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Synchronization of two ball and beam systems with neural compensation

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Abstract: Ball and beam system is one of the most popular and important laboratory models for teaching control systems engineering. There are two problems for ball and beam synchronized control: 1) many laboratories use simple controllers such as PD control, and theory analysis is based on linear models, 2) nonlinear controllers for ball and beam system have good theory results, but they are seldom used in real applications. Almost nobody realize synchronized control for ball and beam systems. In this paper we first use PD control with nonlinear exact compensation for the cross-coupling synchronization. Then a RBF neural network is applied to approximate the nonlinear compensator. Two types of controller are proposed: parallel and serial PD regulators. The synchronization stability of two ball and beam systems are discussed . Real experiments are applied to test our theory results.

1. INTRODUCTION

Ball and beam system is widely used because many important classical and modern design methods can be studied based on it. It has a very important property: open loop unstable, because the system output (the ball position) increases without limit for a fixed input (beam angle). The control job is to automatically regulate the position of the ball by changing the position of the motor. This is a difficult control task because the ball does not stay in one place on the beam when it is tilted. This standard experiment can be approximated by a linear model, many universities use it for education of classical control theory. Linear feedback control or PID control can be applied, the stability analysis are based on linear state-space model or transfer function [18].

Resent results show that the stabilization problem of the ball and beam can be solved by nonlinear controllers. Approximate input-output linearization used state feedback to linearize ball and beam system first, the a tracking controller based on the approximates system can stabilize the ball and beam system [9]. But this controller is very complex for real application. In order to solve transient performance problem, energy shaping method uses a nonlinear static state feedback that is derived from the interconnection and damping assignment [8]. But it requires the kinetic and potential energies shaping [17]. Sliding mode controller can overcome the problem associated with singular states [11]. Some intelligent controllers for ball and beam can also be found, such as fuzzy control [23], sliding mode fuzzy control, neural control [3], fuzzy neural control [5], *etc.*

Synchronization can be defined as the mutual time conformity of two or more processes [1]. Different kinds of synchronization can be defined based on the type of interconnections in the system [6][19][22]. In case of disconnected systems, synchronous behavior is called natural synchronization. If the synchronization is achieved by proper interconnections, i.e., without any artificially introduced external action, then the system is called self-synchronized. If there exist external actions (controls) and/or artificial interconnections then the system is called controlled synchronization. It has two formulation: internal (mutual) synchronization and external synchronization. For the first type, all synchronized objects occur on equal terms in the unified multi-composed system, e.g., cooperative systems. For the second type, one object is more powerful than the others and its motion can be considered as independent of the motion of the other objects, e.g., master–slave systems. The idea of cross-coupling control was first introduced by [12]. There are some examples for machine tools, such as in [6], [13], [22], [21] and [19].

This paper first focuses on internal controlled synchronization of under actuated mechanical systems: ball and beam system. A synchronization controller based on the so-called cross-coupling control plus a nonlinear compensator is proposed, coupling errors are used to induce the mutual synchronization behavior. We analyze the stability of the internal controlled synchronization, with the complete nonlinear models of the ball and beam systems. Since the dynamic equations of ball and beam systems are not suitable for Lyapunov method, some special transformation are applied.

But unfortunately the above controller requires a ball and beam model to compensate for the uncertainties, and the compensator is very complex. In this paper, a new modified algorithm is proposed which overcomes this limitation of nonlinear ball and beam control. A RBF neural networks is used to estimate the compensator. Unlike other work which used neural networks to compensate the uncertainties [21], a new proof of stability is presented using

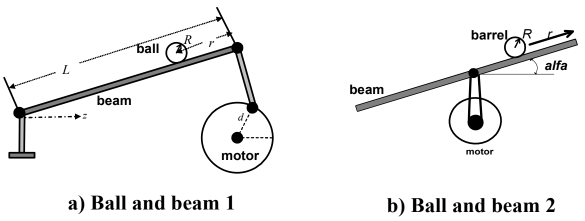

Fig. 1. Ball and beam systems

Lyapunov analysis. Finally, results from experimental tests carried out to validate the controller are presented.

2. SYNCHRONIZATION OF TWO BALL AND BEAM SYSTEMS

The control objective of ball and beam systems described schematically in Fig.1 is to turn the angle of gear θ, and the angle of the beam α, such that the ball can stay in a position. When the angle is changed from the horizontal position, gravity causes the ball to roll along the beam. The synchronization control problem is to design a controller which computes the applied voltage U_i for the motor i to move the ball i in such a way that the synchronization error reaches zero. The electrical and mechanical subsystems are coupled to each other through an algebraic torque equation

$$\begin{aligned} & U = L_m \dot{I}_m + R_m I_m + K_b \dot{\theta} \\ & \frac{1}{K_g}\left(J_m \ddot{\theta} + B_m \dot{\theta}\right) = \tau_m \\ & \tau_m = K_m I_m \end{aligned} \quad (1)$$

where U is input voltage, I_m is armature current, R_m and L_m are the resistance and inductance of the armature, K_b is back emf constant, $\dot{\theta}$ is angular velocity. Compared to $R_m I_m$ and $K_b \dot{\theta}$, the term $L_m \dot{I}_m$ is very small. In order to simplify the modeling and as most DC motor modeling methods, we neglected the term $L_m \dot{I}_m$, K_g is gear ratio, J_m is the effective moment of inertia, B_m is viscous friction coefficient, τ_m is the torque produced at the motor shaft, K_m is torque constant of the motor. Assume that there is no backlash or electric deformation in the gears, the work done by the load shaft equals to the work done by the motor shaft, $\tau = \frac{1}{K_g}\tau_m = \tau_m$, here τ is the toque on the frame of ball and beam system. In the absence of friction or other disturbances, the dynamics of the ball and beam system can be obtained by Lagrangian method, the mathematical model of the ball and beam system is given by

$$\begin{aligned} & \left(J + J_e + mr^2\right)\ddot{\alpha} + 2mr\dot{r}\dot{\alpha} + \zeta\cos\alpha = \tau \\ & k_4 \ddot{r} - r\dot{\alpha}^2 + g\sin\alpha = 0 \end{aligned} \quad (2)$$

where J is the moment of inertia of the beam, $\dot{\alpha}$ is angle velocity of the beam, α is the angle of the beam, J_e is the moment of inertia of the ball, $\dot{r}$ is the velocity of the ball, r is the position of the ball, m is mass of the ball, M is mass of the frame, L is longitude of the beam. The beam angle α and motor position θ, could not be the same, a general relation between them is given by $\alpha\gamma = \theta$.

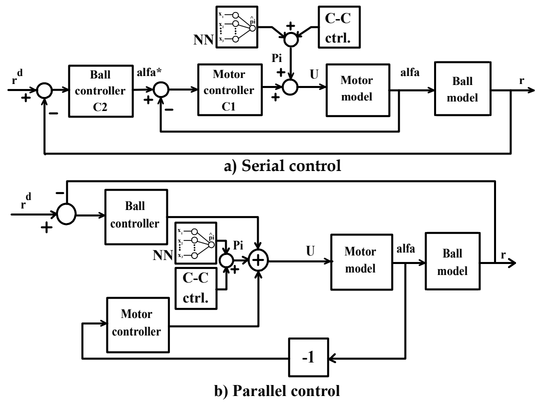

Fig. 2. PD NN cross-coupling control for synchronization of ball and beam system

For the ball and beam system 1 in Fig.1 (a), $\zeta = mgr + \frac{L}{2}Mg$, because the rotation center of the beam is on one end side (the gravity of the beam cannot be neglected [24]).

For the ball and beam system 2 in Fig.1 (b), $\zeta = mgr$ [9].

Firstly, we discuss the control of one ball and beam system. Two types of PD controllers will be designed for this system. The first one is serial PD control which is shown in Fig.2 a). It has the following form

$$U = k_{pm}(\alpha^* - \alpha) + k_{dm}(\dot{\alpha}^* - \dot{\alpha}) + \pi \quad (3)$$

where π is a compensator which can assure asymptotically stable, $\alpha^* = -k_{pb}\left(r^d - r\right) - k_{db}\left(\dot{r}^d - \dot{r}\right)$. The parallel PD control has the following form

$$U = \left(-k_{pm}\alpha - k_{dm}\dot{\alpha}\right) - \left[k_{pb}\left(r^d - r\right) + k_{db}\left(\dot{r}^d - \dot{r}\right)\right] + \pi \quad (4)$$

For regulation problem the control aim is to stabilize the ball in a desired position r^d, so $\dot{r}^d = 0$. The two PD controllers can be rewritten in a unique form

$$U = -a_1\tilde{r} + a_2\dot{r} + a_3\ddot{r} - a_4\alpha - a_5\dot{\alpha} + \pi \quad (5)$$

where for serial PD control $a_1 = k_{pm}k_{pb}$, $a_2 = k_{pm}k_{db} + k_{dm}k_{pb}$, $a_3 = k_{dm}k_{db}$, $a_4 = k_{pm}$, $a_5 = k_{dm}$, for parallel PD control $a_1 = k_{pb}$, $a_2 = k_{db}$, $a_3 = 0$, $a_4 = k_{pm}$, $a_5 = k_{dm}$, $a_i > 0$ $(i = 1 \cdots 5)$. In this section, PD regulation for ball and beam system is proposed. By (1) we have

$$\frac{K_m}{R_m}\left(U - K_b\dot{\theta}\right) = \tau$$

The whole ball and beam system is (1) and (2)

$$\begin{aligned} & \left(mr^2 + k_1\right)\ddot{\alpha} + 2mr\dot{r}\dot{\alpha} + \left(mgr + \frac{L}{2}Mg\right)\cos\alpha = k_2U - k_3\dot{\alpha} \\ & k_4\ddot{r} - r\dot{\alpha}^2 + g\sin\alpha = 0 \end{aligned} \quad (6)$$

where $k_1 = \frac{R_m J_m}{K_m K_g}\frac{L}{d} + J_1$, $k_2 = 1 + \frac{K_m}{R_m}$, $k_3 = \frac{L}{d}\left(\frac{K_m K_b}{R_m} + K_b + \frac{R_m B_m}{K_m K_g}\right)$, $k_4 = \frac{7}{5}$, $k_i > 0$ $(i = 1 \cdots 4)$. We define

$$x = \left[\,\alpha \;\; r\,\right]^T, \dot{x} = \left[\,\dot{\alpha} \;\; \dot{r}\,\right]^T, \tilde{x} = \left[\,-\alpha \;\; \tilde{r}\,\right]^T$$

The closed-loop system with PD controller (5) is

$$M(x)\ddot{x} + C(x,\dot{x})\dot{x} + G(x) = B\widetilde{\ddot{x}} + D \quad (7)$$

where

$$M(x)=\begin{bmatrix} k_1+mr^2 & -k_2a_3 \\ k_2a_3 & k_4 \end{bmatrix},$$

$$C(x,\dot{x})=\begin{bmatrix} k_2a_5+k_3 & -k_2a_2+2mr\dot{\alpha} \\ -r\dot{\alpha} & 0 \end{bmatrix}$$

$$B=\begin{bmatrix} k_2a_4 & -k_2a_1 \\ -k_2a_1 & 1 \end{bmatrix},\quad D=\begin{bmatrix} k_2\pi \\ k_2a_3\ddot{\alpha}-k_2a_1\alpha-\tilde{r} \end{bmatrix},$$

$$G(x)=\begin{bmatrix} \left(mgr+\frac{L}{2}Mg\right)\cos\alpha \\ g\sin\alpha \end{bmatrix}$$

We have the following stability theorem for one ball and beam system control.

Theorem 1. The serial or parallel control (5) with the compensator as if

$$\pi=\begin{cases} \frac{\dot{r}}{\dot{\alpha}k_2}\left[k_2a_1\alpha+2g\sin\alpha-\dot{r}+\tilde{r}\right] \\ +\frac{1}{k_2}[\left(mgr+\frac{L}{2}Mg\right)\cos\alpha-k_2a_3\ddot{r} & \dot{\alpha}\neq 0 \\ \quad -(k_1+mr^2)\ddot{\alpha}-2r\dot{r}\dot{\alpha}] \\ \frac{1}{k_2}\left(mgr+\frac{L}{2}Mg\right)\cos\alpha-a_2\dot{r} & \dot{\alpha}=0 \end{cases} \tag{8}$$

if the PD gains in (5) satisfy

$$a_4>k_2a_1^2,\quad k_2a_5+k_3>\frac{1}{4}k_2^2a_2^2 \tag{9}$$

then the closed-loop system is asymptotically stable with initial condition $0\leq r(0)\leq L$

$$\lim_{t\to\infty}\alpha(t)=0,\quad \lim_{t\to\infty}r(t)=0$$

Secondly, we discuss the synchronization of two ball and beam systems. The synchronization error s_i is defined as

$$s_i=r_i-r_i,\quad \dot{s}_i=\dot{r}_i-\dot{r}_i,\quad i=1,2$$

Here we only discuss serial PD synchronization control, see Fig.2 a). For the parallel PD synchronization control (Fig.2 a)), we can get the similar results, because serial and parallel controllers have the same form as in (5). The serial PD synchronization control has the form as $U_i=k_{pmi}(\alpha_i^*-\alpha_i)+k_{dmi}(\dot{\alpha}_i^*-\dot{\alpha}_i)+\beta k_{pbi}s_i+\beta k_{dbi}\dot{s}_i+\hat{\pi}_i$ where π_i is a compensator which can assure asymptotically stable, β is a synchronization constant, $\alpha_i^*=-k_{pbi}\left(r^d-r_i\right)-k_{dbi}\left(\dot{r}^d-\dot{r}_i\right)$. The parallel PD synchronization control has the following form

$$U_i=\left(-k_{pmi}\alpha_i-k_{dmi}\dot{\alpha}_i\right)-[k_{pbi}\left(r^d-r_i\right)$$
$$+k_{dbi}\left(\dot{r}^d-\dot{r}_i\right)]+\beta k_{pbi}s_i+\beta k_{dbi}\dot{s}_i+\pi_i$$

The two synchronization controllers can be rewritten

$$U_i=-a_{1i}\tilde{r}_i+a_{2i}\dot{r}_i-a_{4i}\alpha_i-a_{5i}\dot{\alpha}_i+a_{6i}s_i+a_{7i}\dot{s}_i+\pi_i \tag{10}$$

where for serial synchronization control $a_{1i}=k_{pmi}k_{pbi}$, $a_{2i}=k_{pmi}k_{dbi}+k_{dmi}k_{pbi}$, $a_{3i}=k_{dmi}k_{dbi}$, $a_{4i}=k_{pmi}$, $a_{5i}=k_{dmi}, a_{6i}=\beta k_{pbi}, a_{7i}=\beta k_{dbi}$ for parallel PD control $a_{1i}=k_{pbi}$, $a_{2i}=k_{dbi}$, $a_{4i}=k_{pmi}$, $a_{5i}=k_{dmi}$, $a_{6i}=\beta k_{pbi}, a_{7i}=\beta k_{dbi}$, $a_{ji}>0$, $j=1...7$, $i=1,2$, $\beta>0$, $\tilde{r}_i=r^d-r_i$. We define

$$x_i=\left[\,\alpha_i\ r_i\ s_i\,\right]^T,\quad \tilde{x}_i=\left[\,-\alpha_i\ \tilde{r}_i\ -s_i\,\right]^T$$

The closed-loop system with PD controller (10) is

$$M_i(x_i)\ddot{x}_i+C_i(x_i,\dot{x}_i)\dot{x}_i+G_i(x_i)=B_i\widetilde{x_i}+D_i \tag{11}$$

where

$$M_i(x)=\begin{bmatrix} k_{1i}+m_ir_i^2 & -k_{2i}a_{3i} & 0 \\ -k_{2i}a_{3i} & k_{4i}+1 & -1 \\ 0 & -1 & 1 \end{bmatrix},$$

$$B_i=\begin{bmatrix} k_{2i}a_{4i} & -k_{2i}a_{1i} & -k_{2i}a_{6i} \\ -k_{2i}a_{1i} & 2 & -1 \\ -k_{2i}a_{6i} & -1 & 1 \end{bmatrix}$$

$$C_i(x,\dot{x})=\begin{bmatrix} k_{2i}a_{5i}+k_{3i} & -k_{2i}a_{2i}+2m_ir_i\dot{\alpha}_i & -k_{2i}a_{7i} \\ -r_i\dot{\alpha}_i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$D_i=\begin{bmatrix} k_{2i}\pi_i \\ \ddot{r}_{i|2}-k_{2i}a_{3i}\ddot{\alpha}_i-2\tilde{r}_i-k_{2i}a_{1i}\alpha_i-s_i \\ r^d-r_{i|2}-\ddot{r}_{i|2}-k_{2i}a_{6i}\alpha_i \end{bmatrix},$$

$$G_i(x)=\begin{bmatrix} \left(m_igr_i+\frac{L_i}{2}M_ig\right)\cos\alpha_i \\ g\sin\alpha_i \\ 0 \end{bmatrix}$$

$i|2=2$ when $i=1$, $i|2=1$ when $i=2$. We have the following stability theorem for the synchronization control of two ball and beam systems.

Theorem 2. The serial or parallel PD synchronization control (10) with the compensator as

$$\pi_i=\begin{cases} \frac{1}{k_{2i}\dot{\alpha}_i}[-\dot{r}_i\dot{r}_{i|2}+k_{2i}a_{1i}\alpha_i\dot{r}_i+2\tilde{r}_i\dot{r}_i \\ \quad +k_{2i}a_{3i}\ddot{\alpha}_i\dot{r}_i-\ddot{r}_{i|2}\dot{r}_i+g\sin\alpha_i\dot{r}_i \\ \quad -\dot{s}_i\tilde{r}_{i|2}+\ddot{r}_{i|2}\dot{s}_i+k_{2i}a_{6i}\alpha_i\dot{s}_i-\dot{s}_i^2] & \dot{\alpha}_i\neq 0 \\ \quad +\frac{1}{k_{2i}}[\left(m_igr_i+\frac{L_i}{2}M_ig\right)\cos\alpha_i \\ \qquad +(m_i-1)r_i\dot{\alpha}_i\dot{r}_i] \\ \frac{1}{k_{2i}}(m_igr_i+\frac{L_i}{2}M_ig)\cos\alpha_i-a_{2i}\dot{r}_i & \dot{\alpha}_i=0 \end{cases} \tag{12}$$

If the PD gains in (10) satisfy

$$a_{4i}>\max\left[\frac{k_{2i}a_{1i}^2}{2},a_{1i}^2k_{2i}+2a_{6i}^2k_{2i}+2\left(a_{1i}a_{6i}\right)k_{2i}\right]$$
$$k_{1i}k_{4i}>k_{2i}^2a_{3i}^2$$

then the closed-loop system and the synchronization error are asymptotically stable with initial condition $0\leq r(0)\leq L$

$$\lim_{t\to\infty}\alpha_i=0,\quad \lim_{t\to\infty}\tilde{r}_i=0,\ \lim_{t\to\infty}s_i=0\quad i=1,2 \tag{13}$$

3. SYNCHRONIZATION WITH NEURAL NETWORK COMPENSATION

Since the nonlinear compensators (8) and (12) needs the complete information of the ball and beam system. It is very difficult to realize these compensations. We will use Radial Basis Function (RBF) neural networks to approximate these compensators. The advantages of the RBF approach, such as the linearity in the parameters and the availability of the fast and efficient training methods, have been noted in several publications [10]. RBF neural networks has one hidden layer and a linear output layer. The output of neural networks may be presented as

$$y=\sum_{j=1}^{N}w_j\phi_j(Vx)+b \tag{14}$$

where N is hidden nodes number, w_j is the weight connecting hidden layer and output layer. x is input vector $x\in\Re^m$ (m is input node number), $V\in\Re^{N\times m}$ is the weight matrix in hidden layer, b is the threshold. The significance of the threshold is that the output values have nonzero mean. It can be combined with the first term as $w_{0,j} = b$, $\phi_0(Vx) = 1$, so $y = \sum_{j=0}^{N} w_j\phi_j(Vx)$. $\phi_j(Vx)$ is radial basis function which we select it as Gaussian function

$$\phi_j(V_i x_{inn}) = \exp\left\{\frac{\|V_i x_{inn} - c_j\|}{2\sigma_j^2}\right\} \tag{15}$$

where c_j and σ_j^2 represent the center and spread of the basis function,

$$x_{inn} = \left[\alpha_i, \dot{\alpha}_i, \ddot{\alpha}_i, r_i, \dot{r}_i, \tilde{r}_{i|2}, \dot{r}_{i|2}, \ddot{r}_{i|2}, \dot{s}_i, s\right]^T$$

RBF neural networks compensation does not require structure information of the uncertainties [10]. According to the Stone-Weierstrass theorem [4], the nonlinear compensator π_i in (12) can be written as neural networks (14) form

$$-\hat{\pi}_i(x_{inn}) - s^2 = W_i^{*T}\phi_i(V_i^{*T}x_{inn}) + \eta_i, \quad i = 1,2 \tag{16}$$

where W_i^*, V_i^* are some fixed bounded weights, η_i is the approximated error, whose magnitude also depends on the values of $\hat{W}_t$ and $\hat{V}_t$, η_i is assumed to be quadratic bounded as

$$\eta_i^2 \leq \bar{\eta}_i \tag{17}$$

where $\bar{\eta}_i$ is a positive constant, π_i can be estimated by $\hat{W}_{it}^T\phi_i(\hat{V}_{it}^T x_{inn})$, $\hat{W}_{it}^T$ and $\hat{V}_{it}^T$ are time-varying weights of the neural networks. It is clear that all Gaussian function, commonly used in neural networks, satisfy Lipschitz condition

$$\tilde{\phi}_i = \phi_i(V_{it}^{*T}x_{inn}) - \phi_i(\hat{V}_{it}^T x_{inn}) = D_{\sigma i}\tilde{V}_{it}^T x_{inn} + \nu_{\sigma i} \tag{18}$$

where $\tilde{V}_{it} = V_{it}^* - \hat{V}_{it}$, $D_{\sigma i} = \frac{\partial \phi_i^T(Z)}{\partial Z}\mid_{Z=\hat{V}_t^T x_{inn}}$

$$\|\nu_{\sigma i}\|_{\Lambda_{\sigma i}}^2 = \nu_{\sigma i}^T \Lambda_{\sigma i}\nu_{\sigma i} \leq \bar{\eta}_{\sigma i} \tag{19}$$

where $\bar{\eta}_\sigma$ is a positive constant. We have the following relation

$$\begin{aligned} & W_i^{*T}\phi_i(\hat{V}_i^{*T}x) - \hat{W}_i^T\phi_i(\hat{V}_i^T x) \\ & = \tilde{W}_i^T\phi_i(\hat{V}_i^T x_{inn}) + W_i^{*T}\tilde{\phi}_i \\ & = \tilde{W}_i^T\phi_i(\hat{V}_i^T x_{inn}) + W_i^{*T}D_{\sigma i}\tilde{V}_i^T x_{inn} + W_i^{*T}\nu_{\sigma i} \\ & = \tilde{W}_i^T\phi_i(\hat{V}_i^T x_{inn}) + \hat{W}_i^T D_{\sigma i}\tilde{V}_i^T x_{inn} \\ & + \tilde{W}_i^T D_{\sigma i}\tilde{V}_i^T x_{inn} + W_i^{*T}\upsilon_{\sigma i} \end{aligned} \tag{20}$$

where $\tilde{W}_i = W_i^* - \hat{W}_i$. In regulation case , *i.e.* $x_2^d = 0$, the synchronization control with RBF neural network compensation can be expressed as

$$\begin{aligned} U_i = & -a_{1i}\tilde{r}_i + a_{2i}\dot{r}_i + a_3\ddot{r} - a_{4i}\alpha_i \\ & -a_{5i}\dot{\alpha}_i + a_{6i}s_i + a_{7i}\dot{s}_i - \hat{W}_{it}^T\phi_i(\hat{V}_{it}^T x_{inn}) \end{aligned} \tag{21}$$

First we consider a simple case, $\hat{V}_{it} = I$. The following theorem gives a stable learning algorithm of the neural network compensator.

Theorem 3. The PD neural network cross-coupling control, serial or parallel as in (21) with the following adaptation law

$$\dot{\hat{W}}_i = k_{2i}\dot{\alpha}_i\Gamma_i\phi_i(x_{inn}) \tag{22}$$

where Γ_i is the learning rate, $\Gamma_i > 0$,

$$x_{inn} = \left[\alpha_i, \dot{\alpha}_i, \ddot{\alpha}_i, r_i, \dot{r}_i, \tilde{r}_{i|2}, \dot{r}_{i|2}, \ddot{r}_{i|2}, \dot{s}_i\right]^T$$

and the condition:

$$a_{4i} > \max\left[\frac{k_{2i}a_{1i}^2}{2}, a_{1i}^2 k_{2i} + 2a_{6i}^2 k_{2i} + 2(a_{1i}a_{6i})k_{2i}\right],$$
$$k_{1i}k_{4i} > k_{2i}^2 a_{3i}^2, \quad k_{2i}a_{5i} + k_{3i} > \frac{k_{2i}}{2}$$

can guarantee stability of the ball and beam system (6) and synchronization error converges

$$\lim_{T\to\infty} sup \frac{1}{T}\int_0^T s^2 dt \leq \frac{k_{2i}}{2}\bar{\eta}_i \tag{23}$$

from any well defined set of initial conditions.

Now we consider multilayer case, $\hat{V}_t \neq I$, i.e., there exist hidden layers. The following theorem gives a stable learning algorithm of the neural compensator.

Theorem 4. The PD neural network cross-coupling control, serial or parallel as in (21) with the following adaptation law

$$\begin{aligned} \dot{\hat{W}}_i &= \Gamma_{wi}[\dot{\alpha}_i k_{2i}\phi_i(\hat{V}_i^T x_{inn}) + \dot{\alpha}_i k_{2i}D_{\sigma i}\tilde{V}_i^T x_{inn}] \\ \dot{\hat{V}}_i &= \Gamma_{vi}[\dot{\alpha}_i k_{2i}x_{inn}\hat{W}_i^T D_{\sigma i}] \end{aligned} \tag{24}$$

where Γ_{vi}, Γ_{wi} are learning rates, $\Gamma_{vi}, \Gamma_{wi} > 0$, and the condition:

$$\begin{aligned} & a_{4i} > \frac{k_{2i}{a_{1i}}^2}{2}, \\ & a_{4i} > k_{2i}^2 a_{3i}^2 \\ & k_{2i}a_{5i} + k_{3i} > \frac{k_{2i}}{2} \end{aligned}$$

can guarantee stability of the ball and beam system (6) and synchronization error converges to

$$\lim_{T\to\infty} sup \frac{1}{T}\int_0^T s^2 dt \leq \frac{k_{2i}}{2}\bar{\eta}_i + \bar{\eta}_{\sigma i} \tag{25}$$

from any well defined set of initial conditions.

Remark 1. The error will converge to the ball radius the upper bounded of $\frac{k_{2i}}{2}\bar{\eta}_i + \bar{\eta}_{\sigma i}$, and it is influenced by the prior known matrices W^* and V^*. Theorem 3 shows that W^* and V^* do not influence the stability property, we may select any value for W^* and V^* at first. From Theorem 2 we know the algorithm (24) can make the identification error convergent. W^* and V^* may be selected by following off-line steps:

(1) Start from any initial value for W^* and V^*
(2) Do on-line identification with W^* and V^*
(3) Let W_t and V_t as new initial conditions, *i.e.*, $W^* = W_t$, and $V^* = V_t$
(4) If the identification error decreases, repeat the identification process, go to 2. Otherwise, stop off-line identification, now W_t and V_t are final values for W^* and V^*.

4. EXPERIMENTAL CASE STUDY

The experiment is carried out on two ball and beam systems of the Quanser [18] and the Balance Control, see Fig.3. The configurations of the two ball and beam systems are different, see Fig.1. The input to the system is motor control voltage U_i, outputs are the positions of motor (θ_i) and ball (r_i). Power modules are Quanser PA-0103 with ±12V and 3A output. A/D-D/A board is based on a a Xilinx FPGA microprocessor, which is a multifunction analog and digital timing I/O board dedicated to real-time data acquisition and control in the Windows XP environment. The board is mounted in a PC Pentium-III 500MHz host computer. Because Xilinx FPGA chip supports real-time operations without introducing latencies caused by the Windows default timing system, the control program is

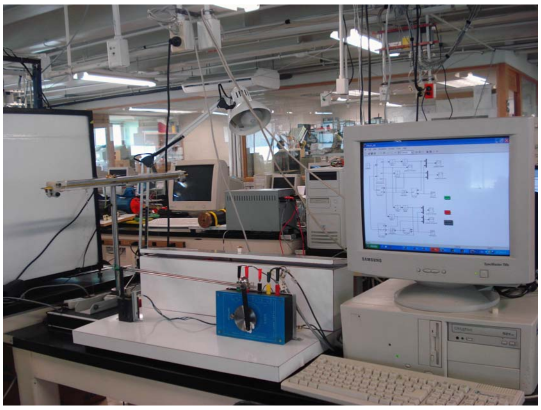

Fig. 3. Two ball and beam controled systems

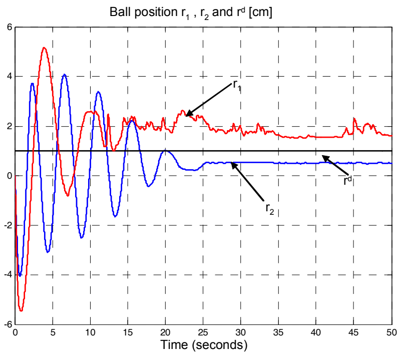

Fig. 4. Control without synchronization

operated in Windows XP with Matlab 6.5/Simulink. The sampling time is about 10ms.

Because of the PD controllers require direct velocity measurements and they are unavailable. We use derivative block of Simulink to calculate them. This require position signals are smooth enough, first order low-pass filters are applied. For motor position we use the following first-order filter $G_1(s) = \frac{7}{s+7}$. For ball position we use the following first-order filter $G_2(s) = \frac{8}{s+8}$. For the experimental case study we use the PD cross-coupling serial control with the following parameters $k_{pm1} = .7$, $k_{dm1} = 0.1$, $k_{pb1} = 0.15$, $k_{db1} = 0.09$, $k_{pm2} = .9$, $k_{dm2} = 0.065$, $k_{pb2} = 0.35$, $k_{db2} = 0.015, k_{21} = 2, k_{22} = 1, \Gamma_1 = diag\{.5\}, \Gamma_2 = \{1.8\}, \sigma_1 = \sigma_2 = 1, N_1 = 50, N_2 = 10$, $\beta = 0.5$, a constant reference signal $r^d = 1$. The responses of normal PD synchronization control and stable synchronization control proposed in this paper are shown in Fig.4 and Fig.5

We can see that synchronization between the ball and beam systems is achieved, bounded synchronization error is obtained and that synchronization error is smaller and converges faster to a minimum in comparison with the uncoupled case without compensator. Since in the last there is not any interaction between the systems.

5. CONCLUSION

In this paper, a stable PD control with RBF neural network compensation is proposed for internal synchronization of two under actuated mechanical systems, two ball and beam systems. It has been shown that the proposed

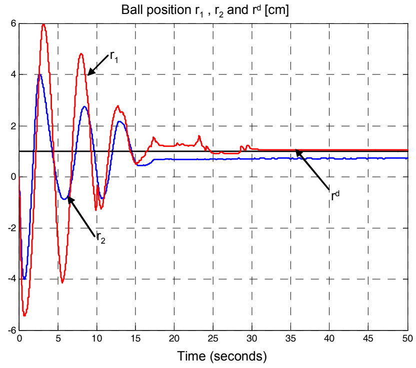

Fig. 5. Stable synchronization control with neural network compensation

synchronization control guarantees stability of the multi-composed system and synchronization errors, for a well defined set of initial conditions by using Lyapunov's method with the complete nonlinear models and neural approximation. Two types of synchronization controllers have been presented for regulation case. Experimental results are presented to illustrate the control system stability and performance.

REFERENCES

[1] I. I. Blekhman, P. S. Landa, and M. G. Rosenblum, Synchronization and chaotization in interacting dynamical systems, *ASME Appl Mech. Rev.*, vol. 48, pp. 733–752,

[2] I. I. Blekhman, A. L. Fradkov, H. Nijmeijer, and A. Y. Pogromsky, On self-synchronization and controlled synchronization, *Syst. Control Lett.*, vol. 31, pp. 299–305, 1997.

[3] Y.C.Chu, J.Huang, A neural-network method for the nonlinear servomechanism problem, *IEEE Trans. Neural Networks*, Vol.10, No.6, 1412-1423, 1999.

[4] G.Cybenko, Approximation by Superposition of Sigmoidal Activation Function, *Math.Control, Sig Syst*, Vol.2, 303-314, 1989

[5] P.H.Eaton, D.V.Prokhorov, D.C. Wunsch II, Neurocontroller alternatives for "fuzzy" ball-and-beam systems with nonuniform nonlinear friction, *IEEE Trans. Neural Networks*, Vol.11, No.2, 423-435, 2000.

[6] L. Feng, Y. Koren, and J. Borenstein, Cross-coupling motion controller for mobile robots, *IEEE Control Syst. Mag.*, vol. 13, pp. 35–43, Dec. 1993.

[7] S.S. Ge, T.H. Lee, C.J. Harris, *Adaptive neural network control of Robot Manipulators*, World Scientific, 1998

[8] F.Gordillo, F.Gómez-Estern, R.Ortega, and J.Aracil, On the ball and beam problem: regulation with guaranteed transient performance and tracking periodic orbits, *International Symposium on Mathematical Theory of Networks and Systems*, University of Notre Dame, IN, USA. August, 2002.

[9] J.Hauser, S.Sastry and P.Kokotovic, Nonlinear control via approximate input-output linearization: ball and beam example, *IEEE Trans. on Automatic Control*, Vol.37, No.3,392-398., 1992.

[10] S.Haykin, *Neural Networks- A comprehensive Foundation*, Macmillan College Publ. Co., New York,

1994.ol.2, 303-314, 1989

[11] R. M. Hirschorn, Incremental sliding mode control of the ball and beam, *IEEE Trans. on Automatic Control*, Vol.47, No.10, 1696-1700, 2002.

[12] Y. Koren, Cross-coupled biaxial computer controls for manufacturing systems, *ASME J. Dynam. Syst., Meas. Control*, vol. 102, pp. 265–272, 1980.

[13] P. K. Kulkarni and K. Srinivasan, Cross-coupled control of biaxial feed drive servomechanisms, *ASME J. Dynam. Syst., Meas. Control*, vol. 112, no. 2, 1990.

[14] H. C. Lee and G. J. Jeon, A neuro-controller for synchronization of two motion axes, *Int. J. Intell. Syst.*, vol. 13, pp. 571–586, 1998.

[15] Y. Li S. Qiang, X. Zhuang, O. Kaynak, Robust and adaptive backstepping control for nonlinear system using RBF neural networks, *IEEE Trans. on Neural networks*, vol. 15, pp. 693-701, 2004

[16] K.S. Narendra,A.M. Annaswamy, A new adaptive law for robust adaptation without persistent excitation, *IEEE Trans. Automat. Contr*, vol. AC-32, pp. 134-145,1987.

[17] R. Ortega, M.W. Spong, F. Gómez-Estern, G. Blankenstein, Stabilization of a class of underactuated mechanical systems via interconnection and damping assignment, *IEEE Trans. on Automatic Control*, Vol.47, No.8, 1218 -1233, 2002.

[18] *Ball and Beam-Experiment and Solution*, Quanser Consulting, 1991.

[19] A.Rodriguez-Angeles, H.Nijmeijer, Mutual Synchronization of Robots via Estimated State Feedback: A Cooperative Approach, *IEEE Transactions on Control Systems Technology*, vol. 12, No. 4, 542-554, 2004.

[20] E.D.Sontag and Y.Wang, On characterization of the input-to-state stability property, *System & Control Letters*, Vol.24, 351-359, 1995

[21] D. Sun and J. Mills, Adaptive synchronized control for coordination of two robot manipulators, *in Proc. 2002 IEEE Int. Conf. Robotics and Automation*, Washington, DC, 2002, pp. 976–981.

[22] M. Tomizuha, J. S. Hu, and T. C. Chiu, Synchronization of two motion control axes under adaptive feedforward control, *ASME J. Dynam. Syst., Meas. Control*, vol. 114, no. 6, pp. 196–203, 1992.

[23] L.X.Wang, Stable and optimal fuzzy control of linear systems, *IEEE Trans. on Fuzzy Systems*, Vol.6, No.1, 137 -143, 1998.

[24] W.Yu and F.Ortiz, Stability analysis of PD regulation for ball and beam system, I*EEE Conference on Control Applications*, CCA'05, Toronto, Canada, 517-522, 2005

6. APPENDIX

Proof. [Proof of Theorem 1] Substitute (5) into (2)

$$(mr^2+k_1)\ddot{\alpha}+2mr\dot{r}\dot{\alpha}+\left(mgr+\frac{L}{2}Mg\right)\cos\alpha$$
$$=-k_2a_1\tilde{r}+k_2a_2\dot{r}+k_2a_3\ddot{r}-k_2a_4\alpha-k_2a_5\dot{\alpha}+k_2\pi-k_3\dot{\alpha}$$
$$k_2a_3\ddot{\alpha}+k_4\ddot{r}-r\dot{\alpha}^2+g\sin\alpha$$
$$=k_2a_3\ddot{\alpha}-k_2a_1\alpha+\tilde{r}+k_2a_1\alpha-\tilde{r} \tag{26}$$

From (7) we know $M(x)$ and B are positive definite by the condition $a_4>k_2a_1^2$. Define a Lyapunov function as

$$V(x_1,x_2)=x_2^TM(x)x_2+\frac{1}{2}x_1^TBx_1=\dot{x}^TM(x)\dot{x}+\frac{1}{2}\tilde{x}^TB\tilde{x} \tag{27}$$

So

$$\dot{V}=\ddot{x}^TM(x)\dot{x}+\dot{x}^TD-\dot{x}^TC(x,\dot{x})\dot{x}-\dot{x}^TG(x)+\dot{x}^T\dot{M}(x)\dot{x}$$

Using $k_4\ddot{r}=-g\sin\alpha+r\dot{\alpha}^2$,

$$\dot{V}=-\dot{x}^TQ\dot{x}+\dot{\alpha}k_2\pi+\dot{r}\left[\dot{r}-k_2a_1\alpha-2g\sin\alpha-\tilde{r}\right]$$
$$+\dot{\alpha}[(k_1+mr^2)\ddot{\alpha}+k_2a_3\ddot{r}-\left(mgr+\frac{L}{2}Mg\right)\cos\alpha+2r\dot{r}\dot{\alpha}] \tag{28}$$

where $Q=\begin{bmatrix} k_2a_5+k_3 & -\frac{1}{2}k_2a_2 \\ -\frac{1}{2}k_2a_2 & 1 \end{bmatrix}$, it is positive definite matrix when $k_2a_5+k_3>\frac{1}{4}k_2^2a_2^2$. If we choose the compensator as

A) If $\dot{\alpha}\neq 0$ when

$$\pi=-\frac{\dot{r}}{\dot{\alpha}k_2}\left[\dot{r}-k_2a_1\alpha-2g\sin\alpha-\tilde{r}\right]-\frac{1}{k_2}[(k_1+mr^2)\ddot{\alpha}+k_2a_3\ddot{r}$$
$$-\left(mgr+\frac{L}{2}Mg\right)\cos\alpha+2r\dot{r}\dot{\alpha}]$$

(28) becomes

$$\dot{V}=-\dot{x}^TQ\dot{x}\le 0$$

B) If $\dot{\alpha}=0$ and $\ddot{\alpha}=0$, we assume $\alpha\neq 0$. From (2) and (5) we know

$$\left(mgr+\frac{L}{2}Mg\right)\cos\alpha=-k_2a_1\tilde{r}+k_2a_2\dot{r}+k_2a_3\ddot{r}-k_2a_4\alpha+k_2\pi$$

We select $\pi=-a_2\dot{r}+\frac{1}{k_2}\left(mgr+\frac{L}{2}Mg\right)\cos\alpha$, since $k_4\ddot{r}+g\sin\alpha=0$,

$$\left(mgr+\frac{L}{2}Mg\right)\cos\alpha=-k_2a_1\tilde{r}-k_2a_3\frac{g\sin\alpha}{k_4}$$
$$-k_2a_4\alpha+\left(mgr+\frac{L}{2}Mg\right)\cos\alpha \tag{29}$$
$$\tilde{r}=-\frac{1}{k_2a_1}\left[k_2a_3\frac{g\sin\alpha}{k_4}+k_2a_4\alpha\right]$$

So $\tilde{r}$ is a constant, but it is impossible for $\alpha\neq 0$, because the ball must move with $\alpha\neq 0$. So when $\dot{\alpha}=0$ and the compensator is $\pi=-a_2\dot{r}+\frac{1}{k_2}\left(mgr+\frac{L}{2}Mg\right)\cos\alpha$ imply $\alpha=0$. $\tilde{r}$ is a constant means $\dot{r}=0$ and $\pi=0$. So $\dot{V}\le 0$. We can conclude that $\dot{\alpha}=0$ and $\alpha=0$ imply $x_1=\tilde{x}$ and $x_2=-\dot{x}$ are always bounded.

Now we use LaSalle lemma, define

$$\Psi=\left\{[x_1,x_2]:\dot{V}=0\right\}$$

The only possible solution for α is $\alpha=0$, otherwise the ball has to move. For any $\alpha\neq 0$, $\tilde{r}$ cannot be a constant, so (29) has no solution. When $\alpha=0$, from (29) we know $\tilde{r}=0$. Because $\alpha^*=0$, this allows us to conclude $[\alpha,r]=\left[\alpha^*,r^d\right]$ is the unique solution for (27). The invariant set $\dot{V}=0$, is defined by $x_1=0,x_2=0$, so x_1 and x_2 are asymptotically stable. ■