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Adaptive Tracking Control of Coordinated Nonholonomic Mobile Manipulators

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Abstract: In this paper, adaptive control of multiple mobile manipulators carrying a common object in a cooperative manner with unknown inertia parameters and disturbances has been investigated. firstly, A complete dynamics consisting of the dynamics of mobile manipulators and the object is presented for coordinated multiple mobile manipulators. Then, adaptive control has been designed for compensating parametric uncertainties and suppressing disturbances. The control ensures that the output tracking errors of the system converge to zero whereas the internal force tracking error remains bounded and can be made arbitrarily small. Simulation studies show the effectiveness of the proposed scheme.

1. INTRODUCTION

Coordinated systems of multiple mobile manipulators can be much more flexible than single mobile manipulator in accomplishing complex and changeable tasks. However, such systems may also bring complexity in control when mobile manipulators cooperate closely and form a closed-chain mechanism. In recent years, the control of coordinated mobile manipulators has received considerable attention(Khatib et al. (1996), Yamamoto and Fukuda (2002), Sugar and Kumar (2002), Tanner et al. (1998), Tanner et al. (2003)). Research difficulty lies in when kinematic and dynamic constraints act on coordinated multiple mobile manipulators, such as their positions and velocities. The degree of freedom of the whole system decreases, and the generated internal forces need to be controlled.

To solve control problems of such system, some control methods, such as hybrid position force control (See Khatib et al. (1996), Yamamoto et al. (2004), Tanner et al. (2003)); and leader-follower method (See Sugar and Kumar (2002), Trebi-Ollennu et al. (2002), Hirata et al. (2004)), have been proposed. The drawback of aforementioned schemes for coordinated control of multiple mobile manipulators is their dependance on the precise knowledge of the complex dynamics of the system. To deal with the uncertainties in the dynamics, adaptive and robust coordinated control schemes should be led-in. Some recent works have been successfully doing so. Adaptive control was proposed for trajectory/force control of mobile manipulators subjected to holonomic and nonholonomic constraints with unknown inertia parameters (See Dong (2002)). In Li et al. (2007a), a unified robust adaptive force-motion control was proposed for single mobile manipulator subjected to holonomic and nonholonomic constraints with unknown inertia parameters and disturbances. In Li et al. (2007b) and Li et al. (2007c), robust adaptive controls for coordinated system of multiple mobile manipulators was proposed.

In this paper, motivated by previous works (See Li et al. (2007a), Li et al. (2007b), Li et al. (2007c)), we further consider the situation when multiple mobile manipulators are grasping an object in a cooperative manner as shown in Fig. 1. The purpose of controlling such coordinated system is to control the object in the desired motion. Meanwhile, internal forces which do not contribute to system motion are maintained under desired values.

This paper is organized as follows. In section 2, some descriptions and assumptions of the system is briefly introduced. In section 3, the dynamics of inter-connected system including coordinated mobile manipulators' dynamics plus object dynamics and the interaction between object and environments is developed. In section 4, the adaptive control law based on full dynamic model is presented and this is followed by a simulation study in section 5. Finally, some conclusion remarks are stated in section 6.

2. SYSTEM DESCRIPTION AND ASSUMPTION

Consider m mobile manipulators holding a common rigid object in a task space. Different coordinate frames have been established for system modeling, in which $OXYZ$ is the inertial reference frame in which the position and orientation of the mobile manipulator end-effectors and the object are referred, $O_oX_oY_oZ_o$ is the object coordinate frame fixed at the center of mass of the object, and $O_{ie}X_{ie}Y_{ie}Z_{ie}$ is the end-effector frame of the ith manipulator located at the grasp point.

To facilitate the dynamic formulation, the following assumptions are made.

Assumption 2.1 All the end-effectors of the manipulators are rigidly attached to the common object so that there is no relative motion between the object and the end-effectors.

Assumption 2.2 The object is rigid, that is, the object does not get deformed with the application of forces.

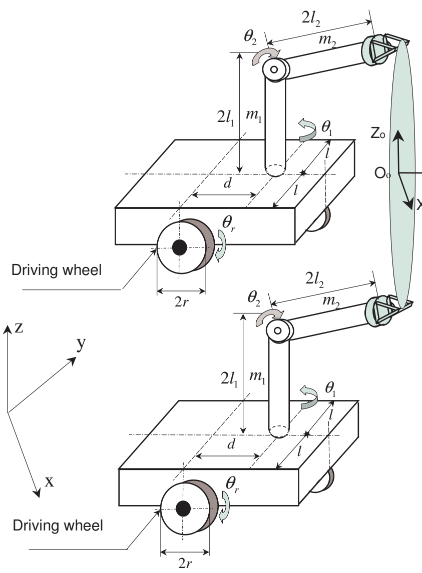

Fig. 1. Two coordinated mobile manipulators

Assumption 2.3 Each manipulator is non-redundant and operating away from any singularity.

3. DYNAMICS OF SYSTEM

3.1 Dynamics of Multiple Mobile Manipulators

The dynamics of the ith mobile manipulator in joint space is given by

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) + d_i = B_i(q_i)\tau_i + J_i^T f_i \quad (1)$$

where $q_i = [q_{iv}^T, q_{ia}^T]^T \in R^n$ with $q_{iv} \in R^{n_v}$ describing the generalized coordinates for the mobile platform and $q_{ia} \in R^{n_a}$ denoting the generalized coordinates of the manipulator, and $n = n_v + n_a$. $M_i(q_i) \in R^{n\times n}$ is the symmetric positive definite inertia matrix, $C_i(\dot{q}_i, q_i) \in R^{n\times n}$ presents the Centrifugal and Coriolis effects, $G_i(q_i) \in R^n$ presents the gravitational torque vector, $d_i(t) \in R^n$ presents the external disturbances and $\tau_i \in R^k$ presents the control inputs. $B_i(q_i) = \text{diag}[B_{iv},\ B_{ia}] \in R^{n\times k}$ is a full rank input transformation matrix for the mobile platform and the robotic manipulator, $J_i^T \in R^{n\times n}$ is Jacobian matrix and f_i are the constraint forces.

In this paper, the mobile platform is subjected to nonholonomic constraints, and the holonomic constraint force is measured by the force sensor mounted on each mobile manipulator's end-effector. The l non-integrable and independent velocity constraints can be expressed as

$$A_i(q_i)\dot{q}_i = 0 \quad (2)$$

where $A_i = [A_{i1}^T(q_i),\ \ldots,\ A_{il}^T(q_i)]^T : R^n \to R^{l\times n}$ is the kinematic constraint matrix. Thus, we have $H^T(q_i)A_i^T(q_i) = 0$, $H(q_i) = [H_1(q_i),\ \ldots,\ H_{n-l}(q_i)] \in R^{n\times(n-l)}$ which means a set of $(n-l)$ smooth and linearly independent vector fields. Constraints (2) implies the existence of vector $\eta_i \in R^{n-l}$, such that

$$\dot{q}_{iv} = H(q_i)\dot{\eta}_i \quad (3)$$

Consider the nonholonomic constraints (2) and (3) and its derivative, the dynamics of a mobile manipulator (1) can be expressed as

$$M_i^1(\zeta_i)\ddot{\zeta}_i + C_i^1(\zeta_i,\dot{\zeta}_i)\dot{\zeta}_i + G_i^1(\zeta_i) + d_i^1 = u_i + J_{ie}^T f_{ie} \quad (4)$$

where

$$M_i^1 = \begin{bmatrix} H^T M_{iv} H & H^T M_{iva} \\ M_{iav} H & M_{ia} \end{bmatrix}, \zeta_i = \begin{bmatrix} \eta_i \\ q_{ia} \end{bmatrix}, G_i^1 = \begin{bmatrix} H^T G_{iv} \\ G_{ia} \end{bmatrix}$$

$$C_i^1 = \begin{bmatrix} H^T M_{iv}\dot{H} + H^T C_{iv} H & H^T C_{iva} \\ M_{iav}\dot{H} + C_{iav} H & C_{ia} \end{bmatrix}, u_i = B_i^1 \tau_i$$

$$J_{ie} = \begin{bmatrix} 0 & 0 \\ J_{iv} H & J_{ia} \end{bmatrix}, B_i^1 = \begin{bmatrix} H^T B_{iv} & 0 \\ 0 & B_{ia} \end{bmatrix}, d_i^1 = \begin{bmatrix} H^T d_{iv} \\ d_{ia} \end{bmatrix}$$

The dynamics of m mobile manipulators from (4) can be expressed concisely as

$$M(\zeta)\ddot{\zeta} + C(\zeta,\dot{\zeta})\dot{\zeta} + G(\zeta) + D = U + J_e^T F_e \quad (5)$$

where $M(\zeta) = $ block diag $[M_1^1(\zeta_1),\ \ldots,\ M_m^1(\zeta_m)] \in R^{m(n-l)\times m(n-l)}$; $\zeta = [\zeta_1^T,\ \ldots,\ \zeta_m^T]^T \in R^{m(n-l)}$; $U = [(B_1^1\tau_1)^T,\ \ldots,\ (B_m^1\tau_m)^T]^T \in R^{m(n-l)}$; $G(\zeta) = [G_1^{1T}(\zeta_1), \ldots, G_m^{1T}(\zeta_m)]^T \in R^{m(n-l)}$; $F_e = [f_{1e}^T,\ \ldots,\ f_{me}^T]^T \in R^{m(n-l)}$; $C(\zeta,\dot{\zeta}) = $ block diag $[C_1^1(\zeta_1,\dot{\zeta}_1),\ \ldots,\ C_m^1(\zeta_m,\dot{\zeta}_m)] \in R^{m(n-l)\times m(n-l)}$; $D = [d_1^{1T},\ \ldots,\ d_m^{1T}]^T \in R^{m(n-l)}$; $J_e^T = $ block diag$[J_{1e}^T, \ldots, J_{me}^T]^T \in R^{m(n-l)\times m(n-l)}$

3.2 Dynamics of the Common Object

Let $x_o \in R^{n_o}$ the position/orientation vector of the object, the equation of motion of the object is written by the resultant force vector $F_o \in R^{n_o}$ acting on the center of mass of the object, the symmetric positive definite inertial matrix $M_o(x_o) \in R^{n_o\times n_o}$ of the object, the Corioli and centrifugal matrix $C_o(x_o,\dot{x}_o) \in R^{n_o\times n_o}$, and the gravitational force vector $G_o(x_o) \in R^{n_o}$ as

$$M_o(x_o)\ddot{x}_o + C_o(x_o,\dot{x}_o)\dot{x}_o + G_o(x_o) = F_o \quad (6)$$

Define $J_o(x_o) \in R^{m(n-l)\times n_o}$ as $J_o(x_o) = [J_{1o}^T(x_o),\ \ldots,\ J_{mo}^T(x_o)]^T$ with the Jacobian matrix $J_{io}(x_o)$ from the object frame $O_oX_oY_oZ_o$ to the ith mobile manipulator's end-effector frame $O_{ie}X_{ie}Y_{ie}Z_{ie}$. Then the relationship between the resultant force F_o, the end-effector force F_e and the internal force F_I can be written as following equation (See Jean an Fu (1993))

$$F_e = -(J_o^T(x_o))^+ F_o - F_I \quad (7)$$

where $(J_o^T(x_o))^+ \in R^{m(n-l)\times n_o}$ is the pseudo-inverse matrix of $J_o^T(x_o)$, $F_I \in R^{m(n-l)}$ satisfies $J_o^T(x_o)F_I = 0$.

From Xi et al. (1996), we have F_I can be parameterized by the vector of Lagrangian multiplier $\lambda_I \in R^{n_\lambda}$ as $F_I = \mathcal{J}^T\lambda_I$. Let $\mathcal{J}^T = I - (J_o^T(x_o))^+ J_o^T(x_o)$, where $\mathcal{J}^T \in R^{m(n-l)\times n_\lambda}$ is Jacobian matrix for the internal force satisfying $J_o^T(x_o)\mathcal{J}^T = 0$.Also considering (6), we have

$$F_e = -(J_o^T(x_o))^+(M_o(x_o)\ddot{x}_o + C_o(x_o,\dot{x}_o)\dot{x}_o + G_o(x_o)) - \mathcal{J}^T\lambda_I \quad (8)$$

3.3 Dynamics of System

Let $x_{ie} \in R^{n-l}$ denote the position and orientation vector of the ith end-effector. Then $\dot{x}_{ie}$ is related to $\dot{\zeta}_i$ the Jacobian matrix $J_{ie}(\zeta_i)$ as $\dot{x}_{ie} = J_{ie}(\zeta_i)\dot{\zeta}_i$. The relationship between $\dot{x}_{ie}$ and $\dot{x}_o$ is given by $\dot{x}_{ie} = J_{io}(x_o)\dot{x}_o$. As it is assumed that the manipulators work in a nonsingular region, thus the inverse of the Jacobian matrix $J_{ie}(\zeta_i)$ exists. Considering all the manipulators acting on the object at the same time, it yields

$$\dot{\zeta} = J_e^{-1}(\zeta)J_o(x_o)\dot{x}_o \tag{9}$$

Using equations (9) and its derivative, and considering $J_o^T(\zeta)\mathcal{J}^T = 0$, the dynamics of multiple manipulators system (5), coupled with the object dynamics (6), is then given by

$$\mathcal{M}\ddot{x}_o + \mathcal{C}\dot{x}_o + \mathcal{G} + \mathcal{D} = \mathcal{U} \tag{10}$$

$$\lambda_I = Z(U - C^*\dot{x}_o - G^* - D) \tag{11}$$

where

$$\begin{aligned}
L &= J_e^{-1}(\zeta)J_o(x_o)\\
\mathcal{M} &= L^T M(\zeta)L + M_o(x_o)\\
\mathcal{C} &= L^T M(\zeta)\dot{L} + L^T C(\zeta,\dot{\zeta})L + C_o(x_o,\dot{x}_o)\\
\mathcal{G} &= L^T G(\zeta) + G_o(x_o)\\
\mathcal{D} &= L^T D\\
\mathcal{U} &= L^T U\\
M^* &= M(\zeta) + J_e^T(\zeta)(J_o^T(x_o))^+ M_o(x_o)(J_o(x_o))^+ J_e(\zeta)\\
Z &= (\mathcal{J}J_e(\zeta)(M^*)^{-1}J_e^T(\zeta)\mathcal{J}^T)^{-1}\mathcal{J}J_e(\zeta)(M^*)^{-1}\\
C^* &= M(\zeta)\dot{L} + C(\zeta,\dot{\zeta})L + J_e^T(\zeta)(J_o^T(x_o))^+ C_o(x_o,\dot{x}_o)\\
G^* &= G(\zeta) + J_e^T(\zeta)(J_o^T(x_o))^+ G_o(x_o)
\end{aligned}$$

The dynamic equation (10) has following structure properties, which can be exploited to facilitate the control system design.

Assumption 3.1 The external disturbance is bounded, i.e. $\|\mathcal{D}\| \le c_d$, c_d is an unknown constant.

Property 3.1 The matrix M^2 is symmetric positive definite, and is bounded, i.e., $\lambda_{min}(\mathcal{M})I \le \mathcal{M} \le \lambda_{max}(\mathcal{M})I$, where $\lambda_{min}(\mathcal{M})$ and $\lambda_{max}(\mathcal{M})$ denote the minimum and maximum eigenvalues of $\mathcal{M}$.

Property 3.2 The matrix $\dot{\mathcal{M}} - 2\mathcal{C}$ is skew-symmetric, that is, $r^T(\dot{\mathcal{M}} - 2\mathcal{C})r = 0$ for any vector $r \in R^n$.

Property 3.3 All Jacobian matrices are uniformly bounded and uniformly continuous if ζ and $x_o$is uniformly bounded and continuous, respectively.

Property 3.4 From Assumption 3.1, for any differentiable x_o, $\mathcal{M}\ddot{x}_o + \mathcal{C}\dot{x}_o + \mathcal{G} + \mathcal{D} = Y(\zeta,\dot{\zeta},x_o,\dot{x}_o)\vartheta$, where $Y(\zeta,\dot{\zeta},x_o,\dot{x}_o)$ is a known matrix containing $\zeta,\dot{\zeta},x_o,\dot{x}_o$. ϑ is an unknown constant vectors (See Dong (2002), Lian et al. (2002)).

4. ADAPTIVE CONTROL DESIGN

Given a desired motion trajectory $x_{od}(t)$ and a desired internal force λ_{Id}, since the system is inter-connected, we can obtain the desired motion trajectory $q_d(t)$. therefore, the trajectory and internal force tracking control is to determine a control law such that for any $(x_o(0),\dot{x}_o(0)) \in \Omega$, $x_o, \dot{x}_o$, λ_I converge to a manifold specified as Ω where

$$\Omega_d = \{(x_o,\dot{x}_o,\lambda_I)|x_o = x_{od}, \dot{x} = \dot{x}_{od}, \lambda_I = \lambda_{Id}\} \tag{12}$$

Assumption 4.1 The desired reference trajectorie $x_{od}(t)$ is assumed to be bounded and uniformly continuous, and has bounded and uniformly continuous derivatives up to the third order. The desired internal force λ_{Id} is also bounded and uniformly continuous.

Let $e_o = x_o - x_{od}$, $\dot{x}_{or} = \dot{x}_{od} - K_o e_o$, $r = \dot{e}_o + K_o e_o$ with K_o is diagonal positive definite, $e_I = \lambda_I - \lambda_{Id}$.

Decoupled generalized position and constraint force separatively is introduced. Considering the control input U as the form:

$$U = U_a + J_e^T(\zeta)\mathcal{J}^T U_b \tag{13}$$

then, (10) and (11) may be changed to

$$\mathcal{M}\ddot{x}_o + \mathcal{C}\dot{x}_o + \mathcal{G} + \mathcal{D} = L^T U_a \tag{14}$$

$$\lambda_I = Z(U_a - C^*\dot{x}_o - G^* - D) + U_b \tag{15}$$

Consider the following control laws:

$$L^T U_a = -K_p r + Y(\zeta,\dot{\zeta},x_{or},\dot{x}_{or})\hat{\vartheta} \tag{16}$$

$$U_b = -\lambda_{max}(\mathcal{M})\|Z(L^T)^+\|\ddot{x}_{od} + \lambda_{Id} - K_f e_I \tag{17}$$

$$\dot{\hat{\vartheta}} = -\Gamma Y^T(\zeta,\dot{\zeta},x_{or},\dot{x}_{or})r \tag{18}$$

where K_p, K_f are positive definite, and $(L^T)^+ = J_e^T(\zeta)$ $(J_o^T(x_o))^+$. Define $\nu = \lambda_{min}(K_p)/\lambda_{max}(\mathcal{M}) > 0$.

Theorem 1. Considering the mechanical system described by (4), using the control law (16) and (17), the following holds for any $(x_o(0),\dot{x}_o(0)) \in \Omega$:

(i) r converges to a small set containing the origin with the convergence rate at least as fast as $e^{-\nu t}$;
(ii) e_o and $\dot{e}_o$ asymptotically converge to 0 as $t \to \infty$, and
(iii) e_I and τ are bounded for all $t \ge 0$.

Proof. (i) Integrating (13) into (14), the closed-loop system dynamics can be rewritten as

$$\mathcal{M}\dot{r} = L^T U_a - (\mathcal{M}\ddot{x}_{or} + \mathcal{C}\dot{x}_{or} + \mathcal{G} + \mathcal{D}) - \mathcal{C}r \tag{19}$$

Substituting (16) into (19), the closed-loop dynamic equation is obtained

$$\mathcal{M}\dot{r} = -K_p r + Y(\zeta,\dot{\zeta},x_{or},\dot{x}_{or})\hat{\vartheta} - \mu - \mathcal{C}r \tag{20}$$

where $\mu = \mathcal{M}\ddot{x}_{or} + \mathcal{C}\dot{x}_{or} + \mathcal{G} + \mathcal{D}$.

Consider the Lyapunov function candidate,

$$V = \frac{1}{2}r^T\mathcal{M}r + \frac{1}{2}\tilde{\vartheta}\Gamma^{-1}\tilde{\vartheta} \tag{21}$$

where $\tilde{\vartheta} = \hat{\vartheta} - \vartheta$,then

$$\dot{V} = r^T(\mathcal{M}\dot{r} + \frac{1}{2}\dot{\mathcal{M}}r) + \tilde{\vartheta}\Gamma^{-1}\dot{\tilde{\vartheta}} \tag{22}$$

From Property 3.1, we have $\frac{1}{2}\lambda_{min}(\mathcal{M})r^T r \leq V \leq \frac{1}{2}\lambda_{max}(\mathcal{M})r^T r$. By using Property 3.2, the time derivative of V along the trajectory of (20) is

$$\begin{aligned}\dot{V} &= -r^T K_p r - r^T \mu + r^T Y(\zeta, \dot{\zeta}, x_{or}, \dot{x}_{or})\hat{\vartheta} + \tilde{\vartheta}\Gamma^{-1}\dot{\tilde{\vartheta}} \\ &\leq -r^T K_p r + r^T Y(\zeta, \dot{\zeta}, x_{or}, \dot{x}_{or})\tilde{\vartheta} + \tilde{\vartheta}\Gamma^{-1}\dot{\tilde{\vartheta}} \\ &\leq -r^T K_p r \\ &\leq -\lambda_{min}(K_p)\|r\|^2\end{aligned}$$

Therefore, we arrive at $\dot{V} \leq -\nu V + \delta$. Thus, r converges to a set containing the origin with a rate at least at fast as $e^{-\nu t}$.

Integrating both sides of the above equation gives

$$V(t) - V(0) \leq -\int_0^t r^T K_p r ds + \rho < \infty \qquad (23)$$

Thus V is bounded, which implies that $r \in L_\infty^n$.

(ii) From (23), V is bounded, which implies that $x_o \in L_\infty^n$. We have $\int_0^t r^T K_p r ds \leq V(0) - V(t) + \rho$, which leads to $r \in L_2^n$. From $r = \dot{e}_o + K_o e_o$, it can be obtained that $e_o, \dot{e}_o \in L_\infty^n$. As we have established $e_o, \dot{e}_o \in L_\infty$, from Assumption 4.0.6, we conclude that $x_o(t), \dot{x}_o(t), \dot{x}_{or}(t), \ddot{x}_{or}(t) \in L_\infty^n$ and $\dot{q} \in L_\infty^{mn}$.

Therefore, all the signals on the right hand side of (19) are bounded, and we can conclude that $\dot{r}$ and therefore $\ddot{x}_o$ are bounded. Thus, $r \to 0$ as $t \to \infty$ can be obtained. Consequently, we have $e_o \to 0, \dot{e}_o \to 0$ as $t \to \infty$. It follows that $e_q, \dot{e}_q \to 0$ as $t \to \infty$.

(iii) Substituting the control (16) and (17) into the reduced order dynamic system model (15) yields

$$(I + K_f)e_I = Z(L^T)^+ \mathcal{M}\ddot{x}_o - \lambda_{max}(\mathcal{M})\|Z(L^T)^+\|\ddot{x}_{od} \qquad (24)$$

Since $\ddot{x}_o$ and Z are bounded, $x_o \to x_{od}$, $Z(L^T)^+ M^2 \ddot{x}_o - \lambda_{max}(\mathcal{M})\|Z(L^T)^+\|\ddot{x}_{od}$ is also bounded, the size of e_I can be adjusted by choosing the proper gain matrix K_f.

Since r, x_o, $\dot{x}_o$, x_{or}, $\dot{x}_{or}$, $\ddot{x}_{or}$, and e_I are all bounded, it is easy to conclude that τ is bounded from (16) and (17).

5. SIMULATION STUDIES

Let us consider two same 2-DOF mobile manipulators shown in Fig. 1. Each mobile manipulator is subjected to the following constraint:

$$\dot{x}_i \sin\theta_i - \dot{y}_i \cos\theta_i = 0$$

Using the Lagrangian approach, we can obtain the standard form (1) with $q_{iv} = [x_i\ y_i\ \theta_i]^T$, $q_{ia} = [\theta_{i1}\ \theta_{i2}]^T$, $q_i = [q_{iv}^T\ q_{ia}^T]^T$, $A_i = [\sin\theta_i\ \ -\cos\theta_i\ 0.0\ 0.0\ 0.0]^T$, and $\zeta = [y_i\ \theta_i\ \theta_{i1}\ \theta_{i2}]^T$, $\dot{\zeta} = [\dot{y}_i\ \dot{\theta}_i\ \dot{\theta}_{i1}\ \dot{\theta}_{i2}]^T$. $\tau_i = [\tau_{il}\ \tau_{ir}\ \tau_{i1}\ \tau_{i2}]^T$ where τ_{il}, τ_{ir}, τ_{i1} and τ_{i2} means the motor torque of left wheel, right wheel, joint one and joint two of ith manipulator respectively,. The dynamics of the ith mobile manipulator is given by (1) where M_i, C_i, G_i, D_i, and B_i are omitted here for the space limit.

The position of end-effector can be given by

$$\begin{aligned}x_{ie} &= x_{if} - 2l_2 \sin\theta_{i2}\cos(\theta_i + \theta_{i1}) \\ y_{ie} &= y_{if} - 2l_2 \sin\theta_{i2}\sin(\theta_i + \theta_{i1}) \\ z_{ie} &= 2l_1 - 2l_2\cos\theta_{i2} \\ \beta_{ie} &= \theta_i + \theta_{i1}\end{aligned}$$

where β_{ie} is the pitch angle for the ith end-effector.

So the mobile manipulator Jacobian matrix J_{ie} is given by

$$\begin{bmatrix}\dot{x}_{ie} \\ \dot{y}_{ie} \\ \dot{z}_{ie} \\ \dot{\beta}_{ie}\end{bmatrix} = \begin{bmatrix}J_{i11} & J_{i12} & J_{i13} & J_{i14} \\ J_{i21} & J_{i22} & J_{i23} & J_{i24} \\ J_{i31} & J_{i32} & J_{i33} & J_{i34} \\ J_{i41} & J_{i42} & J_{i43} & J_{i44}\end{bmatrix}\begin{bmatrix}\dot{y}_i \\ \dot{\theta}_i \\ \dot{\theta}_{i1} \\ \dot{\theta}_{i2}\end{bmatrix}$$

where

$$\begin{aligned}&J_{i11} = \cot\theta_i, J_{i12} = -d\sin\theta_i + 2l_2\sin\theta_{i2}\sin(\theta_i + \theta_{i1}) \\ &J_{i13} = 2l_2\sin\theta_{i2}\sin(\theta_i + \theta_{i1}) \\ &J_{i14} = -2l_2\cos\theta_{i2}\cos(\theta_i + \theta_{i1}) \\ &J_{i21} = 1.0, J_{i22} = d\cos\theta_i - 2l_2\sin\theta_{i2}\cos(\theta_i + \theta_{i1}) \\ &J_{i23} = -2l_2\sin\theta_{i2}\cos(\theta_i + \theta_{i1}) \\ &J_{i24} = -2l_2\cos\theta_{i2}\sin(\theta_i + \theta_{i1}) \\ &J_{i31} = 0.0, J_{i32} = 0.0, J_{i33} = 0.0, J_{i34} = 2l_2\sin\theta_{i2} \\ &J_{i41} = 0.0, J_{i42} = 1.0, J_{i43} = 1.0, J_{i44} = 0.0\end{aligned}$$

Let the position $x_o = [x_{1o}, x_{2o}, x_{3o}, x_{4o}]^T$ be positions to X axis, Y axis , Z axis and rotation angle to Z axis as shown in Fig. 1, Z_o is parallel to the Z axis. $f_{ie} = [f_{ix}\ f_{iy}\ f_{iz}\ \tau_{i\beta}]^T$. The dynamic equation of the object is given by

$$M_o(x_o)\ddot{x}_o + G_o(x_o) = J_{1o}^T(x_o)f_{1e} + J_{2o}^T(x_o)f_{2e} \qquad (25)$$

where M_o,G_o are omitted here, and

$$J_{1o}^T(x_o) = \begin{bmatrix}1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ l_{c1}\sin x_{4o} & -l_{c1}\cos x_{4o} & 0.0 & 1.0\end{bmatrix},$$

$$J_{2o}^T(x_o) = \begin{bmatrix}1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ -l_{c2}\sin x_{4o} & l_{c2}\cos x_{4o} & 0.0 & 1.0\end{bmatrix}$$

We could obtain $\mathcal{J}^T$ by $\mathcal{J}^T = I - (J_o^T(x_o))^+ J_o^T(x_o)$, moreover, F_e can be measured by the force sensor mounted on the end-effector, therefore, we can obtain F_o. From (7), we can obtain F_I, using $\mathcal{J}^T = I - (J_o^T(x_o))^+ J_o^T(x_o)$, we can obtain λ_I.

$$F_I = \begin{bmatrix}\cos x_{4o} & -\sin x_{4o} & 0.0 \\ \sin x_{4o} & \cos x_{4o} & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & l_{c1} & 1 \\ -\cos x_{4o} & \sin x_{4o} & 0.0 \\ -\sin x_{4o} & -\cos x_{4o} & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & l_{c2} & -1\end{bmatrix}\begin{bmatrix}\lambda_{Ix} \\ \lambda_{Iy} \\ \lambda_{I\beta}\end{bmatrix},$$

where λ_{Ix}, λ_{Iy}, and $\lambda_{I\beta}$ present components of compression force, shearing force and bending moment respectively.

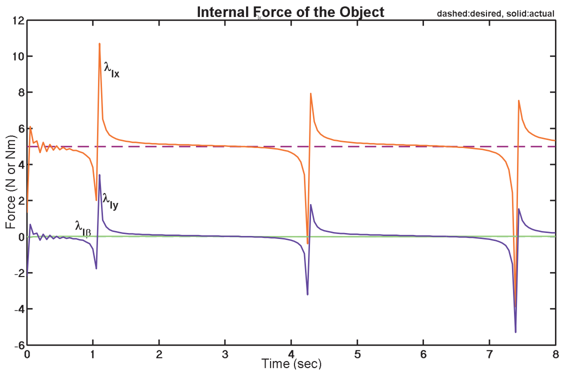

Fig. 2. The internal force of the object

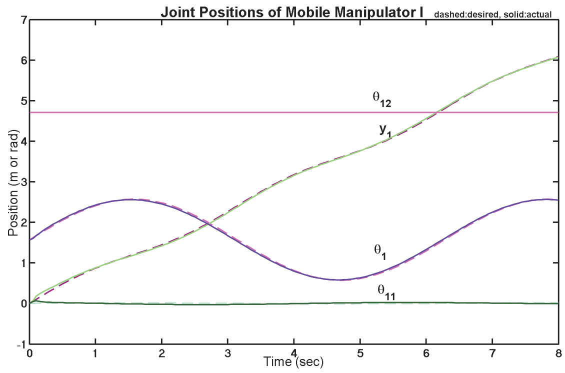

Fig. 3. The joint positions of mobile manipulator I

The desired trajectory for the object and the desired internal force are chosen as $\dot{x}_{od} = [\cos(\sin t + \frac{\pi}{2})\ \sin(\sin t + \frac{\pi}{2})\ 0.0\ \cos t + 0.01\cos t]^T$, $x_{od}(0) = [0.0\ 0.0\ 2l_1\ 0.0]^T$, $\lambda_{Id} = [5.0\ 0.0\ 0.0]^T$.

We can obtain the desired trajectory of each mobile manipulator after θ_{i2} being fixed as $\frac{3\pi}{2}$. Assuming that the parameters are selected as $m_w = 0.0kg$, $m_p = 6.0kg$, $m_1 = m_2 = 1.0kg$, $I_{zp} = 19.0kgm^2$, $I_{yw} = 0.0kgm^2$, $I_{zw} = 6.0kgm^2$, $I_{z1} = I_{z2} = 1.0kgm^2$, $d = 0.0m$, $l = 1.0m$, $r = 0.5m$, $2l_1 = 1.0m$, $2l_2 = 0.6m$, the mass of the object $m_o = 1.0kg$, $I_o = 1.0kgm^2$, $l_{c1} = l_{c2} = 0.5m$. The disturbance of mobile manipulator I d_1 is $[0.1\sin t\ -0.2\sin t\ -0.05\sin t\ 0\ 0]^T$, The disturbance of mobile manipulator II d_2 is $[0.2\sin t\ -0.2\sin t\ 0\ 0\ 0]^T$. We choose ϑ as $[m_{wp12}\ m_{12}\ m_2\ I_{zw}\ I_{zp}\ I_{z12}\ m_o\ I_o\ d_{11}\ d_{12}\ d_{13}\ d_{14}\ d_{15}\ d_{21}\ d_{22}\ d_{23}\ d_{24}\ d_{25}]^T$ and $\hat{\vartheta}(0) = [8.0\ 3.0\ 2.0\ 3.0\ 10.0\ 2.0\ 2.0\ 0.5\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$ are the nominal parameters of the system. $x_o(0) = \dot{x}_o(0) = 0$. The control gains are selected as $K_p = \mathrm{diag}[40.0]$, $K_o = \mathrm{diag}[8.0]$, $K_f = \mathrm{diag}[5000]$, and $\Gamma = \mathrm{diag}[0.5]$. The simulation results for two mobile manipulator system are shown in Figs. 2 - 6.

The simulation results show that the trajectory and internal force tracking errors tend to the desired values, which validates the effectiveness of the control law in Theorem 1. Comparing to the same system without adaptive control, (the joint velocities of mobile manipulator I with different control methods are shown in fig.7-8. Notice that the mass and inertia of the object are varied between $m_o = 10.0kg, I_o = 10.0kgm^2$ and $m_o = 0.1kg, I_o = 0.1kgm^2$ every 2 seconds, demonstrating the situation when the object need to be replaced while moving), we can conclude that the good performances is largely due to the "adaptive" mechanism though the parametric uncertainties and the external disturbances are both introduced into the simulation model. The simulation results demonstrate the effectiveness of the proposed adaptive control in the presence of unknown nonlinear dynamic system and environments. Different motion/force tracking performance can be achieved by adjusting parameter adaptation gains and control gains.

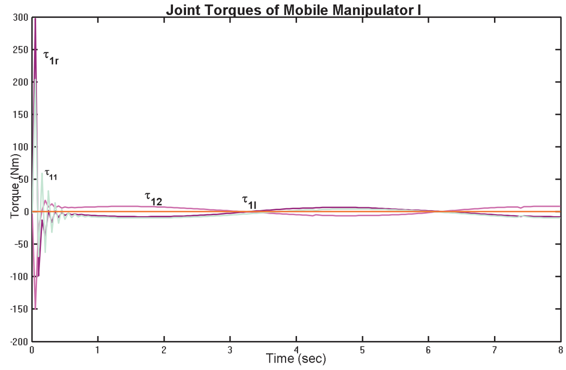

Fig. 4. The joint torques of mobile manipulator I

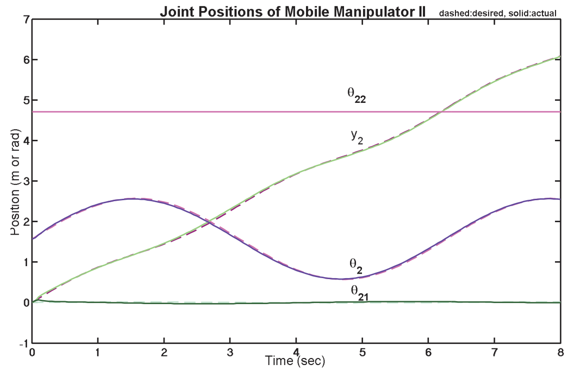

Fig. 5. The joint positions of mobile manipulator II

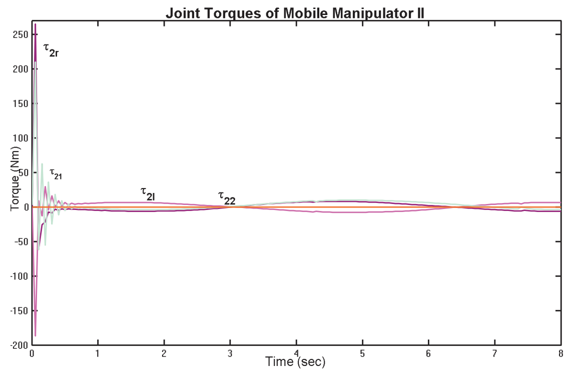

Fig. 6. The joint torques of mobile manipulator II

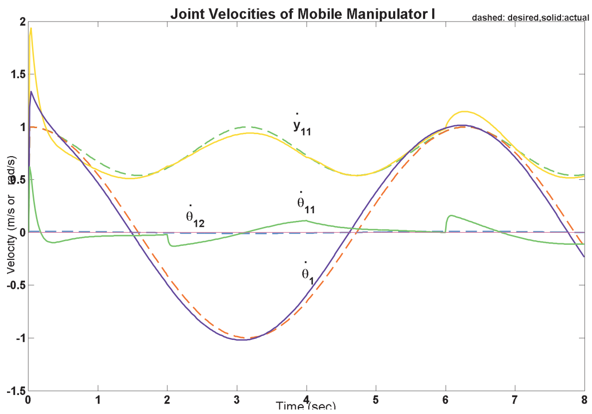

Fig. 7. The joint velocities of mobile manipulator I with adaptive control

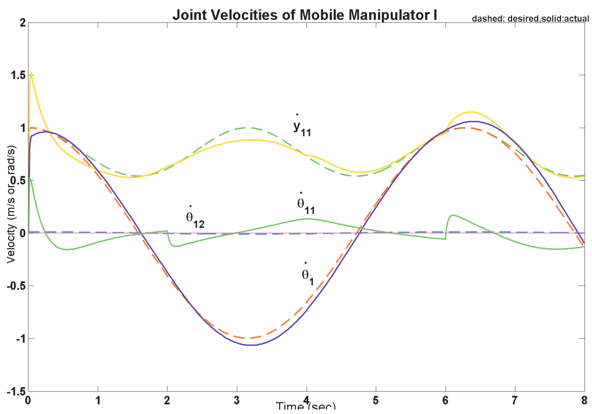

Fig. 8. The joint velocities of mobile manipulator I with nonadaptive control

6. CONCLUSION

In this paper, adaptive control strategies have been presented systematically to control the coordinated multiple mobile manipulators which carry a common object in the presence of uncertainties and disturbances. All control strategies have been designed to drive the system motion converged to the desired manifold and at the same time guarantee the range of the internal force. The proposed controls are non-regressor based and require no information on the system dynamics. Simulation results have shown the effectiveness of the proposed controls.

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