of system studied and objectives of the work, a detailed model is described. This model is then simplified and validated against experimental test data. The control strategy designed from this simple model is then detailed, and experimental validation results are shown.

### 2. SYSTEM DESCRIPTION

The engine considered in this paper is a four cylinder turbocharged gasoline engine. Figure 1 shows the architecture of the system. Fresh air enters in the engine through the compressor which increases the air density. This air is used as a comburant in the cylinder where the combustion occurs, resulting in the production of mechanical torque. The remaining energy contained in the gas in the form

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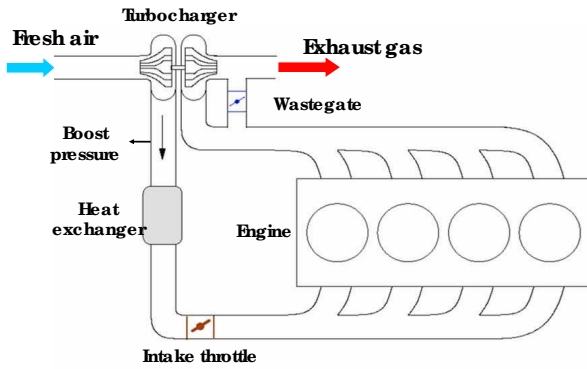


Figure 1. Engine scheme.

of enthalpy exits the system through the turbine. Part of the gas enthalpy is then converted into mechanical power on the turbocharger shaft, whose dynamics are the consequences of the balance between the compressor and turbine energies. Two actuators are available on the air system :

- The intake throttle. It allows to reach low pressure in the intake manifold.
- The wastegate valve. Its function is to divert from the turbine part of the exhaust gas. When this valve is opened, the turbine mass flow and enthalpy are reduced, and so is the energy provided to the turbocharger shaft.

The sensors available on the system are the following :

- Compressor downstream pressure,  $P_{dc}$ .
- Compressor upstream pressure and temperature,  $P_{uc}$  and  $T_{uc}$ .
- Engine intake manifold pressure and temperature,  $P_{man}$  and  $T_{man}$ .

Under turbocharging operation, the throttle is normally opened in order to prevent from losses.

### 3. MODELLING AND CONTROL OBJECTIVES

First of all, the objective of the model development described here is to provide a basis for the design of a control strategy. The difficulty of this task consist in keeping the right level of complexity. Two main criteria will be considered :

- The model has to represent only the main dynamics governing the evolution of the system, the fast dynamics being neglected.
- In an engine, the evolution of a turbocharger depends on the conditions at its boundaries : pressures and temperatures upstream and downstream the compressor and turbine, gas mass flow through these components. Since the model will be used online in a ECU, these influences have to be represented only if it is possible to measure or estimate them with the available sensors.

As a consequence of these two criteria, some assumptions will have to be made. They will be justified by a compari-

son between experimental test data and the results of the model.

The goal of the air path management is to control the intake manifold pressure. The intake throttle allows a fast and direct action on this variable, but is constrained by its upstream pressure at the compressor outlet. The turbocharger control strategy therefore actuates the wastegate in order to follow a compressor downstream pressure set-point. Additionally than tracking the pressure set-point, some constraints have to be followed :

- In steady state, the pressure drop through the throttle has to be minimized in order to avoid energy losses. A consequence is that when operating at high loads the throttle will be fully opened.
- The control law has to be robust with respect to environmental conditions changes : the thermodynamic conditions at the boundaries of the system will affect its behavior.
- The speed of the turbocharger shaft has to be maintained below a maximum value.

In order to satisfy the first constraint, the supercharging pressure set-point will be taken equal to the intake manifold pressure set-point. The throttle will then be automatically opened in steady state. For the following of this paper, we will consider that this condition is true, and that the throttle is open. It follows that  $P_{dc} = P_{man}$ .

## 4. TURBOCHARGED ENGINE MODEL

Most of the equations governing the behavior of the turbocompressor can be found in other publications (see for example Moraal and Kolmanovsky [1999] , Sorenson et al. [2005] or Eriksson [2006]). The novelty of the approach presented here lies in the simplification proposed further and the control strategy designed from the simplified model.

### 4.1 Turbocharger modeling

The turbocharger is composed by a turbine driven by the exhaust gas and connected via a common shaft to the compressor, which compresses the air in the intake. The rotational speed of the turbocharger shaft  $N_t$  can be derived from a power balance between the turbine  $\mathcal{P}_t$  and the compressor side  $\mathcal{P}_c$

$$\frac{d}{dt} \left( \frac{1}{2} J_t N_t^2 \right) = \mathcal{P}_t - \mathcal{P}_c \quad (1)$$

where  $J_t$  is the inertia of the turbocharger.

*Compressor* In order to derive an equation for the compressor power, the first law of thermodynamics is applied. It states that (neglecting heat losses) the compressor power is related to the mass flow through the compressor  $D_c$  and the total change of enthalpy by  $\mathcal{P}_c = D_c c_p (T_{dc} - T_{uc})$ . The compressor efficiency is introduced as the ratio between isentropic and actual compression powers. The compressor power reads

$$\mathcal{P}_c = D_c c_p T_{amb} \frac{1}{\eta_c} \left( \Pi_c^{\frac{\gamma-1}{\gamma}} - 1 \right) \quad (2)$$

where  $\eta_c$  is the compressor efficiency,  $\Pi_c \triangleq \frac{P_{dc}}{P_{uc}}$  the compressor pressure ratio, and  $\gamma$  the specific heat ratio.

The compressor speed, flow, pressure ratio and efficiency are linked. Different representations can be found in the literature, among which a commonly used one consists in mapping the pressure ratio and efficiency against flow and speed. These maps are extrapolated from data measured during characterization tests. Several extrapolation methods have been proposed (for example Jensen et al. [1991]). In order to take account of the variations in the compressor upstream conditions, these variables are corrected as follow

$$D_{c,cor} = D_c \frac{\sqrt{T_{uc}}}{P_{uc}} \quad \text{and} \quad N_{c,cor} = \frac{N_t}{\sqrt{T_{uc}}}$$

and

$$\begin{cases} \Pi_c = \phi_{\Pi_c}(D_{c,cor}, N_{c,cor}) \\ \eta_c = \phi_{\eta_c}(D_{c,cor}, N_{c,cor}) \end{cases} \quad (3)$$

The compressor pressure ratio and efficiency corresponding to the system studied here are represented in Figures 2 and 3.

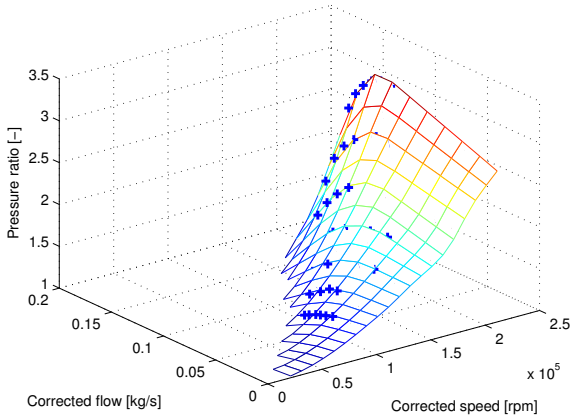


Figure 2. Compressor map. Pressure ratio  $\Pi_c$  at the compressor w.r.t. the corrected flow through the compressor  $D_{c,cor}$  and the corrected turbine crankshaft speed  $N_{c,cor}$ . The blue crosses show the characterization measurements.

*Turbine* Similarly, the turbocharger power is related to the mass flow through the turbine  $D_t$  and the total change of enthalpy. This results in

$$P_t = D_t c_p T_{ut} \eta_t \left( 1 - \Pi_t^{\frac{1-\gamma}{\gamma}} \right) \quad (4)$$

where  $\eta_t$  is the turbine efficiency,  $T_{dt}$  and  $P_{dt}$  are the temperature and pressure after the turbine,  $P_{ut}$  the exhaust manifold pressure,  $\Pi_t \triangleq \frac{P_{dt}}{P_{ut}}$  is the turbine pressure ratio and  $\gamma$  the specific heat ratio. In this case, the corrected turbine flow  $D_{t,cor}$  and isentropic efficiency  $\eta_t$  are mapped versus the pressure ratio across the turbine and the corrected turbocharger shaft speed  $N_{t,cor}$ . As for compressor maps, different methods have been proposed to obtain these maps from test data (see Moraal and Kolmanovsky [1999]). The corrected variables are defined as :

$$D_{t,cor} = D_t \frac{\sqrt{T_{ut}}}{P_{ut}} \quad \text{and} \quad N_{t,cor} = \frac{N_t}{\sqrt{T_{ut}}}$$

and

$$\begin{cases} D_{t,cor} = \phi_{D_t}(\Pi_t, N_{t,cor}) \\ \eta_t = \phi_{\eta_t}(D_{t,cor}, N_{t,cor}) \end{cases} \quad (5)$$

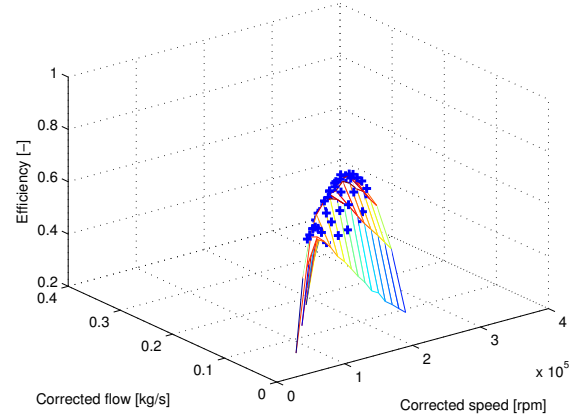


Figure 3. Compressor map. Efficiency  $\eta_c$  at the compressor w.r.t. the corrected flow through the compressor  $D_{c,cor}$  and the corrected turbine crankshaft speed  $N_{c,cor}$ . The blue crosses show the characterization measurements.

This can be rearranged in the following form, for more commodity :

$$D_t = \phi_{D_t}(\Pi_t, N_{t,cor}) \Pi_t \frac{P_{dt}}{\sqrt{T_{ut}}} \quad (6)$$

*Wastegate flow* The discharge valve, named waste gate, is used to control the flow through the turbine, and thus the turbocharger. It can be modelled with the standard equations of compressible gas flow through an orifice.

$$D_{wg} = S_{wg} \phi_{wg}(\Pi_t) \frac{P_{dt}}{\sqrt{T_{ut}}}$$

where

$$\phi_{wg}(\Pi_t) \triangleq S_{wg} \frac{\Pi_t}{\sqrt{R}} \sqrt{\frac{2\gamma}{\gamma-1} \left( \Pi_t^{\frac{-2}{\gamma}} - \Pi_t^{\frac{-\gamma-1}{\gamma}} \right)}$$

#### 4.2 Engine modelling

Conventionally (see Heywood [1988] for exemple), we assume that the aspirated flow  $D_{asp}$  can be computed as

$$D_{asp} = \eta_v \Pi_c \Psi \quad (7)$$

where

$$\Psi \triangleq \frac{V_{cyl} P_{uc} N_e}{RT_{int} 120}$$

and  $V_{cyl}$  is the cylinder volume.  $\eta_v$  is the volumetric efficiency. Classically, it is experimentally derived and, eventually, given by a look-up table  $\eta_v(P_{dc}, N_e)$ . This also depends on the engine valve timing. Indeed, the variation of camshaft timing change the exhaust manifold pressure and the residual gas mass trapped inside the cylinder at intake valve closing. Since the engine is operated at stoichiometry, the fuel mass flow  $D_f$  can be given by :

#### 4.3 Intake and exhaust modelling

We consider the exhaust and intake manifolds as a fixed volume for which the thermodynamics states (pressure,

temperature, composition) are assumed to be homogeneous. Under turbocharging operations, the air throttle is wide open. The entire volume between the compressor and the engine can be lumped into a single volume. The mass balance in this volume and in the exhaust manifold leads to

$$\begin{cases} \frac{dP_{ut}}{dt} = \frac{RT_{ut}}{V_{ut}}(D_{asp} + D_f - D_t - D_{wg}) \\ \frac{dP_{dc}}{dt} = \frac{RT_{dc}}{V_{dc}}(D_c - D_{asp}) \end{cases} \quad (8)$$

#### 4.4 Summary

Gathering equations (1)-(8) leads to the following dynamics

$$\begin{cases} \frac{d}{dt} \left( \frac{1}{2} J_t N_t^2 \right) = \phi_{D_t}(\Pi_t, N_t, cor) c_p P_{dt} \sqrt{T_{ut}} \eta_t \phi_t(\Pi_t) \\ \quad - D_c c_p T_{uc} \frac{1}{\eta_c} \phi_c(\Pi_c) \\ \frac{dP_{ut}}{dt} = \frac{RT_{ut}}{V_{ut}} \left( (1 + \lambda_s) \eta_v \Pi_c \Psi \right. \\ \quad \left. - (\phi_{D_t}(\Pi_t, N_t) \Pi_t + S_{wg} \phi_{wg}(\Pi_t)) \frac{P_{dt}}{\sqrt{T_{ut}}} \right) \\ \frac{dP_{dc}}{dt} = \frac{RT_{dc}}{V_{dc}} (D_c - \eta_v \Pi_c \Psi) \end{cases} \quad (9)$$

where  $\phi_t(\Pi) \triangleq \Pi(1 - \Pi^{\frac{1-\gamma}{\gamma}})$  and  $\phi_c(\Pi) \triangleq \Pi^{\frac{\gamma-1}{\gamma}} - 1$ . This model takes account of parameters external to the turbocharger itself : temperatures upstream the compressor and turbine, pressure downstream the turbine. However, it contains three states. Since the ultimate purpose of this work is to design a model based control law, further simplifications have to be undertaken. Different types of assumptions will be made and verified empirically.

### 5. MODEL REDUCTION

The first type of assumptions concern the dynamics. The second type concern the steady state dependencies. The purpose is to keep only the relevant dynamics of the system, and parameters that can be measured or estimated from the available sensors.

#### 5.1 Model simplification by singular perturbation

The third order nonlinear system (9) accurately describes the dynamics of the system. However, one can notice that the turbocharger speed is much slower than the pressure dynamics. Indeed, typically we have  $\frac{V_{ut}}{RT_{ut}} \simeq 5e - 9$ ,  $\frac{V_{dc}}{RT_{dc}} \simeq 5e - 8$  and  $J_t = 3e - 5$ . This suggests to simplify these dynamics with a singular perturbation method Khalil [1992]. Let  $\epsilon \triangleq \frac{V_{ut}}{RT_{ut}}$  be a scalar that represents all the small parameters to be neglected. The reference dynamics (9) has the form of the standard singularly perturbed system

$$\begin{cases} \dot{z}_1 = \phi(z_1, z_2, u) \\ \epsilon \dot{z}_2 = \psi(z_1, z_2, \epsilon) \end{cases} \quad (10)$$

where  $z_1 \triangleq N_t$ ,  $z_2 \triangleq [P_{ut} \ P_{dc}]^T$ . Noting the time constants  $\tau_{dc} \frac{V_{dc}}{RT_{dc}} = \mathcal{O}(\epsilon)$  i.e.  $\tau_{dc} = k_{dc}(\epsilon)\epsilon$  with  $\lim_{\epsilon \rightarrow 0} k_{dc}(\epsilon) = \bar{k}_{dc} > 0$ , we have

$$\psi(z_1, z_2, \epsilon) = \begin{bmatrix} (1 + \lambda_s) \eta_v \Pi_c \Psi - (\phi_{D_t}(\Pi_t, N_t) \Pi_t + S_{wg} \phi_{wg}(\Pi_t)) \frac{P_{dt}}{\sqrt{T_{ut}}} \\ \frac{1}{k_{dc}(\epsilon)} (D_c - \eta_v \Pi_c \Psi) \end{bmatrix}$$

The equation  $\psi(z_1, z_2, 0) = 0$  has a unique root of interest  $z_2 = h(z_1)$ . In details, it is

$$\begin{cases} (1 + \lambda_s) \eta_v \Pi_c \Psi = (\phi_{D_t}(\Pi_t, N_t) \Pi_t + S_{wg} \phi_{wg}(\Pi_t)) \frac{P_{dt}}{\sqrt{T_{ut}}} \\ D_c = \eta_v \Pi_c \Psi \end{cases}$$

To ensure the validity of the simplification, we can check the uniform stability of the Jacobian of  $\psi$  Kokotović et al. [1999][Assumption 3.2 p11]. For that, we consider  $\partial_{z_2} \psi|_{z_2=h(z_1)}$  and compute its eigenvalues

$$\partial_{z_2} \psi|_{z_2=h(z_1)} = \begin{bmatrix} -(\phi_{D_t}(\Pi_t) + S_{wg} \phi'_{wg}(\Pi_t)) (1 + \lambda_s) \partial_{\Pi_c} \eta_v \Pi_c \Psi \\ 0 & -\partial_{\Pi_c} \eta_v \Pi_c \Psi \end{bmatrix}$$

Since  $S_t$  is strictly positively bounded, there exists  $c > 0$  such that  $\mathcal{R}e(\partial_{z_2} \psi|_{z_2=h(z_1)}) < -c$ . The reduced dynamics writes

$$\begin{cases} \dot{\bar{z}}_1 = \phi(\bar{z}_1, h(\bar{z}_1), u) \\ \bar{z}_2 = h(\bar{z}_1) \end{cases} \quad (11)$$

From Khalil [1992][Th 11.1], the following proposition holds

*Proposition 1.* Consider the singularly perturbed system (10) and  $z_2 = h(z_1)$  the isolated root of  $\psi(z_1, z_2) = 0$ . There exists a positive constant  $\epsilon^* > \epsilon > 0$  such that (10) possesses a unique trajectory  $z_1(t, \epsilon)$ ,  $z_2(t, \epsilon)$ , and

$$\begin{aligned} z_1(t, \epsilon) - \bar{z}_1(t) &= \mathcal{O}(\epsilon) \\ z_2(t, \epsilon) - h(\bar{z}_1(t)) &= \mathcal{O}(\epsilon) \end{aligned}$$

hold when  $\epsilon < \epsilon^*$ .

Thus, the new reference system writes

$$\begin{cases} \frac{d}{dt} \left( \frac{1}{2} J_t N_t^2 \right) = \phi_{D_t}(\Pi_t, N_t) c_p P_{dt} \sqrt{T_{ut}} \eta_t \phi_t(\Pi_t) \\ \quad - \eta_v \Pi_c \Psi c_p T_{uc} \frac{1}{\eta_c} \phi_c(\Pi_c) \\ (1 + \lambda_s) \eta_v \Pi_c \Psi = (\phi_{D_t}(\Pi_t, N_t) \Pi_t + S_{wg} \phi_{wg}(\Pi_t)) \frac{P_{dt}}{\sqrt{T_{ut}}} \end{cases} \quad (12)$$

#### 5.2 Turbine flow simplification

The turbine can be considered as a restriction on the exhaust gas flow. However, the standard equation for compressible flow across an orifice cannot be applied in this case. Modified versions of this equation have been proposed which fit better the experimental results, based on various assumptions (see Eriksson [2006]). Most of them neglect the influence of the turbine speed. The formula kept in our case is given below, the justification being that it shows a good correlation with the characterization data (see Figure 4).

$$D_t = \frac{P_{dt}}{\sqrt{T_{ut}}} \phi_{turb}(\Pi_t)$$

where

$$\phi_{turb}(\Pi_t) \triangleq S_t \frac{\Pi_t^{\frac{3}{2}}}{\sqrt{R}} \sqrt{\frac{2\gamma}{\gamma-1} \left( \Pi_t^{\frac{-2}{\gamma}} - \Pi_t^{\frac{-\gamma-1}{\gamma}} \right)}$$

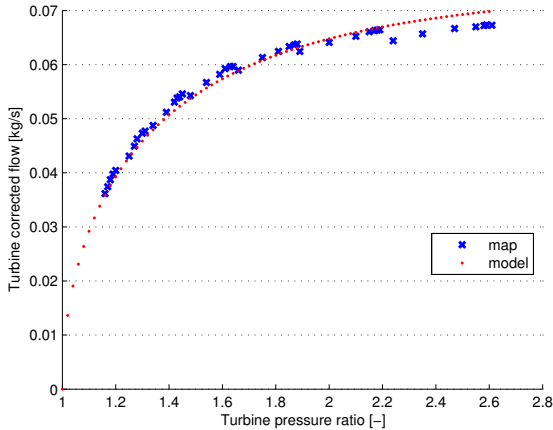


Figure 4. Comparison between turbine characterization data and simplified model.

### 5.3 Correlation between the turbocharger speed and the intake pressure

For given engine operating conditions, the turbocharger speed and the intake pressure are very correlated. Since we consider that the mass flow through the compressor is equal to the aspirated mass flow ( $D_c = D_{asp}$ ), it is therefore interesting to consider the combination of (3) and (7). The corrected compressor flow depends on the compressor pressure ratio, the engine speed and the operating conditions, and :

$$\Pi_c = \phi_{\Pi,c}(\eta_v(\Pi_c P_{uc}, N_e)\Pi_c \Psi, \frac{N_t}{\sqrt{T_{uc}}}) \quad (13)$$

This expression is remarkable since it shows a direct dependency between the compressor pressure ratio and the engine speed. The influence of the intake temperature is of second order and will be neglected. The following graph shows experimental measurements. As experimentally rep-

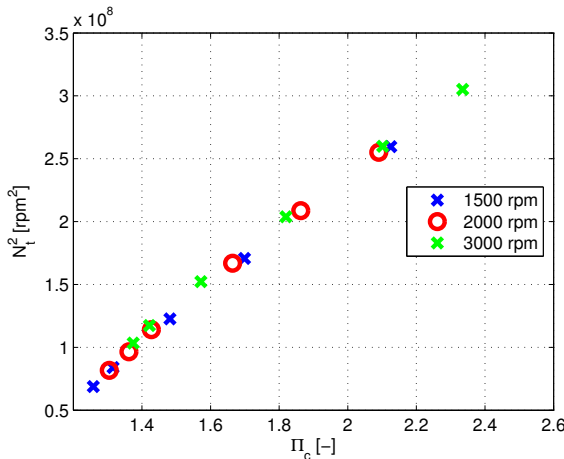


Figure 5. Experimental results at steady state. Variation of the turbocharger speed square  $N_t^2$  w.r.t. the compressor pressure ratio  $\Pi_c$ .

resented on Figure 5, we can estimate the turbocharger speed square  $N_t^2$  linearly w.r.t. the compressor pressure ratio  $\Pi_c$ , i.e.

$$N_t^2 = a\Pi_c + b$$

### 5.4 Steady state assumptions

The system dynamics depend on a lot of different variables that physically are related to the engine (engine speed, volumetric efficiency) or the environment (compressor upstream pressure and temperature, turbine downstream pressure). Since they are external to the turbocharger, we will make the assumption that they depend on the operating point of the engine. They can either be measured or estimated based on steady state maps.

The only remaining unknown terms in the system of equations 12 are  $\eta_c$  and  $\eta_t$ . Since they vary in small proportions on the engine operating points, we will also consider that they can be mapped as functions of the engine operating conditions. It is difficult to validate it in transient since it is not possible to measure the efficiencies in this case. The correct behavior of the control law designed from these assumptions will validate them a posteriori.

### 5.5 Reference system

The linear correlation between compressor pressure ratio and turbocharger kinetic energy (13) considerably simplifies the studied system. The state variable can be chosen as the compressor pressure ratio, and the turbocharger speed does not appear any more in the equations.

The reference system writes

$$\begin{cases} \dot{\Pi}_c = \alpha_1 \psi_t(\Pi_t) - \alpha_2 \psi_c(\Pi_c) \\ \Pi_c = \alpha_3 (\phi_{turb}(\Pi_t) + u S_{wg,max} \phi_{wg}(\Pi_t)) \end{cases} \quad (14)$$

where  $\alpha_i$  depend on the engine operating conditions :

$$\begin{cases} \alpha_1 = c_p \sqrt{T_{ut}} P_{dt} \eta_t \frac{2}{J_t a} \\ \alpha_2 = \eta_v \Psi N_e C_p T_{uc} \frac{1}{\eta_c} \frac{2}{J_t a} \\ \alpha_3 = \frac{P_{dt}}{\sqrt{T_{ut}} (1 + \lambda_s) N_e \Psi \eta_v} \end{cases}$$

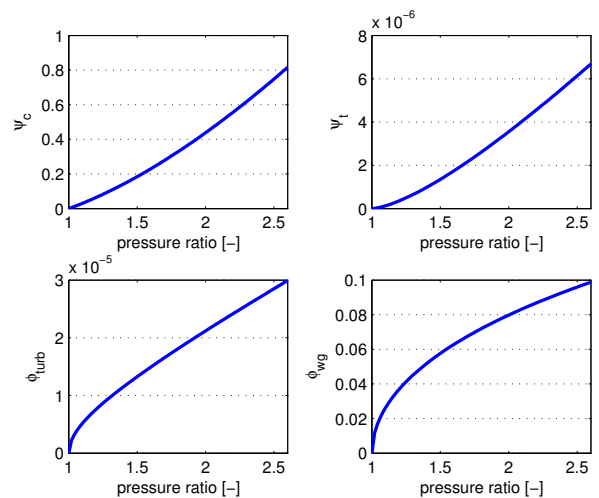


Figure 6. Functions  $\psi_c$ ,  $\psi_t$ ,  $\phi_{turb}$ ,  $\phi_{wg}$ .

The first equation of system 14 represents the balance between compressor and turbine mechanical power, giving the dynamics of the system. The second equation represents the mass conservation in the exhaust manifold, the dynamics being neglected. The wastegate section  $S_{wg}$  is the command of the system. It has been normalized for simplification :  $S_{wg} \triangleq u S_{wg,max}$ , with the constraints :  $0 \leq u \leq 1$ .

The functions  $\phi_{turb}$ ,  $\phi_{wg}$ ,  $\psi_c$  and  $\psi_t$  are represented in Figure 6. These functions are nonlinear but invertible. This property is very important and will be used when designing the control law.

The coefficients  $\alpha_i$  can be computed from sensors available on the engine. In the following figures they are plotted for different engine operating points extracted from a test campaign corresponding to the complete turbocharging zone, for which the assumptions made before are valid. The  $\alpha_i$  are represented as functions of engine speed because this variable is the most relevant to define the operating point of the system. It can be noticed that for a given engine speed the variation of these coefficients is limited.

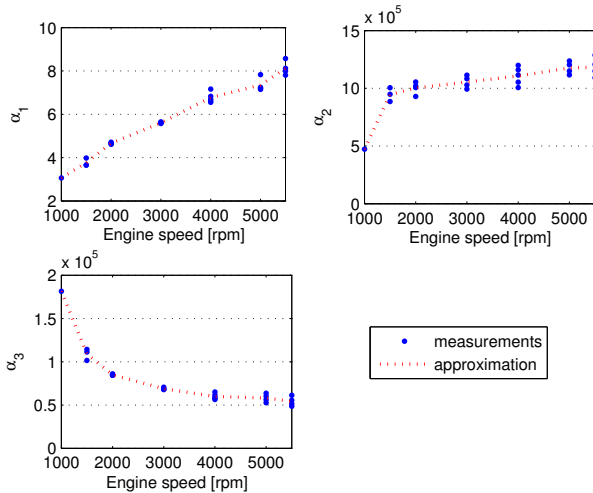


Figure 7. Coefficient  $\alpha_i$  computed from test data for different turbocharging operating points.

### 5.6 Model validation

Figure 8 shows a comparison between experimental data and results obtained from equations (14) during a turbocharger transient. The experiment was performed on an engine testbed, at a constant engine speed of 2000rpm. The input of the model are the sensors available on the engine, and the command applied on the wastegate. The model is valid after the throttle is closed ( $t = 1$ ). The response time of the turbocharger is well represented, even if some higher dynamics are missing, and

## 6. CONTROL STRATEGY

We now focus on the controller design. We proceed in two steps. First, we design an open loop controller, and then, we complement it with a tracking feedback controller.

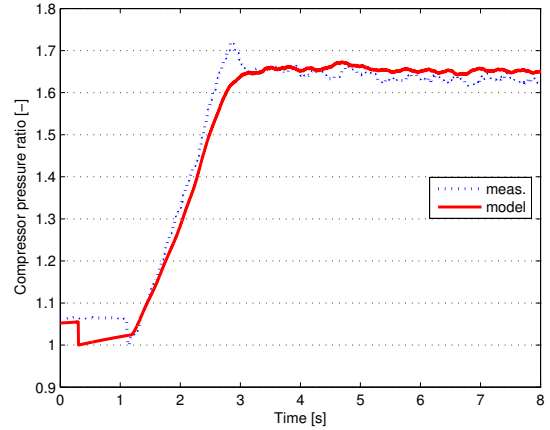


Figure 8. Transient validation : comparison between the compression ratio measured on a test bench and obtained from the dynamics equation of the model.

### 6.1 Feedforward design

*Set points* The driver's request considered here is the accelerator position. First, taking into account the gear box configuration, this request is turned into a torque control objective under the form of an IMEP (Indicated Mean Effective Pressure) set point. Then, the set points for the pressure  $P_{dc}^{sp}$  is inversely given by experimentally calibrated static map on the  $(IMEP^{sp}, N_e)$  operating range. The engine speed  $N_e$  is not modelled but directly measured.  $P_{dc}^{sp}$  is defined as  $P_{dc}^{sp} \triangleq f_P(IMEP^{sp}, N_e)$  and thus, we have  $\Pi_c^{sp} \triangleq \frac{f_P(IMEP^{sp}, N_e)}{P_{uc}}$ ,  $P_{uc}$  being measured or estimated.

*Motion planning* Because  $IMEP^{sp}$  is arbitrarily specified by the driver,  $t \mapsto \Pi_c^{sp}(t)$  may not be smooth nor monotonous. These signals must be filtered to correspond to feasible trajectories of (14) constrained by the boundaries on the system input. This can be done by many methods (including filtering with tunable transfer functions). Here, we propose the following approach that, among several properties, is easy to handle in a convergence analysis process<sup>1</sup>. It addresses only the case of transients from one steady state to another. From a current steady state  $\underline{\Pi}_c$  to a target  $\bar{\Pi}_c$  an interpolation formula is used. Note  $T$  a positive constant, let

$$\phi(t, T) = \begin{cases} 0 & \text{for } 0 \geq t \\ (\frac{t}{T})^2(3 - 2\frac{t}{T}) & \text{for } 0 \leq t \leq T \\ 1 & \text{for } T \leq t \end{cases} \quad (15)$$

The considered interpolation is

$$\Pi_c^{mp}(t) = \underline{x} + (\bar{x} - \underline{x})\phi(t, T) \quad (16)$$

*Model inversion* System (14) is fully actuated and invertible. Thus, an analytic expression of the input can be derived from the state variables and their first derivatives histories. Indeed, from  $(\Pi_c, \dot{\Pi}_c)$ , we can compute

$$\Pi_t = \psi_t^{-1}\left(\frac{1}{\alpha_1}(\dot{\Pi}_c + \alpha_2\psi(\Pi_c))\right) \quad (17)$$

<sup>1</sup> One can refer to Chauvin et al. [2006] for a similar analysis on the airpath of a Diesel HCCI engine.

Then we can compute the desired command using the static equality

$$u = \frac{\Pi_c - \alpha_3 \phi_{turb}(\Pi_t)}{\alpha_4 \phi_{wg}(\Pi_t)} \quad (18)$$

Finally, gathering the two previous equations lead to

$$S_{wg} \triangleq g(\Pi_c, \dot{\Pi}_c)$$

The unique open-loop control law ( $u^{mp}$ ) corresponding to any desired ( $x^{mp}$ ) trajectory (defined by formulas (16)) is

$$S_{wg}^{mp} = g(\Pi_c^{mp}, \dot{\Pi}_c^{mp}) \quad (19)$$

## 6.2 Feedback design

The control strategy is directly obtained from the simplified model proposed above. We want the system to follow the dynamics described by :

$$\dot{y} = -\mu_p y - \mu_i \int_0^t y(\tau) d\tau \quad (20)$$

with  $y = \Pi_c^{mp} - \Pi_c$

From the same approach as described in the previous section to compute a feed forward strategy, the closed loop command can be deduced from the model ensuring a convergence to the desired set-point. An additional integral term is introduced in order to take account of modelling uncertainties.

$$u^{mp} = g(\Pi_c^{mp} + \mu_p(\Pi_c^{mp} - \Pi_c) + \mu_i \int_0^t (\Pi_c^{mp} - \Pi_c) d\tau, \dot{\Pi}_c^{mp}) \quad (21)$$

The designed control strategy satisfies the requirements and constraints exposed in Section 3 :

- The manifold pressure is controlled to its set point.
- In steady state, the throttle is opened, minimizing the pressure drop across this component.
- The environment conditions are taken into account in the model.
- Since the speed of the turbocharger shaft is directly linked to the compressor pressure ratio, a limit on the set-point will ensure the safety of the system by maintaining the shaft speed below a maximum value.

## 7. EXPERIMENTAL RESULTS

### 7.1 Engine set-up

We now briefly discuss the engine we conduct experimentations on, and, more generally, the downsizing context. The downsizing technique aims at replacing a given engine with one with smaller displacement volume without loss of performance or efficiency. At IFP, we achieve this by combining turbocharging, homogeneous direct injection and variable cam timing on both camshafts. A twin-scroll turbine is used to maximize kinetic energy recovery through the strong pulsating exhaust pressures which are accentuated by small exhaust manifolds (see Pagot et al. [2002]). The main characteristics of the engine are:

- Four cylinder SI engine.
- Waste-gate turbocharger with twin-scroll turbine.
- Homogeneous direct gasoline injection.
- Variable valve timing on intake and exhaust camshafts (valve lift invariable).
- Stroke x Bore: 93 x 82.7 mm.
- Compression ratio: 10.5 : 1.

Experimental results were obtained with this engine fitted in a Renault VelSatis vehicle.

### 7.2 Experimental results

The results of an engine test stand are presented to portray the capabilities of this concept. The control was tested on a tip in (50 Nm to 280 Nm) at fixed engine speed (1500 rpm). All unmeasured disturbances act upon the plant and the degree of freedom (the parameter  $\mu$ ) was calibrated online on the engine test stand to set it for the actual disturbances. In this case calibration of  $\mu$  was performed manually. Most model parameters were identified off-line using manufacturer data and additional measurements. In Figure 9, measurement results on the engine test stand are shown using the proposed controller as introduced above. Figure 10, we show the actuators solicitations. The fig-

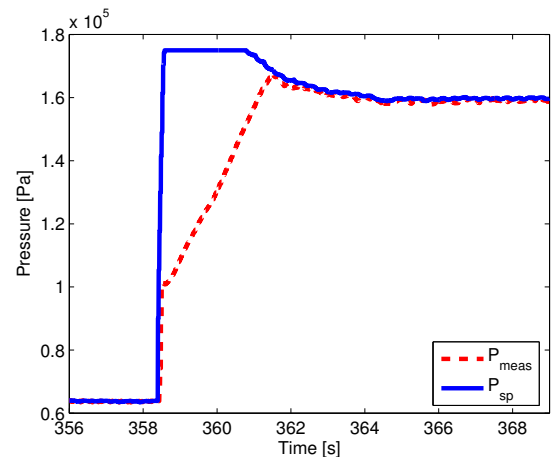


Figure 9. Experimental pressure control. Tip in 50 to 280 Nm at 1500rpm. The red (dotted) curve is the pressure measurement  $P_{dc}$  and the blue one is the pressure set point  $P_{dc}^{sp}$ .

ures portray the closed-loop performance after calibration. The control of the pressure is achieved with no overshoot and a high solicitation of the actuator. The results are therefore very promising since the basic properties of the turbocharger modelling are retained despite the nonlinearity of the plant and controller. The modeling error due to the simplified mean-value filling-and-emptying method or unmeasured disturbances are compensated for by the integrator term of the controller ensuring no steady-state error. Performance of the closed-loop is at least comparable to the common calibrated gain-scheduled PID control structure. However, calibration of a proposed controller is significantly more efficient. The main advantage of this controller is the trade-off between calibration and performance. Indeed, the proposed controller has very good performances (no overshoot, fast response time) while the

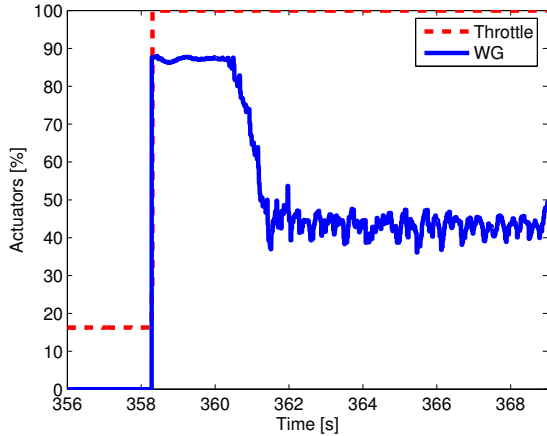


Figure 10. Experimental pressure control. Tip in 50 to 280 Nm at 1500rpm. The red (dotted) curve is the intake throttle position and the blue curve is the wastegate position ( $WG = 0$  means  $u = 0$  and  $WG = 100$  means  $u = 1$ ).

calibration task is small. Model based control allow to take the turbocharger dynamics and the external conditions into account in order to provide a rapid and robust control.

## 8. CONCLUSION

This paper describes the development of a very simple model representing of a turbocharger in a gasoline engine. The purpose was to obtain a representation that can be used as a basis for the design of a control law. The approach consisted in designing a first model based on well known physical principles, and simplifying it by removing the fastest dynamics and making some steady state assumptions. The final model was validated against experimental data. The control strategy deduced from this model is detailed, and test results are shown in order to validate the whole approach.

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Var.	Quantity	Unit
$D_{asp}$	Flow aspirated into the cylinder	$\text{kg.s}^{-1}$
$D_c$	Flow through the compressor	$\text{kg.s}^{-1}$
$D_f$	Flow of fuel injected	$\text{kg.s}^{-1}$
$D_t$	Flow through the turbine	$\text{kg.s}^{-1}$
$D_{wg}$	Flow through the waste gate	$\text{kg.s}^{-1}$
$J_t$	Turbocharger inertia	
$N_t$	Turbocharger speed	$\text{rad.s}^{-1}$
$N_e$	Engine speed	rpm
$\mathcal{P}_c$	Compressor power	-
$\mathcal{P}_t$	Turbine power	-
$P_{uc}$	Upstream compressor pressure	Pa
$P_{dc}$	Downstream compressor pressure	Pa
$P_{ut}$	Upstream turbine pressure	Pa
$P_{dt}$	Downstream turbine pressure	Pa
$P_{man}$	Intake manifold pressure	Pa
$T_{uc}$	Ambient temperature	K
$T_{dc}$	Downstream compressor temperature	K
$T_{ut}$	Upstream turbine temperature	K
$T_{man}$	Intake manifold temperature	K
$\eta_c$	Compressor efficiency	-
$\eta_t$	Turbine efficiency	-
$\eta_v$	Volumetric efficiency	-
$\Pi_c$	Compressor pressure ratio	-
$\Pi_t$	Turbine pressure ratio	-