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A Method For Guidance of a Wheeled Mobile Robot Based on Received Radio Signal Strength Measurements ⋆

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Abstract: We propose a new guidance law for the problem of wheeled mobile robot (WMR) navigation towards an unknown target based on Received Signal Strength (RSS) Information. Given a mobile robot moving with a constant linear velocity, we use the miss-distance derivative as a measure for the angle at which the robot approaches the target. Miss-distance derivative is estimated from RSS, using a Robust Extended Kalman Filter (REKF). Having applied the proposed steering control law, termed Equiangular Navigation Guidance (ENG), the robot approaches the stationary or maneuvering target along a semi-equiangular spiral and eventually goes into a circular trajectory around it. In order to avoid circling, the algorithm is modified to decrease the robot's linear velocity to that of the target as it approaches the moving target and follow it in a smooth trajectory while preserving a safety distance from the target. Eventually, the performance of the guidance law and its effectiveness is confirmed with an extensive simulation study.

1. INTRODUCTION

The use of radio-signal-based approaches for the problem of localization or tracking of a target has gained significant interest in different applications such as surveillance, transportation systems, emergency services and so on. This is because of the distinct advantages of this strategy over all of the other systems. First, it is an economical solution to this problem, as wireless networks already exits as part of the communications infrastructure (e.g. in all laptop computers, PDAs and mobile phones it can be used as a positioning system). Second, it covers a large area compared with other indoor positioning systems. GPS systems can provide accurate location information in outdoor settings, however they fail in indoor navigation where GPS signals can not be reliably received. Video or IR based systems on the other hand, are restricted to line-of-sight limitations or poor performance with fluorescent lighting, direct sunlight and lack of light situations. Furthermore, in applications where the target is too small to appear in an image frame, radio-signal-based approaches are most practical.

There is a huge body of literature on robot navigation, however, in most of the current methods, target velocity, position, moving direction or line-of-sight angle (the angle between the reference line and the imaginary straight line starts at the robot's reference point and is directed towards the target's position) are considered given which are not always available in practice.

In this paper, considering dynamic constraints of the mobile robot, i.e. bounded linear and angular velocities, we propose a new algorithm, termed Equiangular Navigation Guidance (ENG), for the problem of robot guidance towards an unknown stationary or maneuvering target based on received signal power, which can be achieved by measuring the strength of the signal transmitted by the target and received at the robot position. Using RSS information, which is a function of miss-distance, we employ a robust estimator to estimate the miss-distance and its derivative. Since the target kinematic states are unknown, their effects are modeled as uncertainties during the estimation. With a robot moving with constant linear velocity, we use the estimated miss-distance derivative as a measure for the angle at which the robot approaches the target. Having applied the proposed idea, the robot moves towards the stationary target in a semi-equiangular spiral whose arc-length and curvature are subjects to change with a control parameter and eventually goes into a circular trajectory around the target.

This paper is organized as follows: problem statement and basic kinematic equation of a WMR are introduced in section 2. Measurement model of the system is defined in section 3. Section 4, presents an overview of the steering logic. Section 5, introduces the basic equations of REKF, which is emerged from the work of Petersen and Savkin [1999]. Proposed guidance law and its modification for smooth following of a maneuvering target is stated in section 6. REKF formulation for Adhoc system is stated in section 7. Simulation results for stationary and maneuvering targets are shown in 8. Finally, the paper is concluded in section 9.

2. PROBLEM STATEMENT

Let us consider a three-wheeled, non-holonomic mobile robot of Dubin's car type which moves in a horizontal

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plane and obstacle-free environment. In a two-dimensional space, the position of the robot can be represented by a triplet $P_R = (X_R, Y_R, \theta_R)$ where (X_R, Y_R) is the location of the middle of the wheel base and θ_R is the heading angle with respect to the reference line. Let V_R be the linear velocity and ω_R the angular velocity of mobile robot. A rolling-without-slippage model is assumed for the robot. The evolution model is classically given by:

$$\begin{aligned} \dot{X}_R &= V_R cos(\theta_R) \\ \dot{Y}_R &= V_R sin(\theta_R) \\ \dot{\theta}_R &= \omega_R \end{aligned} \tag{1}$$

with $U = [V_R \ \omega_R]^T$ as the control vector of the mobile robot, $U = V \times [-\omega_{max} \ \ \omega_{max}]$ with $V, \omega_{max} > 0$. We consider the target as another nonholonomic vehicle and represent its position and orientation with $P_T = (X_T, Y_T, \theta_T)$ which has the same kinematic equation as (1) with (V_T, ω_T) as linear and angular velocities, respectively.

The target may be stationary or moving in any direction. No information of the target motion is available. We assume the following conditions are satisfied:
- The robot is faster than target
- The robot has a higher level of maneuverability than target

Both mobile robot and target are equipped with a RF-transceiver. The transmitted signal by the target is measured at the mobile robot. The voltage measured by a receivers received signal strength indicator circuit, is defined as received signal strength (RSS). Given a model of radio signal propagation in an environment, received signal strength (RSS) can be used to estimate the relative distance between the target and mobile robot.

The objective is to design a guidance law in relative coordinates which allows the robot to approach the target and stay on target position or follow it as close as possible using the RSS measurements, only. We chose the relative coordinate system due to its simplicity in calculations and also nonnecessity to other sensors to measure the robot or target absolute positions.

3. MEASUREMENT MODEL

The received signal can be modeled as a function, composed of two effects: due to pass lost and due to shadow fading, Liu et al. [1998]. Fast fading is negligible, assuming a low pass filter is used to attenuate Rayleigh or Rician fade. The ensemble mean power at distance d from the target is typically modeled as, Xia [1996]

$$P(t) = P_o - 10\varepsilon log_{10} d(t) + v(t) \tag{2}$$

where P_o is a constant determined by the transmitted power, wavelength, and antenna gain of the mobile robot. ε is a slope index (typically 2 for highways and 4 for microcells in the city), $v(t)$ is the logarithm of the shadowing component, which is considered as an uncertainty in the measurement, and $d(t)$ represents the distance between the mobile robot and target. Measurement model is a function $d(t)$ and ε. To have an accurate estimation of d throughout the experiment, we consider ε as an unknown constant and augment a state to the system to estimate this variable through the following equation:

$$\dot{\varepsilon} = 0 \tag{3}$$

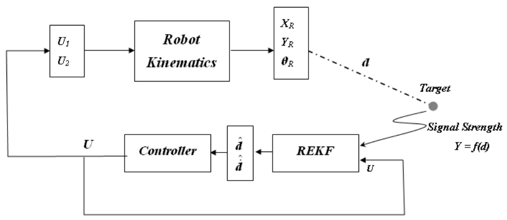


Fig. 1. Block diagram of the closed loop control system

Since, the strength of the signal is a function of miss-distance, we use the received signal as an input to an estimator to estimate the miss-distance and its derivative. We consider a stationary target and superimpose an additional term to the dynamic equation as an input disturbance to model uncertainty due to target movement. This assumption plus other noises and uncertainties in the system, motivate us to apply a robust estimator in the system.

4. AN OVERVIEW OF THE GUIDANCE LOGIC

Measured in decibels at the mobile robot, RSS can be used to estimate the distance between the robot and target. Previously, Kalman filter has been widely used in localization and tracking problems in wireless networks (see e.g. Pathirana et al. [2005]. In this paper, we apply the measured RSS to a robust extended Kalman filter (REKF). In addition to providing the satisfactory results, REKF eliminates the necessity of the knowledge of the measurement noise in standard kalman filter; It is also considerably more computation and memory efficient than more adaptive Bayesian filter, Pathirana et al. [2005]. The output of estimator is the input to a steering control law, which guides the robot towards the target. Fig. (1) shows a simplified block diagram of the system. Given the robot position and orientation with respect to the target posture in the polar coordinate system, we define the relative distance between the robot and target, d, and the angle between the front-direction and the target direction, λ, as shown in Fig. (2)

$$\begin{aligned} d &= \sqrt{d_X^2 + d_Y^2} \\ \lambda_R &= \psi_R - \theta_R \\ \lambda_T &= \psi_R - \theta_T \end{aligned} \tag{4}$$

where θ_R and θ_T are the robot and target heading angles, respectively. ψ_R is the line-of-sight angle and $|\lambda_R| \leq \pi$, $|\lambda_T| \leq \pi$. The robot-target motions is expressed by

$$\dot{d} = -V_R cos(\lambda_R) + V_T cos(\lambda_T) \tag{5a}$$

$$\dot{\lambda}_R = -\omega_R + \frac{V_R}{d} sin(\lambda_R) - \frac{V_T}{d} sin(\lambda_T) \tag{5b}$$

Note that the kinematic equations (5) are only valid for non-zero values of the miss-distance, since λ_R is undefined for $d = 0$. With constant robot's linear velocity, considering (5a), the angle λ_R has the main role to guide the robot towards the target. Since d is the only available information, we use the miss-distance variation as a measure for the angle at which the robot approaches the target. With a fixed $\dot{d}$ smaller than V_R, given (5a), the robot approaches the stationary target with a fixed $\lambda_R = \lambda_o$ along the trajectory, where $0 < |\lambda_o| < \frac{\pi}{2}$. We propose a new steering

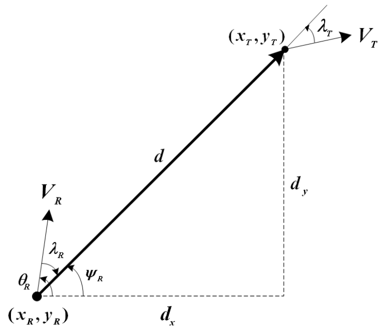


Fig. 2. Robot position and orientation with respect to the target

control law which drives the robot towards the target in a semi-equiangular spiral. However, it is required beforehand to present a brief explanation of REKF.

5. SET-VALUE STATE ESTIMATION WITH A NONLINEAR SIGNAL MODEL

Due to the nonlinearity of measurement equation, we consider a nonlinear uncertain system of the form: Petersen and Savkin [1999]

$$\begin{aligned} \dot{x} &= A(x,u) + B_2 w \\ z &= K(x,u) \\ y &= C(x) + v \end{aligned} \tag{6}$$

defined on a finite time interval $[0, s]$. Here $x(t) \in R^n$ denotes the state of the system, $y(t) \in R^l$ is the *measured output* and $z(t) \in R^q$ is *uncertainty output.* the uncertainty inputs are $w(t) \in R^p$, $v(t) \in R^l$. Also $u(t) \in R^m$ is the known control input. We assume that all of the functions appearing in (6) are with continues and bounded partial derivatives. Additionally, we assume that $K(x,u)$ is bounded. This was assumed to simplify the mathematical derivations and can be removed in practice. The matrix B_2 is assumed to be independent of x, and is of full rank. The uncertainty in the system is described by *nonlinear integral constraint* which has been studied in many papers and books on robust control, (see e.g. Petersen et al. [2000] and references therein) :

$$\phi(x(0)) + \int_0^s L_1(w(t), v(t))dt \le a + \int_0^s L_2(z(t))dt \tag{7}$$

where $a \ge 0$ is a positive real number. Here Φ, L_1 and L_2 are bounded non-negative functions with continues partial derivatives satisfying growth conditions of the type

$$\|\phi(x) - \phi(\acute{x})\| \le \beta(1 + \|x\| + \|\acute{x}\|)\|x - \acute{x}\| \tag{8}$$

where $\|.\|$ is the euclidian norm with $\beta > 0$ and $\phi = \Phi, L_1, L_2$. Uncertainty inputs $w(.), v(.)$ satisfying this condition are called *admissible uncertainties.* We consider the problem of characterizing the set of all possible states χ_s of the system (6) at time $s \ge 0$ which are consistent with a given control input $u^0(.)$ and a given path $y^0(.)$; i.e., $x \in \chi_s$ if and only if there exist admissible uncertainties such that if $u^0(t)$ is the control input and $x(.)$ and $y(.)$ are resulting trajectories, then $x(s) = x$ and $y(t) = y^0(t)$, for all $0 \le t \le s$.

5.1 The State Estimator

The state estimation set χ_s is characterized in terms of level sets of the solution $V(x,s)$ of the PDE

$$\begin{aligned} &\frac{\partial}{\partial t}V + max_{w \in R^m}\{\nabla_x V.(A(x,u^0) + B_2 w) \\ &- L_1(w, y^0 - C(x)) + L_2(K(x,u^0))\} = 0 \end{aligned} \tag{9}$$

$$V(\cdot, 0) = \Phi.$$

The PDE (9) can be viewed as a filter, taking observations $u^0(t)$, $y^0(t)$, $0 \le t \le s$ and producing the set χ_s as an output. The state of this filter is the function $V(.,s)$; thus V is an information state for the state estimation problem.

Theorem 1.1: Assume the uncertain system (6), (7) satisfies the assumptions given above. Then the corresponding set of possible states is given by

$$\chi_s = x \in R^n : V(x,s) \le a \tag{10}$$

where $V(x,t)$ is the unique viscosity solution of (9) in $C(R^n \times [0,s])$.(see Petersen and Savkin [1999] for the proof, originally presented in James and Petersen [1998])

5.2 A Robust Extended Kalman Filter

The aim of REKF is increasing the robustness of the state estimation process and decreasing the chance that a small deviation from the Gaussian process in the system noise causes a significant negative impact on the solution.

We consider an approximation to the PDE (9) which leads to a Kalman filter like characterization of the set χ_s. This was presented in Petersen and Savkin [1999] as an Extended Kalman filter version of the solution to the set value state estimation problem for a linear plant with the uncertainty described by an Integral Quadratic Constraint (IQC)(see e.g. Moheimani et al. [1998] and Petersen and Savkin [1999]). This IQC is also presented as a special case of (7). We consider uncertain system described by (6) and an integral quadratic constraint of the form

$$\begin{aligned} &(x(0) - x_0)' X_0 (x(0) - x_0) \\ &+ \frac{1}{2}\int_0^s (w(t)'Q(t)w(t)) + (v(t)'R(t)v(t))dt \\ &\le a + \frac{1}{2}\int_0^s z(t)'z(t)dt \end{aligned} \tag{11}$$

where $N > 0, Q > 0$ and $R > 0$. For the system (6),(11), the PDE (9) can be written as

$$\begin{aligned} &\frac{\partial}{\partial t}V + \nabla_x V.A(x,u^0) + \frac{1}{2}\nabla_x V B_2 Q^{-1} B_2' \nabla_x V' \\ &-\frac{1}{2}(y^0 - C(x))' R (y^0 - C(x)) + \frac{1}{2}K(x,u^0)'K(x,u^0) = 0 \end{aligned} \tag{12}$$

$$V(x,0) = (x - x_0)' N (x - x_0).$$

Consider a function $\hat{x}(t)$ defined as $\hat{x}(t) \equiv \text{argmin}_x V(x,t)$, and the following equations (13),(14) and (15), we define our approximate solution to the PDE (12):

$$\begin{aligned} \dot{\tilde{x}} &= A(\tilde{x}(t), u^0) + X^{-1}[\nabla_x C(\tilde{x}(t))' R (y^0 - C(\tilde{x}(t))) \\ &+ \nabla_x K(\tilde{x}t, u^0)' K(\tilde{x}(t), u^0)], \quad \tilde{x}(t) = x_0. \end{aligned} \tag{13}$$

$X(t)$ is defined as the solution to the Riccati Differential equation(RDE)

$$\begin{aligned}&\dot{X} + \nabla_x A(\tilde{x}, u^0)'X + X\nabla_x A(\tilde{x}, u^0)\\ &+XB_2Q^{-1}B_2'X - \nabla_x C(\tilde{x}(t))'R\nabla_x C(\tilde{x}(t))\\ &+\nabla_x K(\widetilde{x,u^0})'\nabla_x K(\tilde{x}, u^0) = 0, \;\; X(0) = N\end{aligned} \tag{14}$$

and

$$\begin{aligned}\phi(t) \equiv \frac{1}{2}\int_0^s &[(y^0 - C(\tilde{x}))'R(y^0 - C(\tilde{x}))\\ &-K(\widetilde{x,u^0})'K(\widetilde{x,u^0})]d\tau\end{aligned} \tag{15}$$

The function V(x,t) was approximated by a function of the form

$$\tilde{V}(x,t) = \frac{1}{2}(x - \tilde{x}(t))'X(t)(x - \tilde{x}(t)) + \phi(t) \tag{16}$$

hence, it follows from theorem 1.1 that an approximate formula for the set χ_s is given by

$$\tilde{\chi}_s = \{x \in R^n : \frac{1}{2}(x - \tilde{x}(s))'X(s)(x - \tilde{x}(s)) \le a - \phi(s)\} \tag{17}$$

This amounts to the so called Robust Kalman Filter(REKF) generalization presented in (Petersen and Savkin [1999]). In the application of REKF in this problem, during a corresponding time interval, the system (*robot-target*) is represented by the nonlinear uncertain system in (6) together with the following Integral Quadratic Constraint (IQC) (from equation (11)):

$$\begin{aligned}&(x(0) - x_0)'N(x(0) - x_0)\\ &+\frac{1}{2}\int_0^s (w(t)'Q(t)w(t)) + (v(t)'R(t)v(t))dt\\ &\le a + \frac{1}{2}\int_0^s z(t)'z(t)dt\end{aligned} \tag{18}$$

here $Q > 0, R > 0$ and $N > 0$ are the waiting matrices and the initial state (x_0), is the estimated state of system at startup. With an uncertainty relationship of the form of (18), the inherent measurement noise (see (2)), unknown target maneuver and the uncertainty in the initial condition are considered as bounded deterministic uncertain inputs. In particular, the measurement equation with the standard norm bounded uncertainty can be written as

$$y = C(x) + \delta C(x) + v_0 \tag{19}$$

where $|\delta| \le \xi$ with ξ, a constant indicating the upper bound of the norm bounded portion of the noise. By choosing $z = \xi C(x)$ and $v = \delta C(x)$,

$$\int_0^T |v|dt \le \int_0^T \acute{z}zdt \tag{20}$$

Considering v_0 and the corresponding uncertainty in w as w_0 satisfying the bound

$$\Phi(x(o)) + \int_0^s (w_0(t)'Q(t)w_0(t)) + (v_0(t)'R(t)v_0)(t)dt \le a \tag{21}$$

It is clear that this uncertain system leads to satisfaction of condition in inequality (7) and hence (11)(see Petersen and Savkin [1999]). This more realistic approach removes any noise model assumption in algorithm development and guarantees the robustness.

6. EQUIANGULAR NAVIGATION GUIDANCE (ENG)

ENG has derived from geometry of robot movement combined with the kinematic equation of robot-target tracking system. Considering the target at the origin, the equiangular spiral is a spiral whose polar equation is given by,

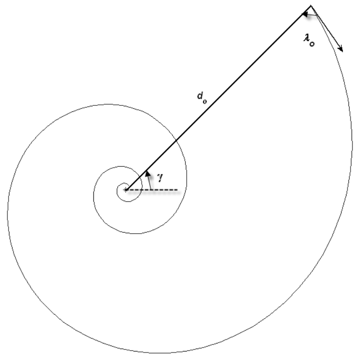


Fig. 3. An equiangular spiral

$$d = d_o e^{-b\gamma} \tag{22}$$

where d and d_o is the current and the initial miss-distance, respectively. $b = \cot(\lambda_o)$, where $0 < |\lambda_o| < \frac{\pi}{2}$ is the approaching angle, and γ is the angle between the x-axis and the vector starts at the target position pointing to the robot position, shown in Fig. (3)(see e.g. Lockwood [1961]). As $\lambda_o \to \frac{\pi}{2}$, $b \to 0$ and as a result the spiral approaches a circle.

With constant robot linear velocity, the miss-distance derivative can be considered as a measure for the angle λ_R at which the robot approaches the target. To approach a stationary target, $\dot{d}$ should be negative and using (5a) we obtain $|\lambda_R| < \frac{\pi}{2}$.

The idea is to approach the target with a fixed $\lambda_R = \lambda_o$ along an equiangular spiral where $|\lambda_o| < \frac{\pi}{2}$. Having fixed the angle λ_R, from (5a) the value of $\dot{d}$ fluctuates around a positive constant $L = V_R cos(\lambda_o)$, which is bounded to robot's linear velocity.

On the other hand, to prevent any dangerous settings of the controls, which would break the gears of the vehicle, the low level motor controllers apply a constraint on robot's angular velocity as $|\omega| \le \omega_{max}$. So, given the maximum turn rate for the robot and based on the argument above the ENG's steering control would be a bang-bang solution, switching between minimum and maximum values of ω. Equiangular Navigation Guidance (ENG), considering the dynamic constraints of the mobile robot, is introduced as follows:

$$\omega_R = -\omega_{max} sgn(L + \dot{d}) \tag{23}$$

where $0 < L < V_R$ and $sgn(\cdot) = +1$, 0 or -1 according as the expression contained in brackets is positive, zero or negative, respectively.

Remarks: Due to the symmetry properties, the steering control law

$$\omega_R = \omega_{max} sgn(L + \dot{d}) \tag{24}$$

has similar performance and characteristics as control law (23).

6.1 Following A Maneuvering Target

For a moving target, we bound the positive constant L to difference of the robot and target linear velocities, $L < (V_R - V_T)$. As we mentioned before, along the trajectory $\alpha < |\lambda|$ and $V_R > V_T$. As a result, with a constant linear velocity the robot inevitably goes into a circular trajectory while following a moving target.

The result is acceptable in applications like Unmanned Arial Vehicles (UAVs) navigation, in which the vehicle's velocity should not goes below the stall speed to keep the altitude constant. However, in other applications like trajectory tracking or target following by a WMR, it is more desirable that the mobile robot decreases the linear velocity to that of the target and follows it in a smooth trajectory, while preserving a safety distance from the target. Considering the safety distance d_s less than one meter, we define the robot velocity as a function of miss-distance as follows

$$V_R = \begin{cases} V_{max} & d > 1 \\ \frac{V_{max}}{(1-d_s)^2}(d - d_s)^2 & d_s < d \leq 1 \\ 0 & d \leq d_s \end{cases} \quad (25)$$

The robot approaches the target with the maximum velocity. When the relative distance between the robot and target is less than one meter, the robot reduces the speed in order to avoid circling. To preserve the safety distance, the speed of the robot converges to zero as the miss-distance tends to d_s. Since the robot velocity decreases as it approaches the target, to have a smooth tracking with a constant approaching angle λ_o, L should change continuously with V_R. We have,

$$L = 0.95(V_R - V_T) \quad (26)$$

7. APPLICATION OF REKF FOR AD-HOC SYSTEM

By taking the derivative of (5a), we obtain:

$$\ddot{d} = V_R\dot{\lambda}_R sin(\lambda_R) - V_T\dot{\lambda}_T sin(\lambda_T) \quad (27)$$

The first term in the right hand side is the input control, u, which models the lateral acceleration of the robot. With no knowledge of target kinematic states, we consider the second term as input uncertainty to the system. Rewriting (27) in a compact form and taking into account (3), we have $\ddot{d} = u - w$, where w denotes the input uncertainty and in matrix form, we have

$$\dot{x} = Ax + B_1u + B_2w \quad (28)$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_1 = -B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (29)$$

where $x(t) = [d(t)\ \dot{d}(t)\ \varepsilon]^T$ and $u = V_R\omega_R$ is the lateral acceleration of the mobile robot. We consider the state space dynamic equations of the system with two input noises: (i) measurement noise (this is standard with any measurement), v in (2) and (ii) w which is disturbance due to unknown target maneuver. The corresponding Riccati differential equation, obtained from (6), (13) and (14), is given by

$$\begin{aligned} \dot{\tilde{x}} = A\tilde{x}(t) + B_1u(t) + X^{-1}(t)[\nabla_x C(\tilde{x}(t))'R(y(t) \\ -C(\tilde{x}(t))) + \xi^2\nabla_x C(\tilde{x}(t))'\nabla_x C(\tilde{x}(t))] \end{aligned} \quad (30)$$

Table 1. Simulation parameters 1

Parameter	Value	Comments
Ts	$0.1s$	Sampling intervals
P_o	$20w$	Target transmission power
R	0.01	Measurement noise covariance
Q	220	Process noise covariance
N	$diag[0.02, 1, 0.2]$	Inverse of error covariance matrix
$x_R(0)$	(0, 0, 0)	Initial robot posture
x_0	[2; 0.1; 2]	Initial state vector
ω_{max}	$1\ rad/s$	Maximum angular velocity
ξ	0.02	Input uncertainty
V_R	$0.2\ m/s$	Robot linear velocity
ε	4	Environmental constant

where $\tilde{x}(0) = x_0$, the initial state of the system. We have,

$$\begin{aligned} \dot{X} + A'X + XA + XB_2Q^{-1}B_2'X \\ -\nabla_x C(\tilde{x}(t))'R\nabla_x C(\tilde{x}(t)) + \xi^2\nabla_x C(\tilde{x}(t))'\nabla_x C(\tilde{x}(t)) = 0, \end{aligned} \quad (31)$$

where $X(0) = N$. In the simulation, we consider the measurement noise as a white gaussian noise with zero mean and a standard deviation equal to $0.05\|y(t)\|$, in which $y(t)$ is the noise free measurement. Considering (2) we have $y(t) = C(\hat{x}(t)) = P_o - 10\hat{x}_3 log_{10}\hat{x}_1 + v(t)$, and

$$\nabla_x C(\hat{x}(t)) = [\frac{-10\hat{x}_3}{ln(10)\hat{x}_1}\ \ 0\ \ -10log_{10}\hat{x}_1] \quad (32)$$

where $\hat{x}(t) = \tilde{x}(t)$ is the solution to the state equation(30).

8. SIMULATION RESULTS

To study the performance of ENG, we simulate a mobile robot equipped with a radio transceiver moving in an obstacle-free area. The target may either be stationary at an unknown location or moving in a trajectory with unknown linear and angular velocities. Simulation parameters have been shown in Table 1.

Approaching an Unknown Stationary Target with different values of L: In the first experiment, we consider an unknown stationary target predefined at $(-20m, 0)$. The robot linear velocity is $0.2m/s$. Applying ENG, the robot moves towards the target with a nearly constant λ_R along a semi-equiangular spiral and eventually goes into a circular trajectory around the target. We initially let the robot moves along the initial heading angle for a short period of time, $5s$, to adjust the REKF estimator. Fig. (4) shows the robot trajectories towards an unknown stationary target with different values of L. In case of availability of true miss-distance, L is a positive constant bounded to robot's linear velocity. However, due to the input uncertainties, measurement noise and estimation error, one should consider smaller values of L depend on noise and uncertainty levels.

Following a maneuvering target: In this experiment, the robot is supposed to follow a moving target with a smaller linear velocity and lower maneuverability. The target initially moves straight with a constant linear velocity $V_T = 0.1\ m/s$ and starts maneuvering after a while with:

$$\omega_T = \begin{cases} 0 & t < 250 \\ 0.02cos(0.01t + 1) & t > 250 \end{cases}$$

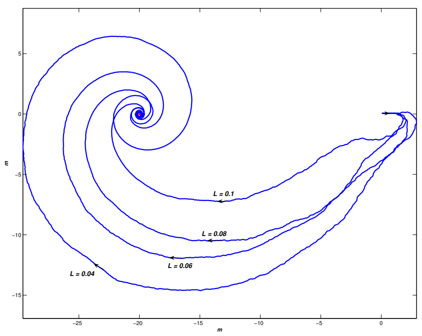
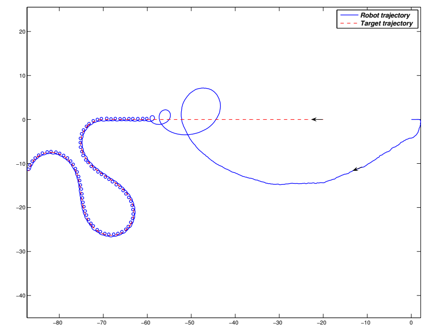
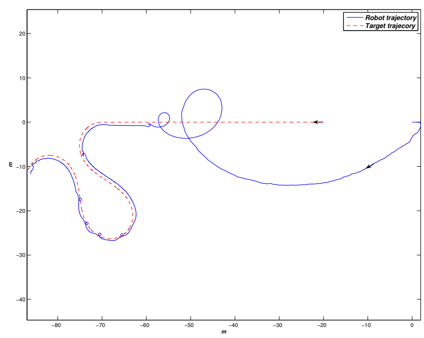


Fig. 4. Moving towards an unknown stationary target



(a) Target following with constant linear velocity



(b) Reducing the speed to avoid circling the target

Fig. 5. Maneuvering target following using ENG

For a moving target, the positive constant L is bounded to difference of the robot and target linear velocities. However, in practice, we consider smaller values of L due to the estimation error, noise and uncertainties in the system. Having applied ENG for $L = 0.09$ with a constant robot linear velocity bigger than target speed, the robot approaches the target and follows it in a circular trajectory, shown in Fig. (5(a)).

In order to reduce the speed as the robot moves towards the target and avoid circling, having applied the modified ENG, the robot follows the target in a smooth trajectory, shown in Fig. (5(b)). Fig. (6) depicts the miss-distance and robot velocity variations with $d_s = 0.5\ m$. Throughout the simulation, except on fast turns, the robot linear velocity converges to the target speed and hovers around

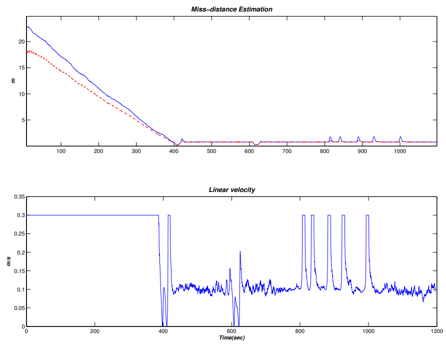


Fig. 6. Robot velocity and miss-distance variation with modified ENG

it. Furthermore, the miss-distance is often constant and the safety distance is preserved.

9. CONCLUSION

In this paper, we proposed a guidance law, which drives the robot towards a stationary or maneuvering target, based on RSS information and using a REKF. Having applied the ENG, the robot approaches the stationary target with constant linear velocity along a semi-equiangular spiral with adjustable arc-length and curvature, and eventually goes into a circular trajectory around the target. Apart from simplicity and ease of use for realtime applications, ENG is also applicable for other nonholonomic vehicles which in specific situations have the same kinematic equation as WMR, such as UAVs and space robots.

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