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Multiple Switching Relay for Identification of Frequency Responses

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Abstract: We discuss a new method for relay-auto tuning of a closed-loop control system. Sustained oscillations are obtained by putting relays in the feedback loop and they provide ultimate periods and gains of processes for tuning PID controllers. The ultimate periods obtained by the conventional relays have relative errors up to 5% for first order plus time delay processes. To improve the estimation of ultimate periods, modified relays such as a saturation relay and a relay with preload can be used. However, these modifications lose the binary (on-off) property of the conventional relay. Here relays with multiple switching which produce pulse-width-modulation (PWM) signals are proposed. They retain the binary property and show improved identification accuracies of ultimate data.

1. INTRODUCTION

A relay in the feedback loop produces a sustained oscillation and ultimate information of process can be extracted (Atherton, 1975). Since Astrom and Hagglund (1984) introduced relay feedback methods in tuning PID controllers, extensive works for theoretical and practical improvements in relay feedback methods have been published (Lee et al., 2007; Hang et al., 2002; Astrom and Hagglund, 1995; Yu, 2006). There are several commercial stand-alone PID controllers implementing these relay feedback autotuning methods (Yu, 2006).

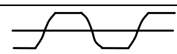
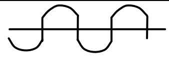
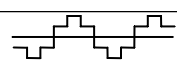
The relay output is binary (on-off) and its response is usually in the form of a square wave. Because the square wave has higher harmonics, the ultimate data obtained can have errors. For the first order plus time delay (FOPTD) processes, the ultimate period errors are as high as 5%. Several methods such as a saturation relay (Yu, 2006), relay with a P control preload (Tan et al., 2006) and a two level relay (Sung et al., 1995) were introduced to obtain more accurate ultimate information of the process by suppressing the effects of high order harmonic terms. However, these methods do not contain the binary nature of the conventional relay.

Higher harmonics in the square wave can be reduced by applying the pulse-width-modulation (PWM) method without losing the binary nature. PWM methods are used in various applications (Mohan et al., 1995). A power inverter produces commercial alternating-current power from direct-current sources such as solar cells and hydrogen fuel cells. Actually, a fast PWM signal can mimic any analog signal and hence any relay feedback method can be implemented in a binary signal through the PWM method. However, a fast PWM which uses fast on and off switching is not practical in control applications. Here PWM signals with minimal on-off switchings (Chiasson et al., 2004) are applied to the relay feedback method. It is shown that the identification error of the ultimate period can be reduced by half compared to the original relay.

2. PWM SIGNALS FOR LINEAR PROCESSES

Relay feedback identification method is based on the square wave forcing an input whose higher harmonics are attenuated by the system. When the system does not reduce the magnitudes of higher harmonics in the square wave, the identified ultimate gain and period can have large errors. A saturation relay (Yu, 2006), a relay with a preload (Tan et al., 2006) and a two level relay (Sung et al, 1995) can reduce higher harmonic terms in the forcing input.

Table 1. Modified relays to improve identification accuracies of the ultimate period and ultimate gain.

Relay Type	Converged Process Input	Characteristics
Saturation Relay (Yu, 2006)		- The slope of the ramp part is very important in improving the estimation accuracy and guaranteeing a limit cycle.
Relay with a Preload (Tan et al., 2005)		- A high preload gain improves the estimation accuracy and shortens the time to reach a stationary oscillation.
Sung et al. (1995)		- An iterative update of the switching times is required.

As shown in Table 1, these forcing inputs are not binary. Here, to reduce the magnitudes of higher harmonics in the forcing input while retaining its binary nature, pulse width modulated (PWM) signals such as shown in Fig. 1 are considered. The PWM signals have been studied in the power electronic area to generate commercial sine wave power from direct-current sources (Mohan et al., 1995).

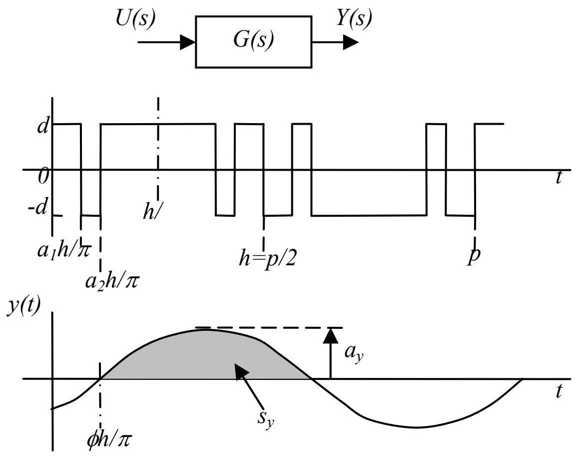


Fig. 1. A PWM signal and its response.

Consider a periodic input signal (period=p) as shown in Fig. 1,

$$u_{PWM}(t)=\begin{cases} d, & t\in[0,a_1h/\pi) \\ -d, & t\in[a_1h/\pi,a_2h/\pi) \\ d, & t\in[a_2h/\pi,h-a_2h/\pi) \\ -d, & t\in[h-a_2h/\pi,h-a_1h/\pi) \\ d, & t\in[h-a_1h/\pi,a_1h/\pi) \end{cases} \tag{1}$$

$$u_{PWM}(t+h)=-u_{PWM}(t)$$
$$u_{PWM}(t+p)=u_{PWM}(t)$$
$$u_{PWM}(t+h/2)=u_{PWM}(-t+h/2)$$

where $h=p/2$. This signal can be expressed as (Chiasson et al., 2004)

$$u_{PWM}(t)=\sum_{i=1}^{\infty}b_{2i-1}\sin((2i-1)\omega t)$$
$$b_{2i-1}=\frac{4d}{\pi(2i-1)}\left(1-2\cos((2i-1)a_1)+2\cos((2i-1)a_2)\right) \tag{2}$$
$$\omega=2\pi/p$$

When $a_1=a_2=0$, $u_{PWM}(t)$ is a square wave,which is the wave form of the original relay feedback system. When

$$a_1=0,\ a_2=\frac{1}{3}\cos^{-1}(0.5)=0.3491 \tag{3}$$

we have $1-2\cos(3a_1)+2\cos(3a_2)=0$ and the third harmonic term can be eliminated ($b_3=0$). By choosing a_1 and a_2 such that

$$\begin{aligned}1-2\cos(3a_1)+2\cos(3a_2)=0\\ 1-2\cos(5a_1)+2\cos(5a_2)=0\end{aligned} \tag{4}$$

we can eliminate the third and fifth harmonics. A solution for Eq. (4) is

$$a_1=0.41268,\ a_2=0.58168 \tag{5}$$

By adding more pulses, the higher harmonic terms can be eliminated. For example, adding one more pulse between phase angles of a_3 and a_4 with $a_1=0$, $a_2=0.2440$, $a_3=0.6499$ and $a_4=0.7438$ removes harmonic terms from the third to the seventh harmonic terms.

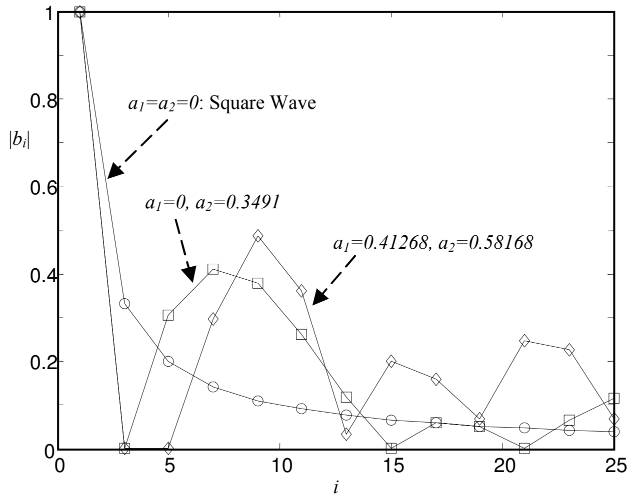


Fig.2. Normalized amplitudes of harmonic terms in PWM signals.

Fig. 2 shows the normalized amplitude of each harmonic component. We can see that the PWM signal of Eq. (3) eliminates the third harmonic term and that of Eq. (5) eliminates the third and fifth harmonic terms. However, amplitudes of some higher harmonic terms increase. However, they are expected to be attenuated by the process.

For the above input, the output of a process whose transfer function is $G(s)$ is

$$\begin{aligned}y(t)=b_1|G(j\omega)|\sin(\omega t+\angle G(j\omega))+\\ b_3|G(j3\omega)|\sin(3\omega t+\angle G(j3\omega))+\cdots\end{aligned} \tag{6}$$

When

$$\frac{b_{2i-1}|G(j(2i-1)\omega)|}{b_1|G(j\omega)|}<<1,\ i=2,3,\cdots \tag{7}$$

we can approximate

$$y(t)\approx b_1|G(j\omega)|\sin(\omega t+\angle G(j\omega)) \tag{8}$$

and obtain

$$\angle G(j\omega)\approx\phi \tag{9}$$

$$|G(j\omega)|\approx\frac{a_y}{b_1}=\frac{\pi a_y}{4d(1-2\cos(a_1)+2\cos(a_2))} \tag{10}$$

where and a_y are measured variables as shown in Fig. 1. As discussed in Lee et al. (2007), because

$$\begin{aligned}y_I(t)&\equiv\int_0^t y(t)dt\\ &=\bar{y}_I-\frac{b_1}{\omega}|G(j\omega)|\cos(\omega t+\angle G(j\omega))-\\ &\quad\frac{b_3}{3\omega}|G(j3\omega)|\sin(3\omega t+\angle G(j3\omega))+\cdots\end{aligned} \tag{11}$$

we have $b_1|G(j\omega)|/\omega\approx s_y/2$, where s_y is the area shown in Figure 1 and $\bar{y}_I$ is the mean value of integral of $y(t)$, and Eq. (10) can be improved as

$$|G(j\omega)| \approx \frac{s_y \omega}{2b_1} = \frac{\pi\omega s_y}{8d(1-2\cos(a_1)+2\cos(a_2))} \qquad (12)$$

In finding the amplitude ratio and phase angle of *G(j)* by Eqs. (9), (10) and (12), conditions of Eq. (7) are important. For high order overdamped processes, the attenuation rates of amplitudes are high (Seborg et al., 2004) and condition (7) is effective for the ultimate frequency of . where $\angle G(j\omega) = -\pi$. Hence we consider only the first order plus time delay (FOPTD) process. If the identification errors are small enough for FOPTD processes, those for high order overdamped processes can be expected to be smaller.

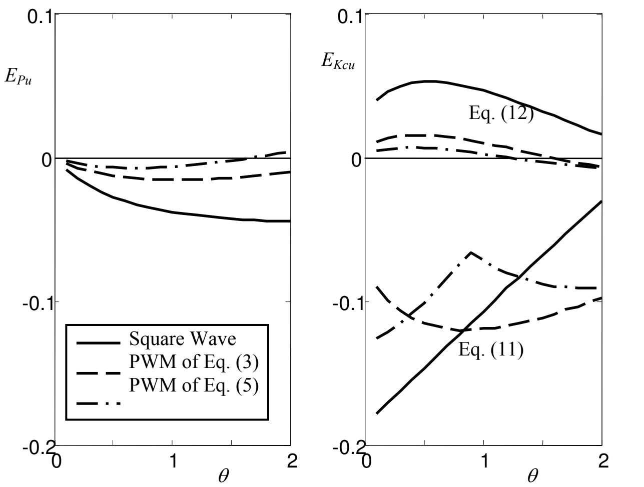


Fig. 3. Relative errors of ultimate periods (*E_{Pu}=(Pu-Pu*)/Pu**, *Pu**=exact ultimate period) and ultimate gains (*E_{Kcu}=(Kcu-Kcu*)/Kcu**, *Kcu**=exact ultimate gain) identified by the PWM signals for FOPTD processes of *G(s)=exp(- s)/(s+1)*.

PWM signals are applied to FOPTD processes of G(s)=exp(- s)/(s+1) with up to 2. The ultimate period, p=2 / . such that =- and the ultimate gain, 1/|G(j)|, are computed. For this, we iteratively solve Eq. (6) with y(0)=0 for . In Eq. (6), 101 terms are used. The exact ultimate period is such that $\angle G(j\omega) = -\theta\omega - \tan^{-1}(\omega) = -\pi$ and its very accurate approximation is given in Lee et al. (2005) as

$$\omega = \frac{2}{\theta}\sqrt{\frac{16\theta+66-\sqrt{181\theta^2+762\theta+3081}}{\theta+17}}\ .$$

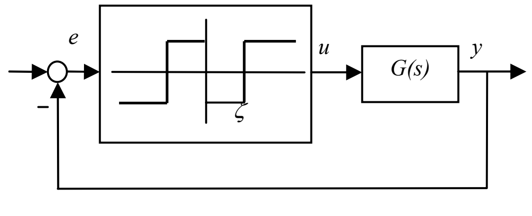


Fig. 4. Block diagram of a multiple switching relay feedback system.

Fig. 3 shows relative errors of the ultimate periods and the ultimate gains. We can see that the identification errors of ultimate periods can be reduced by half even for the simplest PWM signal of Eq. (3). The integrated Eq. (12) is also very useful for accurate identification of the ultimate gains. When Eq. (12) for the ultimate gain is used, the proposed PWM signal can reduce identification errors in half. Errors in identification of the ultimate periods and the ultimate gains are shown in Fig. 3.

3. RELAY FEEDBACK IMPLEMENTATION

The above PWM forcing signal can be realized through the relay feedback. For this, we introduce a switching procedure based on the process output shown in Figure 4.

$$u(t) = \begin{cases} -d, & e \in (-\infty, -\zeta] \\ d, & e \in (-\zeta, 0] \\ -d, & e \in (0, \zeta] \\ d, & e \in (\zeta, \infty) \end{cases} \qquad (13)$$

where is a design parameter. When *e(t)* is a sine wave whose amplitude is a_y,

$$\zeta = a_y \sin(a_2) = a_y \sin(0.3491) \approx a_y / 3 \qquad (14)$$

will generate the PWM signal with $a_1=0$ and $a_2=0.3491$. The PWM signal of Eq. (5) can also be used for better accuracy. However, we use the PWM signal of Eq. (3) because it is simple and its accuracy is sufficient for the controller tuning purpose.

In Eq. (14), a_y is not known in advance. We use a_y of the previous half period and update it iteratively. Figure 5 shows this procedure for $G(s)=2.5/(s+1)^5$. The original relay is used for one or two periods of oscillation and then the multiple switching relay of Eq. (13) is applied until a stable cyclic oscillation is obtained. For all overdamped processes tested in this research, this multiple switching relay feedback system shows a converged cyclic signal.

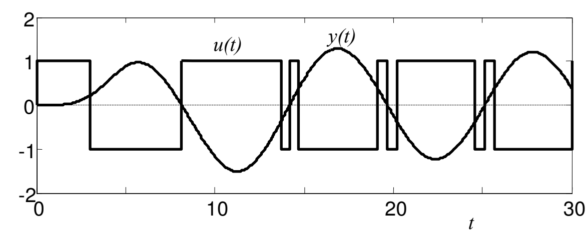


Fig. 5. Responses of a multiple switching relay feedback system for $G(s)=2.5/(s+1)^6$.

For the converged input, the phase of the output becomes $\phi = \angle G(j\omega) \approx -\pi$. Hence the ultimate period, P_u, and ultimate gain, K_{cu}, are identified as

$$P_u \approx p \qquad (15)$$

$$K_{cu} = \frac{1}{|G(j\omega)|} \approx \frac{4d(-1+2\cos(a_2))}{\pi a_y} \qquad (16)$$

Using integrals of *y(t)*, the ultimate gain can be improved as

$$K_{cu} = \frac{1}{|G(j\omega)|} \approx \frac{8d(-1+2\cos(a_2))}{\pi\omega s_y} \quad (17)$$

The design parameter cannot be updated but kept constant for simplicity. Fig. 6 shows the identification errors of ultimate period versus . We can see that improved identification is obtained for a wide range of , usually after two or three cycles of multiple switching relay oscillations.

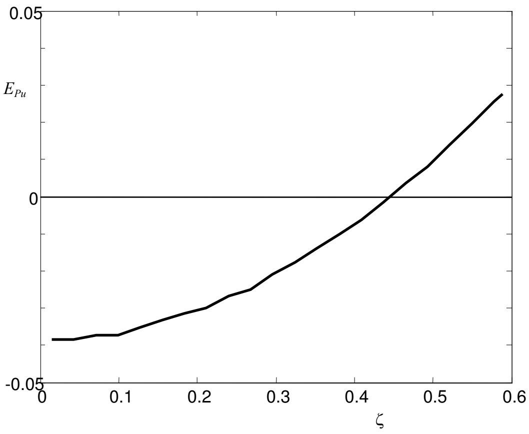


Fig. 6. Relative identification errors of the ultimate period (*E_{Pu}=(Pu-Pu*)/Pu**, *Pu**=exact ultimate period) for FOPTD process *G(s)=exp(-s)/(s+1)*.

4. DESCRIBING FUNCTION ANALYSIS

When is fixed, we can analyze the multiple switching relay feedback system by the describing function method (Atherton, 1982). Let the error signal be

$$e(t) = a\sin(t), \quad a > \zeta \quad (18)$$

Then the relay output is

$$u(t) = b_1 \sin(t) + b_3 \sin(3t) + \cdots$$

$$b_{2i-1} = \frac{4d}{\pi(2i-1)}\left(-1+2\cos((2i-1)a_2)\right) \quad (19)$$

$$a_2 = \sin^{-1}(\zeta / a)$$

Hence, if *a*> , the describing function is

$$N(a) = \frac{b_1}{a} = \frac{4d}{\pi a}\left(-1+2\cos(\sin^{-1}(\zeta / a))\right) \quad (20)$$

The describing function *N(a)* can be considered to be a gain between sine waves of the input and the output.

The relay feedback system is described as in Fig. 7. The characteristic equation is

$$1 + N(a)G(jw) = 0 \quad (21)$$

The oscillation period becomes the ultimate period approximately and the ultimate gain is

$$K_{cu} = 1/|G(jw)| = N(a) \quad (22)$$

Necessary condition for the stability of oscillation is (Atherton, 1982)

$$\frac{dG_I(j\omega)}{d\omega}\frac{dN(a)}{da} < 0 \quad (23)$$

where *G_I(j)* is the imaginary part of *G(j)*. Since $dN(a)/da < 0$ for *N(a)* of Eq. (20), $dG_I(j\omega)/d\omega > 0$ becomes a necessary condition for a stable oscillation. For overdamped processes, *G(j)* cut the negative real axis from the lower half plane to the upper half plane in the Nyquist plane and hence $dG_I(j\omega)/d\omega > 0$. The proposed method will pass the necessary condition for conventional overdamped processes.

5. Conclusions

Binary pulse-width-modulated signal can eliminate the third or the fifth harmonic term in the square wave of the conventional relay feedback system. Multiple switching relays realizing this PWM signal are proposed. They are shown to reduce the estimation error of ultimate periods by 50%, for FOPTD processes with time delays that are up to two times of the size of the time constants.

REFERENCES

Astrom, K.J., “Oscillations in Systems with Relay Feedback”, *IMA Vol. Math. Appl. Adapt. Control, Filter, Signal Processing*, **74**, 1-25 (1995).

Astrom, K.J.; Hagglund, T. “Automatic tuning of simple regulators with specifications on phase and amplitude margins,” *Automatica*, **20**, 645 (1984).

Astrom, K.J.; Hagglund, T., *PID controllers.*, Instrument Society of America, NC (1995).

Atherton, D.P., *Nonlinear Control Engineering*, Van Nostrand Reinhold, London (1982).

Chiasson, J.N.; Tolbert, M.; McKenzie, K.J.; Du, Z., "A Complete Solution to the Harmonic Elimination Problem", *IEEE Trans. Power Electronics*, **19**, 491-499 (2004).

Hang, C.C.; Astrom, K.J.; Wang, Q.G., “Relay feedback auto-tuning of process controllers – a tutorial review”, *J. Process Control*, **12**, 143-162 (2002).

Kreyszig, E., *Advanced engineering mathematics*, Wiley, New York (1999).

Lee, J.; Park, H.C.; Sung, S.W., “Analytic expressions of ultimate gains and ultimate period with phas-optimal approximations of time delays”, *Can. J. Chem. Eng.*, **83**, 990-995 (2005).

Lee, J.; Sung, S.W.; Edgar, T.F., “Integrals of relay feedback responses for extracting process information”, *AIChE J.*, **53**, 2329-2338 (2007).

Mohan, N.; Undeland, T.M.; Robbins, W.P., *Power Electronics, Converters, Applications and Design*, Wiley (1995).

Seborg, D.E.; Edgar, T.F.; Mellichamp, D.A., *Process dynamics and control*, 2nd ed., Wiley, NY (2004).

Sung, S.W.; Park, J.H.; Lee, I., “Modified relay feedback method,” *Ind. Eng. Chem. Res.*, **34**, 4133 (1995).

Tan, K.K.; Lee, T.H.; Huang, S.; Chua, K.Y.; Ferdous, R., “Improved critical point estimation using a preload relay,” *J. Process Control*, **16**, 445 (2005).

Yu, C.C., *Autotuning of PID controllers: A relay feedback approach*, Springer, London (2006).