

## Excitation Signals for Nonlinear Dynamic Modeling of Combustion Engines

W. Baumann\* S. Schaum\* K. Roepke\* M. Knaak\*\*

\* IAV GmbH, Head Office Berlin, Germany .

\*\* IAV Co., Ltd. Japan

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**Abstract:** A crucial point in nonlinear dynamic modeling is the design of a suited excitation signal. It has to fulfill several criteria, like coverage of important amplitude levels and of important frequency range, and for multichannel systems an orthogonal or at least as orthogonal as possible design is desired. Differing from theoretically optimal excitation signals, we suggest the use of smooth signals, better suited for real world automotive applications.

Keywords: Input and Excitation Design, Nonlinear System Identification

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### 1. INTRODUCTION

Whereas at the beginning of the 20th century the focus in engine development was the enhancement of power of spark ignition and diesel engines, it nowadays moves more to a further reduction of emissions. In this respect, the first 50 second of a test driving cycle for gasoline engines are of decisive importance for the overall emission.

To minimize these emissions, there exist several strategies, like fast catalyst heating or to produce as low as possible raw emissions. To find the best compromise, it is necessary to have suited models of the cold start and warm up behaviour of the combustion engine. Due to the highly nonstationary conditions during these phases, the use of nonlinear dynamic models becomes a necessity for such optimization tasks.

### 2. NONLINEAR DYNAMIC MODELING OF COMBUSTION ENGINES

Up to current time, a huge number of different approaches to nonlinear system identification have been developed, and it is therefore important to focus on a specific model structure for a specific application. Our experience shows, that for combustion engines modeling the use of parametric volterra series (see Nelles (2001)) gives good results. Parametric volterra series are characterized by a polynomial nonlinearity with a subsequent recursive part, as e.g.

$$y(k) = \theta_1 y(k-1) + \theta_2 x_1(k) + \theta_3 x_1^2(k) + \theta_4 x_2^2(k) + \theta_5 x_1(k)x_2(k). \quad (1)$$

The main advantage of such an approach is the linearity of parameters which can be exploited for estimation of the coefficients  $\theta$  via a linear regression approach. That is, the model in (1) is written in matrix notation as  $\mathbf{y} = \mathbf{X} \cdot \theta$ , where the least squares solution is given by

$$\hat{\theta} = \mathbf{X}^{-1} \mathbf{y}. \quad (2)$$

Care has to be taken to get the exact solution of equation (2) in case of correlated columns of the regression matrix  $\mathbf{X}$  which leads to ill conditioned problems. Additionally, this type of least square estimation of an ARX model may be

biased and inconsistent. However, there are several ways to avoid this, as e.g. by using the method of instrumental variables (see Ljung (1999)).

### 3. EXCITATION SIGNALS

For the estimation of parameters of the parametric volterra series, i.e. for fitting the model to available datasets, the question arises how to choose excitation signals. In general, there are different aspects that have to be taken into account:

- distribution of amplitude levels, and
- distribution of signal frequency.

For multivariate models we have the additional requirement of as orthogonal as possible excitation signals for a well designed test plan. One has to keep in mind that the linearization step, i.e. putting all nonlinear behaviour in the regressors, can lead to correlated inputs, especially for polynomial nonlinearities.

In theory, the amplitude modulated pseudo random binary noise signal (APRBS) is the optimal excitation signal for nonlinear system identification because of its good amplitude and frequency distribution. However, in practice it is very difficult to apply such a signal to combustion engines, because some parameters are very limited in their adjustment frequency and it is often difficult to do an fast adjustment on ordinary test benches. Especially the big steps that may occur in the APRBS can cause difficulties or even damage the engine. Additionally, the time dependent scaling, which is necessary in excitation of a combustion engine to ensure that boundaries are not violated, can lead to changing amplitude levels of the APRBS.

We therefore propose to use sinus or chirp signals as more slowly varying signals with less significant steps, which show the following advantages

- good applicability to combustion engines
- excitation of important frequency regions
- periodicity can be exploited for noise attenuation
- small crest factor and good scalability.

The simulations in figure 1 show, that for parametric volterra series with single input lag and a pole near one, which is a reasonable model for the temperature behaviour of a combustion engine, the variance of coefficient estimation with sinus or chirp is much less than for the APRBS.

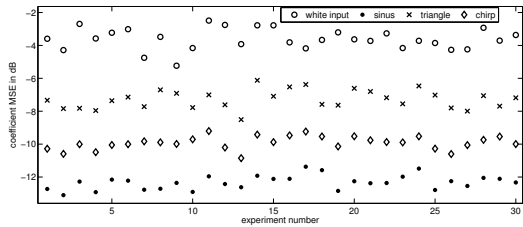


Fig. 1. Comparison of MSE of the coefficient estimation for different input signals with an amplitude range of  $\pm 1$  and an SNR of  $\approx 16$ dB

#### 4. PRACTICAL IMPLEMENTATION

The most crucial points in the practical design of the excitation signal for combustion engine modeling are not only to determine the important frequency ranges but also to avoid a boundary violation. We solve these problems by

- (1) cycle analysis and
- (2) time dependent scaling.

##### 4.1 cycle analysis

Depending on the dynamics of the modeled signal, different frequencies of input sinus may be appropriate. For an exact definition of the interesting frequency range one would need to determine the inputs to the dynamic part of the model, which in this case is solely given by the autoregressive part. This input is a non-measurable variable which is given by the nonlinear transformation of input signals, and therefore depends on the model order. However, it is also possible to examine the behavior of the inputs during a driving cycle of interest (e.g. the FTP cycle) to get the needed information.

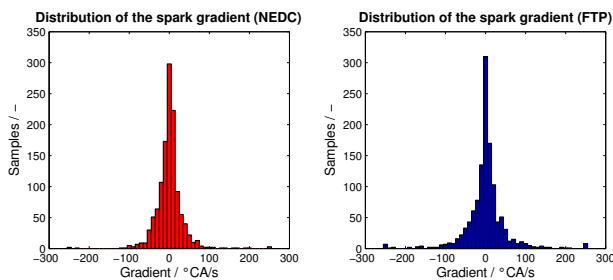


Fig. 2. Distribution of the spark gradient for FTP and NEDC cycles (first 50 s)

Figure 2 exemplarily shows the distributions of spark advance in NEDC and FTP cycle. As for all other inputs, the FTP cycle covers a slightly larger range of gradients. It can be seen, that for spark advance a maximal gradient of  $\approx \pm 50 \frac{^{\circ}CA}{s}$  is sufficient for input excitation. That is, our sinusoidal input signal is designed to reach this value at max.

##### 4.2 time dependent scaling

For a practical use of sinusoidal stimulation of engine input parameters, it is necessary to know all the possible combinations of input signals which don't damage the engine or result in a violation of the safe engine operating mode. For that reason every time step of the test plan is checked with an engine boundary model, like e.g. a convex hull. If necessary, the test setting will be transformed into a range which avoids a boundary violation.

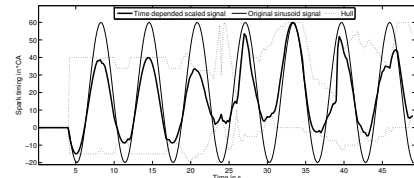


Fig. 3. Time dependent scaling of excitation signal

#### 5. MODELING RESULTS

We carried out cold start experiments with subsequent warming up of the engine for an overall time duration of 50s and used sinusoidal excitation signals as model inputs. Figure 4 shows excitation of speed and torque in the upper and lower plot, respectively.

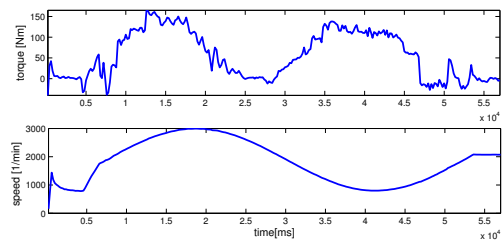


Fig. 4. Excitation of speed and torque with sinus

We performed a total number of 20 experiments, which were split into 14 training and 5 validation data sets, and one outlier dataset was removed. A least square estimation of model parameters was carried out, and figure 5 exemplarily shows a validation result for exhaust temperature where we find a good representation of engine behaviour. Additionally, models of emissions (HC, NO<sub>x</sub>, CO) are currently examined and show promising results.

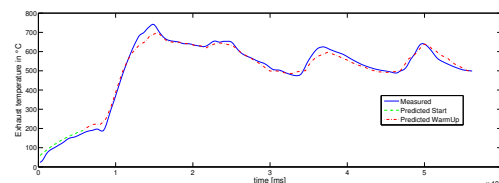


Fig. 5. Validation results of nonlinear dynamic model for exhaust temperature of a SI engine

#### REFERENCES

O. Nelles. Nonlinear System Identification. Springer, Berlin, Heidelberg, New York, 2001.  
 Lennart Ljung. System Identification. Theory for the user. Second edition. Prentice Hall PTR, 1999.