

## Fault Diagnosis in Robotic Manipulators using Joint Torque Sensing

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**Abstract:** Model uncertainty is an important factor hindering reliability of any model-based failure detection and identification (FDI) method. Use of joint torque sensing reduces significantly the complexity of robot modeling by excluding hardly-identifiable link dynamics from overall manipulator dynamics. Application of such model in the proposed FDI filter increases reliability of fault monitoring against modeling uncertainty. The proposed filter is based on a smooth velocity observer of degree  $2n$  where  $n$  stands for the number of manipulator joints. No velocity measurement and assumption on smoothness of faults are used in the fault detection process.

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### 1. INTRODUCTION

With increasing degree of complexity and number of components used in control systems, accurate monitoring of system malfunctioning has become more and more vital in modern control system technology. A classic approach to accommodate system failure is to operate redundant devices in parallel. However, high cost of maintenance together with limitations imposed by weight and volume, make the redundancy approach undesirable in many applications such as space systems or nano technologies. As a result, in the last decade design of reliable failure monitoring and accommodation techniques by using minimum number of spare components has received considerable attention.

An important class of failure detection techniques consists of using a model for the monitored system. Failure in normal operation of a system is typically represented either by abnormal deviation of system parameters from their nominal values or by external disturbance signals called as faults. In the former case standard parameter identification techniques, under sufficient excitation needed for parameter convergence, can ensure localization of failed components. In the latter case, observer based methods aim at generating some residual signals such that each residual is sensitive to a group of faults. Logical combination of residuals can ultimately lead to localization of faults.

A great number of model based techniques for fault detection deal with linear systems, Frank [1990]. For input affine nonlinear systems, geometric methods of Persis and Isidori [2001] and Hammouri et al. [1999] have extended results of Massoumnia [1986] and developed necessary and sufficient conditions for detectability of fault signals together with presenting a procedure for constructing residual generators. To this end, Mattone and De-Luca [2006] have used a geometric approach for a relaxed formulation of fault de-

tection problem for robotic manipulators where the focus is mainly on isolation of a set of faults instead of a single fault. Leuschen et al. [2005] have extended linear analytical redundancy to nonlinear input affine systems with application in robot fault detection. In Filaretov et al. [1999] an observer based fault diagnosis system for flexible robots, and based on the algebra of functions, has been developed. Another observer based fault detector for manipulators by using full state measurements was proposed by Schneider and Frank [1996] where model uncertainty was handled by fuzzy residual evaluation.

Also, in a probabilistic framework, Verma and Simmons [2006] have used a Monte-Carlo approximation method for estimation of robot states and ultimately monitoring the faults. In Dixon et al. [2000] a prediction-error-based approach for fault detection in manipulators in presence of parametric uncertainty has been proposed where the residual signal was considered as the estimation error in filtered input torque. In McIntyre et al. [2005] a nonsmooth nonlinear observer was used for identification of fault signals in a robotic system by using exact knowledge of upperbounds for amplitudes of fault signals and their first and second derivatives.

A major difficulty in model based techniques for general nonlinear systems and in particular robotic systems, is the presence of model uncertainty that can significantly hamper the accuracy and reliability of detection and isolation process. In case of robotic manipulators model accuracy depends reciprocally on manipulator degrees of freedom. This phenomenon is mainly due to the uncertainty in links and actuator mass and inertia parameters. Recently Kosuge et al. [1990], Hashimoto et al. [1993], Aghili and Namvar [2006] have shown that the use of joint torque sensors can significantly reduce the complexity of robot modeling and control by relaxing the need to model link dynamics. In particular, it was shown by Aghili and Nam-

var [2006] that joint torque sensing results in a reduced number of dynamic parameters in an adaptive motion control problem. Moreover, the controlled robot is capable of rejecting perfectly the external force disturbances.

In this paper we use particular model of a robot manipulator resulted from using joint torque sensors to detect and localize a class of additive fault signals. The resulting model has the property that it consists only of dynamical parameters of motor rotors and excludes hardly identifiable parameters related to robot links. This property has a significant impact on reliability of the resulting fault detection process.

Another important property concerns the lower triangular structure of manipulator dynamical equation that gives the ability of localizing faults by using a single  $2n$ th order *smooth* observer.

Finally, the detection process does not require measurement of joint velocities which are usually obtained through some *ad hoc* numerical differentiations. Therefore, in terms of the total number of used sensors, the proposed method employs  $2n$  sensors being  $n$  encoders and  $n$  joint torque sensors. This number is similar to the majority of standard fault detection methods for robotic manipulators.

The proposed method relies only on *a priori* knowledge of an upperbound for joint velocity but does not require known bounds for derivatives of fault signals.

## 2. MANIPULATOR MODEL USING JOINT TORQUE SENSORS

Equations of motion of a rigid manipulator can be written by

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau_m + f \quad (1)$$

where  $M(q) > 0$  stands for inertia matrix and  $C(q, \dot{q})\dot{q}$  represents centrifuge and coriolis forces. Also,  $g(q)$  is the vector of gravitational force and  $\tau_m$  is motor torque. Additive fault signal is denoted by  $f$  and represents a variety of faults in motors. We consider the following assumptions (see Aghili and Namvar [2006]):

- Manipulator is serial and electrically driven,
- Center of mass of all rotors coincide with their rotation axis,
- Rotor inertias about the  $x$ - and  $y$ -axes are identical,
- Principal axes of all rotor inertias are assumed to be parallel to their own joint axes,
- Deformation of the joints are negligible,

Let decompose the inertia matrix by

$$M(q) = M_L(q) + JT(q) + T(q)^T J \quad (2)$$

where  $J = \text{diag}\{J_i\}$  contains polar inertia of rotors and  $M_L(q)$  represents link inertia matrix. Also,  $T(q)$  is a lower triangular full rank matrix given by

$$T(q) = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ z_{2,z_1} & 1 & 0 & \cdots & 0 \\ z_{3,z_1} & z_{3,z_2} & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z_{n,z_1} & z_{n,z_2} & z_{n,z_3} & \cdots & 1 \end{bmatrix} \quad (3)$$

According to Denavit-Hartenberg convention (see Spong et al. [2006]),  $z_k \in R^3$  is a unit vector attached to rotor  $k$  and along with its rotation axis and expressed in fixed frame (see Fig. 1).

Since the rotation of joint  $k$  has no effect on orientations of  $z_1, \dots, z_k$ , it is implied that  $z_k$  is only a function of  $q_1, \dots, q_{k-1}$ . Therefore, each  $T_{ij}$ , for  $i > j$ , is a function of  $q_1, \dots, q_{i-1}$ . It can be shown that if there exist at least  $n-2$  pairs of adjacent parallel joint axes, then  $T$  is independent of  $q$  making (5) a LTI system.

Now defining the joint torque signal by

$$\tau_J := (M_L + T^T J)\ddot{q} + (C - J\dot{T})\dot{q} + g \quad (4)$$

manipulator dynamics (1) can be rewritten as

$$J \frac{d}{dt}(T(q)\dot{q}) = \tau_m - \tau_J + f \quad (5)$$

As shown in Aghili and Namvar [2006],  $[\tau_J]_i$  is the torque sensed at the intersection of link  $i$  with joint  $i$ . By computing the acceleration  $\ddot{q}$  from (1) and replacing in (5), joint torque vector can be expressed by

$$\tau_J = \tau_m + f + JTM^{-1}(-C\dot{q} - g + \tau_m + f) \quad (6)$$

which shows that  $\tau_J$  is a function of manipulator states and external forces. From a theoretical point of view, this fact ensures that measurement of joint torque constitutes a causal and feasible operation.

Equation (5) has two important properties. First, the effect of link dynamics is lumped and represented by  $\tau_J$  and as a result by measuring joint torques there remains no need for modeling link dynamics. Second, lower triangular structure of  $T$  together with particular dependence of its elements to  $q$  are key properties to be used in Section 4 for localization of faults.

Another property of  $T(q)$  is that its derivative with respect to time,  $\dot{T}(q, \dot{q})$ , is linear with respect to  $\dot{q}$ , i.e.

$$\dot{T}(x, y + z) = \dot{T}(x, y) + \dot{T}(x, z) \quad (7)$$

As a result,  $\dot{T}(x, y)$  is linearly bounded with respect to  $y$

$$\|\dot{T}(x, y)\| \leq c\|y\| \quad (8)$$

Moreover,

$$\dot{T}(x, y)z = \dot{T}(x, z)y \quad (9)$$

## 3. VELOCITY OBSERVER

In this section we develop an exponentially convergent velocity observer for (5). Consider the following observer

$$\begin{cases} JT(q)\dot{w} - J\dot{T}(q, v)v + JTk w + JTk^2 q = \tau_m - \tau_J \\ \dot{q} = L\tilde{q} + v \end{cases} \quad (10)$$

where

$$v := w + kq \quad (11)$$

and  $0 < L = \text{diag}(L_i)$ ,  $k > 0 \in R$  are two given constant gains. Note that  $[w \ \hat{q}]$  constitute the states of the observer and  $q$ ,  $\tau_J$  and  $\tau_m$  are given inputs.

*Theorem 1.* Consider the observer (10) together with manipulator dynamics (5). Assume that  $f \equiv 0$  and joint velocity is bounded. Then,  $\tilde{q}$  and  $\dot{\tilde{q}}$  converge to zero exponentially where  $\tilde{q} := q - \hat{q}$ .

**Proof.** Defining the composite error

$$s := \dot{\tilde{q}} + L\tilde{q} \quad (12)$$

it can be verified that  $s = \dot{q} - v$ . Observer (10) can be equivalently written by

$$JT(q)(\dot{w} + kw + k^2q) + \dot{T}(q, v)v = \tau_m - \tau_J$$

However, we have

$$\begin{aligned} \dot{w} + kw + k^2q &= \dot{w} + k(w + kq) \\ &= \dot{w} + kv \\ &= \dot{v} - k\dot{q} + kv \\ &= \dot{v} - ks \end{aligned} \quad (13)$$

Hence (10) is equivalent to

$$JT(q)\dot{v} + \dot{T}(q, v)v = \tau_m - \tau_J \quad (14)$$

Now subtracting (14) from (5) yields

$$JT(q)\dot{s} + J\dot{T}(q, \dot{q})\dot{q} - J\dot{T}(q, v)v + JTKs = f \quad (15)$$

Using property (7) and (9), equation (15) can be equivalently written by

$$JT\dot{s} + J\dot{T}(q, \dot{q})s + J\dot{T}(q, v)s + JTKs = f \quad (16)$$

Define the Lyapunov function

$$V = \frac{1}{2}s^T T^T JTs \quad (17)$$

Left multiplying (16) by  $T^T$  and differentiating  $V$  with respect to time along the resulting error dynamics yields

$$\dot{V} = -ks^T(T^T JTs) - s^T T^T J\dot{T}(q, s)v + s^T T^T f \quad (18)$$

Using the fact that  $v = s - \dot{q}$  and property (8) yields

$$\dot{V} \leq -k\underline{\lambda}(T^T JTs)\|s\|^2 + c\|s\|^2(\|\dot{q}\| + \|s\|) + s^T T^T f \quad (19)$$

where  $\underline{\lambda}$  denotes minimum eigenvalue. As a result when  $f \equiv 0$ , and under the assumption  $\|\dot{q}\| \leq \alpha$ , we obtain

$$\dot{V} \leq -\beta\|s\|^2 \quad (20)$$

provided that  $\|s\|$  is sufficiently small in a sense that

$$\|s\| \leq (\beta + k\underline{\lambda}(T^T JTs))c^{-1} - \alpha \quad (21)$$

Therefore, when  $f \equiv 0$ , the error dynamics (15) is locally exponentially stable and  $s$  converges to zero, exponentially. It is observed that by increasing  $k$  it is possible to widen the domain of attraction and as a result the error dynamics is semi-globally exponentially stable.  $\square$

#### 4. FAULT DIAGNOSIS

The observer (10) is an asymptotic detector for the system (5). In order to investigate the ability of the observer for

localization of faults, we expand (16) and recall the dependency of  $T_{ij}$  to  $q_1, \dots, q_i$  and lower triangular structure of  $T$ . The first equation in (16) reads

$$J_1\dot{s}_1 + J_1k s_1 = f_1 \quad (22)$$

which together with (12) yields

$$\ddot{\tilde{q}}_1 + (k + L_1)\dot{\tilde{q}}_1 + kL_1\tilde{q} = J_1^{-1}f_1 \quad (23)$$

Clearly,  $\tilde{q}_1$  uniquely detects and isolates  $f_1$ . To investigate isolation of other faults, we rewrite equation (15) by

$$\dot{s} = Ys + Zf \quad (24)$$

where  $Y = [y_{ij}]$  and  $Z = [z_{ij}]$  are lower triangular matrices defined by

$$Y := -T^{-1}\dot{T}(q, 2\dot{q} - s) - kI \quad (25)$$

$$Z := (JT)^{-1} \quad (26)$$

The  $i$ th component of  $\dot{s}$  can then be expressed by

$$\dot{s}_i = -ks_i - \sum_{j=1}^{i-1} y_{ij}s_j + \sum_{j=1}^i z_{ij}f_j, \quad i = 1, \dots, n$$

It is clearly seen that  $s_i$  is affected only by  $f_1, f_2, \dots, f_i$  so that when  $s(0) = 0$ ,  $s_i(t)$  will remain identically zero if any of faults  $f_{i+1}, \dots, f_n$  occur. Now, assuming that faults do not occur simultaneously, the signature vector can be constructed by

$$r_i = h_i \cdot \prod_{j=1}^{i-1} \bar{h}_j \quad (27)$$

where

$$h_i = \begin{cases} 1 & \text{if } |s_i| \geq \ell_i \\ 0 & \text{otherwise} \end{cases} \quad (28)$$

and  $\bar{h}_j := \text{NOT}(h_j)$ . Moreover,  $\ell_i$  is the chosen threshold level for the  $i$ th signature signal. In case of zero initial condition ( $s(0) = 0$ ) the threshold level  $\ell_j$  can be set close to zero to account only for the effect of measurement noise.

In practice, if  $s(0)$  is not zero, the effect of nonzero initial condition on  $s$  decays exponentially. By virtue of (17) and (20), it can be inferred that in absence of fault signals,

$$|s_i| \leq \|s\| \leq \left(\frac{\bar{\lambda}}{\underline{\lambda}}\right)^{\frac{1}{2}} \|s(0)\| e^{-\frac{\beta}{\bar{\lambda}}t} \quad (29)$$

where  $\bar{\lambda}$  ( $\underline{\lambda}$ ) denote an upperbound (lowerbound) for the maximum (minimum) eigenvalue of  $T^T JTs$ . Therefore, the threshold level in this case would be the time varying term in the RHS of inequality (29).

##### 4.1 Unmeasured velocity

If velocity is not measured, then instead of (27), the signature signal is generated by using (12). As a result (24) can be expressed in terms of  $\ddot{\tilde{q}}$  and similarly it can be argued that  $\ddot{\tilde{q}}_i$  is affected only by  $f_1, \dots, f_i$ . The signature signal is then given by (27) where  $h$  is now defined by

$$h_i = \begin{cases} 1 & \text{if } |\ddot{\tilde{q}}_i| \geq \ell_i \\ 0 & \text{otherwise} \end{cases} \quad (30)$$

In this case, the threshold level is obtained by combining (29) and (12) as

$$\begin{aligned}
 \tilde{q}_i(t) &= \tilde{q}_i(0)e^{-L_i t} + \int_0^t s_i(\tau)e^{-L_i(t-\tau)} d\tau \\
 &\leq \tilde{q}_i(0)e^{-L_i t} + \int_0^t |s_i(\tau)|e^{-L_i(t-\tau)} d\tau \\
 &\leq \tilde{q}_i(0)e^{-L_i t} + \int_0^t \left(\frac{\bar{\lambda}}{\underline{\lambda}}\right)^{\frac{1}{2}} \|s(0)\| e^{-\gamma\tau} e^{-L_i(t-\tau)} \\
 &\leq \left(\frac{\bar{\lambda}}{\underline{\lambda}}\right)^{\frac{1}{2}} \|\dot{q}(0)\| \frac{1}{L_i - \gamma} (e^{-\gamma t} - e^{-L_i t})
 \end{aligned} \tag{31}$$

where  $\gamma := \frac{2\beta}{\lambda}$ . In the last inequality it is assumed that joint angle measurement is exact or  $\tilde{q}(0) = 0$ . Note that  $\|\dot{q}(0)\|$  is still unknown but knowing that  $\|\dot{q}\| \leq \alpha$ , clearly,  $\dot{q}(0)$  can be chosen such that  $\|\dot{q}(0)\| \leq \alpha$ . Hence,  $\|\dot{q}(0)\| \leq 2\alpha$  and consequently threshold level can be set to

$$\ell_i = \left(\frac{\bar{\lambda}}{\underline{\lambda}}\right)^{\frac{1}{2}} \frac{2\alpha}{L_i - \gamma} (e^{-\gamma t} - e^{-L_i t}) \tag{32}$$

For the threshold level chosen as above we have  $\ell_i(0) = 0$ ,  $\ell_i(\infty) = 0$  and

$$\max_{t \geq 0} \ell_i(t) = \left(\frac{\bar{\lambda}}{\underline{\lambda}}\right)^{\frac{1}{2}} \frac{2\alpha}{\gamma} \left(\frac{L_i}{\gamma}\right)^{\frac{-L_i}{L_i - \gamma}} \tag{33}$$

Therefore, by choosing  $L_i$  large, the threshold level can be effectively minimized and consequently sensitivity of fault detection is increased.

#### 4.2 Effect of torque sensor noise

Assume that torque sensor measurement is contaminated by additive noise  $n(t)$  such that in using torque measurement,  $\tau_j$  in the observer (10) the term  $-n(t)$  adds up to the first equation in (10). We assume that  $n(t)$  is bounded as

$$\|n\| \leq c_n \tag{34}$$

In this case, the RHS of error dynamics (15) and (16) changes into  $f + n$  and consequently under condition (21) and in absence of faults, (20) transforms into

$$\dot{V} \leq -\beta \|s\|^2 + s^T T^T n \tag{35}$$

In view of (34),  $\dot{V}$  is bounded by

$$\dot{V} \leq -\frac{2\beta}{\lambda} V + c_n \left(\frac{2}{\lambda}\right)^{\frac{1}{2}} \|T\| V^{\frac{1}{2}} \tag{36}$$

which is in form of a Bernoulli differential inequality. By the change of variable  $W = V^{\frac{1}{2}}$ , (36) is transformed into a first order differential inequality

$$\dot{W} \leq -\frac{\beta}{\lambda} W + c_n \left(\frac{1}{2\lambda}\right)^{\frac{1}{2}} \|T\| \tag{37}$$

Therefore,

$$W(t) \leq W(0)e^{-\gamma t} + \frac{c_n \bar{\lambda}}{\beta \sqrt{2\lambda}} \|T\| (1 - e^{-\gamma t}) \tag{38}$$

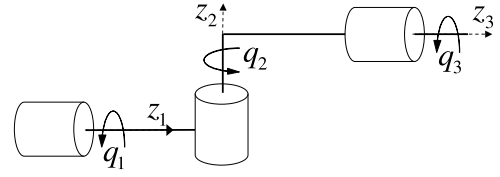


Fig. 1. 3DOF spherical wrist configuration

where  $\gamma = \frac{\beta}{\lambda}$ . Now, in light of (17) it can be inferred that

$$\|s\| \leq \left(\frac{\bar{\lambda}}{\underline{\lambda}}\right)^{\frac{1}{2}} \|s(0)\| e^{-\gamma t} + c_n \left(\frac{\bar{\lambda}}{\beta \lambda}\right) \|T\| (1 - e^{-\gamma t}) \tag{39}$$

By a similar procedure used in deriving (31) and assuming that  $\tilde{q}(0) = 0$  and  $\|\dot{q}(0)\| \leq \alpha$ , we obtain

$$\|\tilde{q}_i\| \leq \left(\frac{\bar{\lambda}}{\underline{\lambda}}\right)^{\frac{1}{2}} \frac{2\alpha}{L_i - \gamma} (e^{-\gamma t} - e^{-L_i t}) + N(t) \tag{40}$$

where

$$N(t) = \frac{A}{L_i} (1 - e^{-L_i t}) - \frac{A}{L_i - \gamma} (e^{-\gamma t} - e^{-L_i t})$$

and  $A = \frac{c_n \bar{\lambda}}{\beta \lambda} \|T\|$ . It can be verified that  $N(t)$  is a non-negative and non-decreasing function of time. As a result, comparing (40) with (32) indicates an increase of threshold level due to torque sensor noise.

## 5. EXAMPLE

We consider a 3-DOF spherical wrist configuration consisting three electric motors shown in Fig. (1). For this system

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \cos(q_2) & 0 & 1 \end{bmatrix}$$

and  $J_i = 10^{-3} \text{kgm}^2$ . It is assumed that manipulator is under motion control such that in absence of faults,  $\|\dot{q}\| \leq 0.1 \text{rad/s}$ . Observer (10) has been implemented with  $L = 10I$  and  $k = 10$  assuming that torque measurement is contaminated by some additive noise  $n(t)$ . Figures 2-4 demonstrate the response of residuals ( $\tilde{q}_i$ ) and signature signals ( $r_i$ ) to step-form faults entered at  $t = 2\text{s}$  and with amplitude of  $10^{-2} \text{Nm}$ . Threshold level is chosen as the RHS of (40). As seen from the figures this choice has made the signature signals insensitive to non-zero initial conditions and torque sensor noise.

## 6. CONCLUSION

Application of joint torque sensing permits detection and localization of additive faults by a single  $2n$ 'th order smooth observer and without a need for bank of observers. Computationally, the resulting observer tends to be less complex than standard observers based on full manipulator dynamics. No assumption was made on smoothness of faults. It is expected that the use of globally convergent observers will further improve the reliability of fault detection.

REFERENCES

F. Aghili and M. Namvar. Adaptive control of manipulators using un-calibrated joint-torque sensing. *IEEE Transactions on Robotics*, 22(4):854–860, 2006.

W.E. Dixon, I.D. walker, D.M. Dawson, and J.P. Hartanft. Fault detection for robot manipulators with parametric uncertainty: A prediction error based approach. *IEEE Trans. Robot. Automat.*, 16(6):689–699, 2000.

V.F. Filaretov, M.K. Vukobratovich, and A.N. Zhirabok. Observer-based fault diagnosis in manipulation robots. *Mechatronics*, 9:929–939, 1999.

P.M. Frank. Fault diagnosis in dynamic systems using analytical and knowledge based redundancy: A survey. *Automatica*, 26:459474, 1990.

H. Hammouri, M. Kinnaert, and E.H. El Yaaghoubi. Observer based approach to fault detection and isolation for nonlinear systems. *IEEE Trans. Automat. Contr.*, 44:1879–1884, 1999.

M. Hashimoto, Y. Kiyosawa, and R.P. Paul. A torque sensing technique for robots with harmonic drives. *IEEE Trans. Robot. Automat.*, 9(1):108–116, 1993.

K. Kosuge, H. Takeuchi, and K. Furuta. Motion control of a robot arm using joint torque sensing. *IEEE Trans. Robot. Automat.*, 6(2):258–263, 1990.

M.L. Leuschen, I.D. Walker, and J.R. Cavallaro. Fault residual generation via nonlinear analytical redundancy. *IEEE Trans. Contr. Sys. Tech.*, 13(3):452–458, 2005.

M.A. Massoumnia. A geometric approach to the synthesis of failure detection filters. *IEEE Trans. Automat. Contr.*, 31:839–846, 1986.

R. Mattone and A. De-Luca. Relaxed fault detection and isolation: An application to a nonlinear case study. *Automatica*, 42:109116, 2006.

M.L. McIntyre, W.E. Dixon, D.M. Dawson, and I.D. walker. Fault identification for robot manipulators. *IEEE Transactions on Robotics*, 21(5):1028–1034, 2005.

C.D. Persis and A. Isidori. A geometric approach to nonlinear fault detection and isolation. *IEEE Trans. Automat. Contr.*, 46(6):853–865, 2001.

H. Schneider and P. M. Frank. Observer based supervision and fault detection in robots using nonlinear and fuzzy logic residual evaluation. *IEEE Trans. Contr. Sys. Tech.*, 4(3):274–282, 1996.

M.W. Spong, S. Hutchinson, and M. Vidyasagar. *Robot modeling and control*. John Wiley and Sons, 2006.

V. Verma and R. Simmons. Scalable robot fault detection and identification. *Robotics and Autonomous Systems*, 54:184191, 2006.

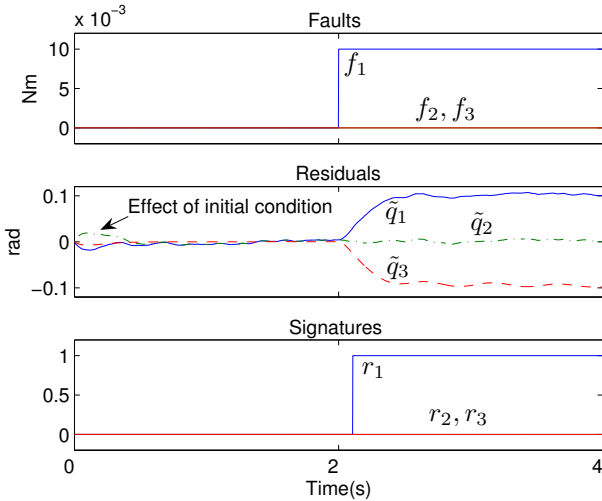


Fig. 2. Detection and localization of  $f_1$

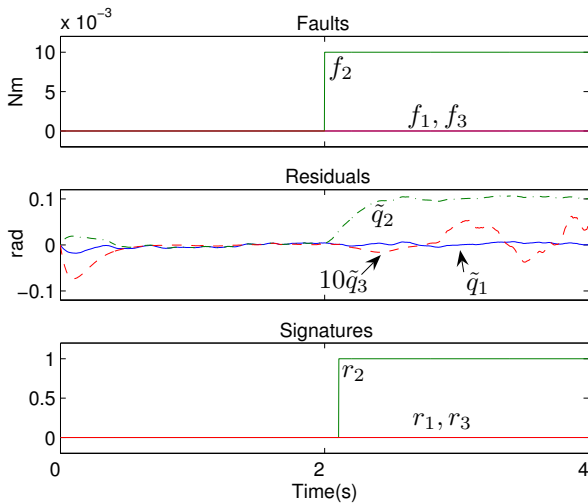


Fig. 3. Detection and localization of  $f_2$ . The residual  $\tilde{q}_1$  is insensitive to  $f_2$ .

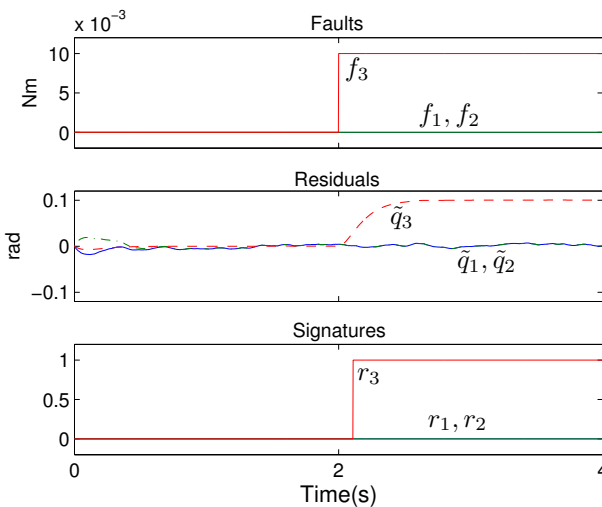


Fig. 4. Detection and localization of  $f_3$ . Both residuals  $\tilde{q}_1$  and  $\tilde{q}_2$  are insensitive to  $f_3$ .