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Nonlinear Dynamic Inversion Based Anti-windup - An Aerospace Application ⋆

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Abstract: A recently suggested general anti-windup (AW) compensation scheme is applied to the nonlinear simulation model of the $\frac{1}{16}$ scaled BAe Hawk aircraft model used for wind tunnel experiments. The Hawk is modelled as a nonlinear affine system subject to input constraints and has a primary control system consisting of an inner-loop nonlinear dynamic inversion controller and an outer-loop linear PID controller. To address the input constraints a recently introduced nonlinear $\mathscr{L}_2$ sub-optimal AW compensation method is applied and compared with a nonlinear version of the internal model control AW scheme. Nonlinear simulation results demonstrate the promise of the approach and indicate the superiority of the optimal AW scheme.

1. INTRODUCTION

Nonlinear Dynamic Inversion (NDI) has emerged as a promising approach for flight control design problems. Perhaps the most appealing aspect of NDI is that the design procedure inherently provides a nonlinear multivariable controller, thereby eliminating the necessity for further gain scheduling. The method has received significant attention from the aerospace community (Bugajski, 1992; Enns et al., 1992; Reiner et al., 1996; Ito et al., 2001; Georgie, 2003; Snell et al., 1992; Snell, 1998; Escande, 1997; Bennani, 1998; G.Papageorgiou, 2001; C.Papageorgiou, 2005; Smith, 2000). The basis of NDI control is the feedback linearisation method which has become classical over the past 30 years (Isidori, 1995).

In the case of unconstrained systems, the underlying rationale behind NDI is to cancel the nonlinear terms in the system dynamics through an appropriate first-step nonlinear state-feedback. In a second step, an appropriate linear control strategy can be employed in an outer loop to enable set point tracking. Perfect knowledge of the system dynamics and accurate sensing of output signals are assumed, which is, of course, impossible in practice. To bestow robust performance properties an additional standard linear outer-loop controller is introduced. From the research mentioned above, it has been concluded that NDI is a promising methodology for the design of automatically gain scheduled multivariable, nonlinear flight controllers.

However, the main impediment to the application of NDI is the invertibility requirement on the plant's input distribution matrix, $G(x)$, which appears in the nonlinear affine system under consideration (Ito et al., 2001; Georgie, 2003). Even with the assumption that the inverse of this matrix exists for all states, x, it is often observed that $G(x)$ becomes 'almost' singular for some x and hence excessively large control signals may occur. However, any practical actuator is subject to limitations and constraints, and thus these excessively large control signals cannot be accommodated in practice, leading to stability and performance problems which are typically associated with actuator saturation (Enns et al., 1992; Snell et al., 1992).

Constrained control problems have been studied for many years in the context of linear control systems subject to input saturation. Controllers are usually designed without explicit knowledge of any actuator constraints, resulting in instability or performance loss once the actuator constraints are violated. Anti-windup (AW) techniques have been proposed which limit the degradation of performance during saturation (see the work by Hanus et al. (1987); Hippe (1999); Glattfelder (2003); Da Silva et al (2002); Crawshaw (2000); Turner (2004) for example). Once saturation of the control signal is detected, usually a linear filter is employed to adjust the control signal by customising the existing controller dynamics and the actual controller output. The linear AW compensator is driven by a signal which determines the extent of actuator constraint violation. Compared to the class of systems comprised of linear plant with isolated nonlinearities, less attention has been paid to nonlinear plants and control systems with actuator limits. At least part of the reason for this is the generality of this class of systems, which makes the problem more complex.

A rigorous anti-windup scheme for nonlinear affine systems has been recently proposed to alleviate actuator saturation in systems which are dynamically invertible and globally exponentially stable (Herrmann et al., 2006; Menon et al., 2006). The proposed anti-windup scheme, 'NDI-AW', follows similar ideas used in linear anti-windup compensation (Weston, 2000; Turner et al., 2004) but generalises these to this special nonlinear setting. In particular, the anti-windup scheme makes use of a coprime-factorization approach involving a pseudo-copy of the plant and a nonlinear state-feedback term as a free parameter. In (Herrmann et al., 2006) it was shown that, for the trivial choice of this state feedback term, the anti-windup compensation scheme reduces to a nonlinear version of the well-known internal model control (IMC) anti-windup strategy. In that paper, an $\mathscr{L}_2$ performance optimisation approach was also suggested to improve the IMC anti-windup design. This optimisation problem contains unknowns which are, effectively, the Lyapunov function, the anti-windup state-feedback control term and a matrix arising from sector bounding the

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saturation nonlinearity. The optimisation problem reduces to a problem containing a matrix partial differential inequality which is similar in form to the linear matrix inequalities derived by Turner et al. (2003); Herrmann et al. (2003); Turner (2004), but containing nonlinear entries. The search space is non-linear and non-convex in nature, i.e. robust nonlinear optimisation methods are necessary. An algorithm for solving this possibly non-convex and nonlinear optimisation problem is provided by Menon et al. (2006); Herrmann et al. (2006).

The main contribution of the current paper is a rigorous application of the developed NDI AW theory to a realistic aerospace example. Due to the popularity of NDI controllers in designing flight control laws, this should be of interest to researchers in the aerospace community. In the case considered here, the AW compensation scheme allows a remarkable recovery of nominal control performance. We also compare the performance of the internal model control compensation with the NDI AW compensation scheme. Note that the theory developed by Herrmann et al. (2006); Menon et al. (2006) are for globally asymptotically stable systems. Although the Hawk model considered in this paper is not of that class, the methods developed in Herrmann et al. (2006); Menon et al. (2006) can be applied in a local manner, allowing local exponential stability to be guaranteed.

2. THE NDI ANTI-WINDUP APPROACH

Notation used throughout the paper is standard but we make extensive use of the following. The saturation $\mathrm{sat}(\cdot)$ and deadzone function $\mathrm{Dz}(\cdot)$ are related by:

$$\mathrm{sat}(u) = u - \mathrm{Dz}(u) \tag{1}$$

$$\mathrm{sat}(u) = \begin{bmatrix} \mathrm{sat}_1(u_1) \\ \vdots \\ \mathrm{sat}_m(u_m) \end{bmatrix} \quad \mathrm{Dz}(u) = \begin{bmatrix} \mathrm{Dz}_1(u_1) \\ \vdots \\ \mathrm{Dz}_m(u_m) \end{bmatrix} \tag{2}$$

where $\mathrm{sat}_i(u_i) = \mathrm{sign}(u_i)\min(|u_i|, \bar{u}_i) \quad \forall i$ and $\mathrm{Dz}_i(u_i) = \mathrm{sign}(u_i)\max(0, |u_i| - \bar{u}_i) \quad \forall i$. Also $\bar{u}_i > 0 \quad \forall i \in \{1, \ldots, m\}$. It follows from the graphs of these functions that for some diagonal matrix $W > 0$, the following inequality holds

$$\tilde{u}' W (u - \tilde{u}) \geq 0 \tag{3}$$

where $\tilde{u} = \mathrm{Dz}(u)$ or $\tilde{u} = \mathrm{sat}(u)$.

We denote the $\mathscr{L}_2$ norm of a signal

$$\|x\|_2 = \sqrt{\int_0^\infty \|x\|^2 dt}$$

and for a (nonlinear) operator, $\mathscr{H}$, we define the induced $\mathscr{L}_2$ norm, or $\mathscr{L}_2$ gain, as $\|\mathscr{H}\|_{i,2} := \sup_{0 \neq x \in \mathscr{L}_2} \frac{\|\mathscr{H}x\|_2}{\|x\|_2}$.

The system considered in the present study falls into the following class of nonlinear affine systems:

$$\dot{x} = Ax + B[f(x) + G(x)u_m] + B_{pd} d \tag{4}$$

$$y = Cx + D_{pd} d \tag{5}$$

where $x \in \mathbb{R}^n$ is the state-vector of the system; $u_m \in \mathbb{R}^m$ is the input to the plant, $u \in \mathbb{R}^m$ is the controller output and $y \in \mathbb{R}^p$ is the output vector used for feedback. The bounded exogenous disturbance signal is represented as $d \in \mathbb{R}^{n_d}$. It is assumed throughout that $f(x)$ is globally Lipschitz and that the matrix $G(x)$ is invertible for all $x \in \mathbb{R}^n$. We also make the assumption that the state x is available for feedback. The range space of the nonlinear part of the model is assumed to be a subspace of the range space of our control input.

The key assumption is that the system (4) is open-loop globally exponentially stable; that is when $u_m = 0$ and $d = 0$, the origin of $\dot{x} = Ax + Bf(x)$ is globally exponentially stable. The assumption of an exponentially stable plant is necessary to ensure global exponential stability of the closed-loop system with input saturation. In our case, we relax this assumption to that of *local exponential* stability. This allows us to apply the results developed in Herrmann et al. (2006); Menon et al. (2006) but with guarantees of local rather than global stability

A two degree of freedom linear controller, K, is used to improve the performance of the nominal NDI scheme:

$$K \sim \begin{cases} \dot{x}_c &= A_c x_c + B_c y + B_{cr} r \\ u_{lin} &= C_c x_c + D_c y + D_{cr} r \end{cases} \tag{6}$$

where $x_c \in \mathbb{R}^{n_c}$ is the linear controller state, $r \in \mathbb{R}^{n_r} \cap \mathscr{L}_p$ represents the disturbance on the controller, normally the reference input, and $u_{lin} \in \mathbb{R}^m$ is the output of the linear controller. Under the condition $u_m = u$, we assume that $\|G(x)^{-1}\|$ is bounded for all x. We also assume that, under the control law

$$u = \underbrace{G(x)^{-1}[-f(x) + u_{lin}]}_{u_{nom}} \tag{7}$$

the closed-loop system (4)-(7) is well-posed and the origin is globally exponentially stable. This is a common but important assumption, characterising the *nominal* (i.e. unsaturated) behaviour of the system.

Remark 1: In this paper, the considered Bristol Hawk model is only locally exponentially stable, so that it is only possible to consider local stability of the control system with AW compensation. In many aerospace applications, the considered air vehicle may be open loop stable in some bounded operating regions, and unstable in others. In this paper, only locally stable regions of the Bristol Hawk model are considered and thus the AW-compensated system can only be guaranteed locally exponentially stable.

The assumption on $\|G(x)^{-1}\|$ guarantees nonsingularity of the matrix $G(x)$ for all $x \in \mathbb{R}^n$. However, this assumption of non-singularity may not prevent 'extreme' cases. For instance, in some portions of the system's state-space, $G(x)$ may be nonsingular, but may be almost singular, meaning that $G(x)^{-1}$ will, in a certain sense, be large and hence by equation (7), the control input may have a very large magnitude. Obviously in practice, the control signal will saturate for sufficiently large values of demanded control. It is also important that the system tracks the reference well, even in the presence of input constraints.

Constrained nonlinear closed loop system: In the presence of input saturation, the closed-loop system satisfies $u_m = \mathrm{sat}(u)$ and hence the system's state equation is

$$\dot{x} = Ax + B[f(x) + G(x)\mathrm{sat}(u)] + B_{pd} d \tag{8}$$

An anti-windup compensator, as given in equations (9) and (10), is introduced to limit the degradation of tracking performance occurring during saturation:

$$\dot{x}_{aw} = Ax_{aw} + B[f(x_{aw}) + G(x)(h(x_{aw}) - \mathrm{Dz}(u))] \tag{9}$$

$$\theta_1 = h(x_{aw}) + G(x)^{-1} f(x_{aw}), \quad \theta_2 = Cx_{aw} \tag{10}$$

Introduction of such a compensator modifies the control input to the system to be

$$u = G(x)^{-1}[-f(x) + u_{lin}] + \theta_1 \tag{11}$$

where u_{lin} is modified from that of the u_{nom} structure shown in equations (7) and (6) as follows:

$$K \sim \begin{cases} \dot{x}_c &= A_c x_c + B_c(y - \theta_2) + B_{cr} r \\ u_{lin} &= C_c x_c + D_c(y - \theta_2) + D_{cr} r \end{cases} \tag{12}$$

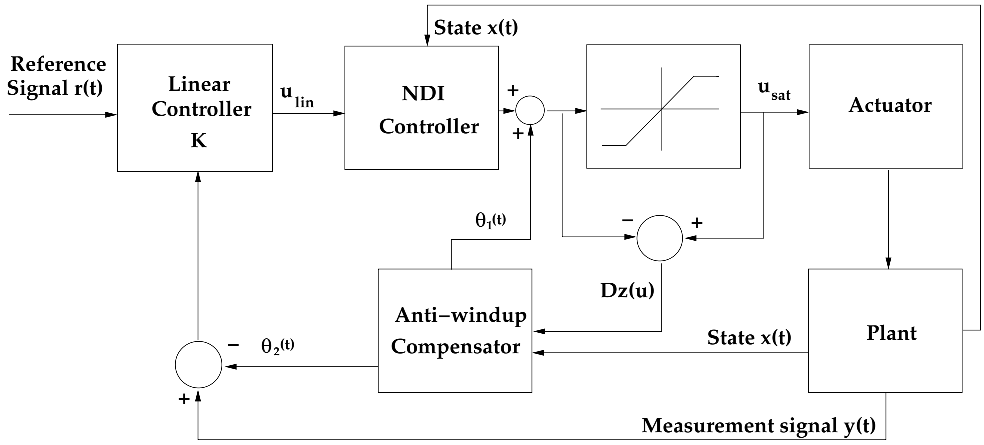

Fig. 1. The closed loop plant with anti-windup block schematic diagram

The architecture of the closed loop system with anti-windup compensator is as shown in figure 1. The form of this compensator is essentially an NDI version of the compensator used by Weston (2000); Turner (2004); Turner et al. (2004); it basically consists of a copy of the nominal plant, driven by the deadzone function and augmented with extra feedbacks (θ_1 and θ_2) to improve the system's behaviour and tracking performance while saturation is active. The free parameter, function $h(\cdot)$, will be chosen so that global stability of the AW scheme is achieved and performance close to nominal control. A simple requirement for $h(\cdot)$ is that, when $\mathrm{Dz}(u) = 0$ (i.e. when saturation does not occur), the system

$$\dot{x}_{aw} = Ax_{aw} + B[f(x_{aw}) + G(x)h(x_{aw})] \tag{13}$$

has to be globally exponentially stable. By the first assumption on the stability of the plant, such a function always exists ($h(x_{aw}) = 0$).

Note that in the formulation of (9)-(10), an additional state-feedback from the nominal system due to the presence of the $G(x)$ term is required in the anti-windup compensator.

IMC outlook: In the case of stable linear systems, a well-known anti-windup technique is that of internal model control (IMC), which essentially consists of a copy of the plant for anti-windup compensator synthesis. Note that if we set the term $h(x_{aw}) = 0$, then we are left with the anti-windup compensator

$$\dot{x}_{aw} = Ax_{aw} + B[f(x_{aw}) - G(x)\mathrm{Dz}(u)] \tag{14}$$

$$\theta_1 = G(x)^{-1} f(x_{aw}), \quad \theta_2 = Cx_{aw} \tag{15}$$

This scheme is similar in structure to the linear IMC scheme (Zheng, 1994). One of the attractive properties of the IMC anti-windup techniques is that stability is guaranteed unconditionally for the open-loop stable (nonlinear) system (Herrmann et al., 2006; Menon et al., 2006).

Despite its ease of construction and guaranteed stability properties, researchers have found the performance of the IMC anti-windup compensator to be typically quite poor (Grimm et al., 2003), even for linear systems with a single isolated nonlinearity. We cannot expect the situation to improve for nonlinear systems with NDI controllers and thus a possible optimisation approach providing anti-windup compensators with better performance is introduced by Herrmann et al. (2006); Menon et al. (2006).

$\mathscr{L}_2$ gain optimisation In order to improve upon the IMC performance, it is necessary to devise some more appealing way of choosing our free parameter, $h(x_{aw})$. As suggested in (Herrmann et al., 2006), we use an $\mathscr{L}_2$ optimisation framework to synthesise $h(x_{aw})$. In this case, a "fictitious" input signal, $w \in \mathbb{R}^m$ and a performance output z, which we assume is linear in the plant states, are introduced; thus our system under consideration becomes

$$\dot{x} = Ax + B[f(x) + G(x)\mathrm{sat}(u) - w] + B_{pd}d \tag{16}$$

$$y = Cx + D_{pd}d \tag{17}$$

$$z = C_z x \tag{18}$$

The same anti-windup compensator described in equations (9)-(10) is used. However, the control law (7) is augmented to include w, that is:

$$u = G(x)^{-1}[-f(x) + u_{lin}] + \theta_1 + w \tag{19}$$

where u_{lin} is the same as given in (12), which is a function of θ_2 generated by the antiwindup compensator (9)-(10). The $\mathscr{L}_2$ gain γ, given in equation (20) is minimised to optimise the performance of the system:

$$\gamma := \sup_{w \neq 0} \frac{\|z\|_2}{\|w\|_2}. \tag{20}$$

A similar type of performance index was used by Herrmann et al. (2004, 2005, 2004) where it was found for linear plants that this optimization approach attempts to design the AW-compensator so that performance of the control system with saturation remains close to that of the nominal closed loop system. This fact has been used to generalize the optimization approach of Turner et al. (2003); Herrmann et al. (2003); Turner et al. (2004) to a non-linear setting (Herrmann et al., 2004, 2005, 2006; Menon et al., 2006). Hence, the design for NDI-AW can be summarized in the following theorem (Herrmann et al., 2006; Menon et al., 2006):

Theorem 1. There exists an anti-windup compensator (9)-(10) which ensures that the origin of the closed-loop system (16)-(19) and (9)-(10) is asymptotically stable when $w = 0$ and the map $\mathscr{T}_p : w \mapsto z$ has $\mathscr{L}_2$ gain less than γ if there exist functions $V(x_{aw}) > 0$ and $h(x_{aw})$, a diagonal matrix $W > 0$ and a scalar $\varepsilon > 0$ such that the following inequality is satisfied for $x_{aw} \neq 0$:

$$\begin{bmatrix} \frac{\partial V}{\partial x_{aw}}[Ax_{aw} + Bf(x_{aw}) + BG(x)h(x_{aw})] + x'_{aw}C'_z C_z x_{aw} & \frac{1}{2}[\frac{\partial V}{\partial x_{aw}}BG(x) - h(x_{aw})'W] & 0 \\ \star & -(W - \frac{\varepsilon}{2}I) & -\frac{1}{2}W \\ \star & \star & -\gamma^2 I \end{bmatrix} < 0 \tag{21}$$

The search for $V(x_{aw})$ and $h(x_{aw})$ is simplified by imposing a structure on $h(x_{aw})$. Taking inspiration from optimal control, an appealing choice is $h(x_{aw}) = -G(x)'B'\partial V(x_{aw})/\partial x_{aw}$.

3. APPLICATION TO THE HAWK MODEL

A mathematical model of the $1/16^{th}$ scaled approximate model of the BAe Hawk aircraft mounted in the 1.1-metre open-jet wind tunnel of the University of Bristol is considered for the present study. The wind tunnel model has a single rotational degree of freedom in pitch. Having one rotational degree of freedom, the focus is limited to the level trim flight condition where angle of attack α is the same as the pitch angle θ. The pitch angle is controlled using the moving tail plane deflection η. The mathematical model developed by Davison et al. (2005) is based on experimental data obtained from the physical model in the wind tunnel. The model acquires the nonlinear dynamics of the physical model, including the fixed point equilibrium solutions, the Hopf bifurcations and limit cycle oscillations. However, the present study is limited to the stable region of the model.

The pitch acceleration is represented in the model as a function of the pitch angle, pitch rate and the tailplane deflection:

$$\ddot{\theta} = M_q(\theta,\dot{\theta},\eta)q + M_\theta(\theta,\dot{\theta},\eta)\theta + M_\eta(\theta,\dot{\theta},\eta)\eta$$

where M_q, M_θ, and M_η are nonlinear parameter dependent coefficients. Hence, this nonlinear model can be represented as an optimised 3-dimensional look-up table, so that M_q, M_θ, and M_η are computed a-priori for all possible combinations of the independent variables, θ, $\dot{\theta}$ and η and stored in a multidimensional look up table. The nonlinearities in the considered model emanate from the parameter dependent coefficients.

The interval of interest for the actuator η signal is $[-3.75^o, 4.75^o]$, i.e. the interval for the actuator limit. In this interval, both the *real* Hawk and the simulation model are stable and the dependency on η is not as strong as for the unstable operating region of the system. Hence, for NDI-control and NDI-AW design, it is feasible to assume that the argument η in the functions $M_q(\theta,\dot{\theta},\eta)$, $M_\theta(\theta,\dot{\theta},\eta)$ and $M_\eta(\theta,\dot{\theta},\eta)$ remains constant at the trim point $\tilde{\eta}$ in the proximity of the constrained actuator range. Thus, the model used for controller design is:

$$\begin{bmatrix} \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} M_q(\theta,\dot{\theta},\eta)|_{\eta=\tilde{\eta}} & M_\theta(\theta,\dot{\theta},\eta)|_{\eta=\tilde{\eta}} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} q \\ \theta \end{bmatrix} + \begin{bmatrix} M_\eta(\theta,\dot{\theta},\tilde{\eta})|_{\eta=\tilde{\eta}} \\ 0 \end{bmatrix} \eta \qquad (22)$$

For the NDI-controller, this creates an a-priori computed trim map lookup table which is quite usual. The nonlinear behaviour of the coefficients is shown in Fig. 2. The left subplots in Fig. 2 provide the surface plots of M_θ, M_q and M_η as function of the pitch angle θ and the pitch velocity q, while the third parameter η is fixed at 4, the level trim condition, $\tilde{\eta} = 4$.

For AW-design, the model fits the compensator synthesis approach by choosing:

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$f(\theta,\dot{\theta}) = M_q(\theta,\dot{\theta},\eta)|_{\eta=\tilde{\eta}}q + M_\theta(\theta,\dot{\theta},\eta)|_{\eta=\tilde{\eta}}\theta,$$
$$G(\theta,\dot{\theta}) = M_\eta(\theta,\dot{\theta},\eta)|_{\eta=\tilde{\eta}} \qquad (23)$$

In the simulation, a second-order actuator model is employed to represent the dynamics of this servo-motor driven actuator. The actuator transfer function is

$$\frac{5\times 10^4}{s^2 + 1250s + 5\times 10^4}.$$

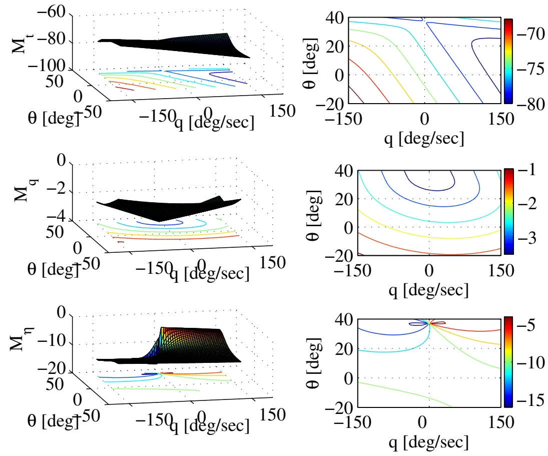

Fig. 2. Nonlinear parameter dependent coefficients

4. RESULTS

A control law structure with NDI inner loop and linear outer loop structure as given in equation (7) is adopted. With the intention of conducting future wind tunnel experiments, the controlled variable is kept as pitch angle θ, and the manipulated variable as tail plane deflection η. The nominal unconstrained control law used is as given in equation (24)

$$u = \frac{1}{M_\eta}[-(M_\theta\theta + M_q q) + u_{lin}] \qquad (24)$$

where u_{lin} is the linear outer loop PID control law ensuring the robust set point tracking:

$$u_{lin} = 15.8\theta_e + 82.8\int \theta_e\, dt + 14.25\frac{d\theta_e}{dt}. \qquad (25)$$

where $\theta_e = (\theta_r - \theta)$ represent the signal generated from the reference θ_r and the pitch measurement θ. The gains are selected to have two complex poles representing the short period dynamics. The NDI inner and linear outer loop combination provide desirable tracking performance.

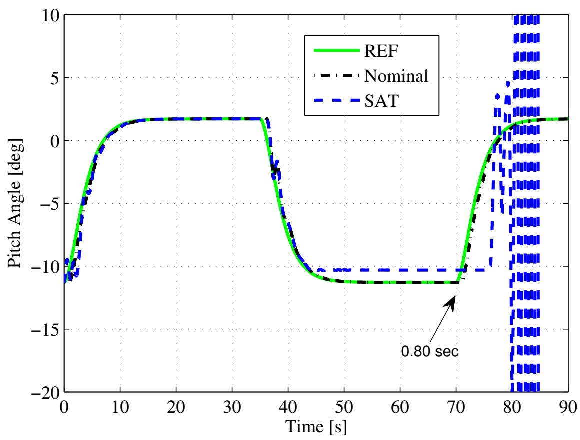

Fig. 3. The reference, nominal and saturated response

The nominal response of the controller to the tracking signal is given in figure (3). In the unconstrained case, the NDI controller tracks the reference command very well with no overshoot. However, there is a 0.8 second “delay” in tracking the reference signal. For studying the anti-windup architectures, actuator saturation with limits $[-3.75^o, 4.75^o]$ is introduced.

Table 1. Optimisation Procedure Pseudo Code

(1) Fix γ starting at a large value, say fixed at 100
SEARCH STEP 1
(2) The initial controller and compensator state are fixed to certain arbitrary value, where $x = x_{aw} \neq 0$
(3) Initialize n number of random Lyapunov candidate matrix entries, depending on the order of the Lyapunov function sought for
(4) While the termination criterion is not satisfied
  (a) Generate n positive definite symmetric matrices P_i, $i = 1,\ldots,n$ $(P_i = Q_i Q_i^T)$
  (b) $\forall P_i$, search for the largest δ satisfying the matrix inequalities; Assign $\delta > 0$ as the fitness of P_i The δ is the slack variable introduced.
  (c) Apply GA operators and continue search for the P_i with largest δ
(5) end of While
SEARCH STEP2
(6) Fix the P, obtained from STEP1
(7) Initialize n random control and compensator state values $([x, x_{aw}])$
(8) While the termination criterion satisfied
  (a) $\forall [x_n, x_{awn}]$, search for the smallest δ satisfying the matrix inequalities; Assign the δ as the fitness of the $[x_n, x_{awn}]$
  (b) Apply GA operators and continue search for the $[x, x_{aw}]$ with smallest δ
(9) end of While
(10) Choose the worst $[x_n, x_{awn}]$ and Proceed to STEP1
(11) Reduce γ and check.

In the presence of saturation, the performance of the nominal nonlinear controller deteriorates to that shown by the bold dashed line in figure 3 (given as case 'SAT'). In this case, the "delay" 'in reference tracking increases to around 4 seconds, due to the control amplitude constraints, and the closed loop system becomes unstable.

The IMC anti-windup compensator (see equations (14) and (15)) is introduced for compensation of the effects due to actuator saturation. The bold dotted line in figure (4) shows the pitch angle response of the closed loop system with the IMC anti-windup compensation scheme. Stability of the closed loop system has been recovered while the saturation nonlinearity is active. However, notice that even with IMC compensation scheme the "delay" in reference tracking increases from 0.8 to 2.2 seconds. The IMC-compensated controller also has oscillatory periods in its response, in contrast to the nominal case. Ideally, AW should recover the nominal performance as much as possible, even when saturation is active.

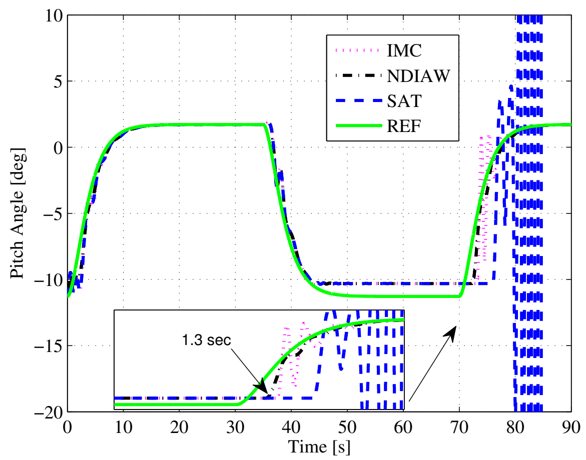

Fig. 4. Comparison of the IMC and NDIAW compensation schemes

The optimal NDI-AW compensation scheme is developed as given in equations (9) and (10). The significant difference from IMC is the introduction of the additional signal $h(x_{aw})$ just before the saturation nonlinearity. The closed loop system with the optimal anti-windup architecture is given in figure (1). The additional signal is $h(x_{aw}) = -G(x)'B'\partial V(x_{aw})/\partial x_{aw}$ and basically depends on the open loop Lyapunov function V. However, to ensure $\mathscr{L}_2$ performance, the compensation synthesis is necessarily formulated as an optimisation procedure, minimising the $\mathscr{L}_2$ gain subject to a performance matrix inequality as given in equation (21). This corresponds to Theorem 1 mentioned in Section 2. The matrix inequality in equation (21) is nonlinear (convexity of the matrix inequality cannot be established due to nonlinear coupling). Hence, we make use of a robust stochastic search procedure based on Genetic Algorithms. Genetic Algorithm (GA) Optimisation, is a general purpose stochastic search and optimisation procedure, based on genetic and evolutionary principles (Goldberg, 1989). Here, we are searching for the entries of the Lyapunov candidate matrix, subject to the matrix inequality constraints guaranteeing stability and performance. A 'δ' term is introduced for optimisation purposes and the matrix inequality given in equation(21) is re-written in the form $[ML] \leq -\delta[MR]$, where $[ML]$ is the left hand side of equation (21) and $[MR]$ is as follows:

$$\begin{bmatrix} -\dfrac{\partial V}{\partial x_{aw}}[Ax_{aw} + Bf(x_{aw})] & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \tag{26}$$

Pseudocode details of the optimisation algorithm for the optimal anti-windup compensator design are given in Table 1. The value of ε is kept very small and fixed at 0.01. Note that in 'STEP2', the states of controller and compensator are bounded.

Using the optimisation algorithm described in Table 1, we designed an anti-windup compensator which minimised the $\mathscr{L}_2$ performance bound, γ for

$$z = C_z x = \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix} = \begin{bmatrix} q \\ \theta \end{bmatrix}.$$

In this case, a quadratic, positive definite Lyapunov function was sufficient and the value returned by the optimisation procedure was

$$V(x_{aw}) = [\,x_{aw}\,]' \underbrace{\begin{bmatrix} 1.179 & 1.156 \\ 1.156 & 2.3726 \end{bmatrix}}_{P} [\,x_{aw}\,]$$

The associated value of γ was $\gamma = 0.37$ and the search for this Lyapunov function was confined to the box $|x_i|, |x_{aw,i}|$ for $i = 1, 2$ using the intervals

$$\theta \in [-20, 40],\ q \in [-150, 150]$$

as bounds for θ and q. Although a quadratic Lyapunov function was satisfactory in this case, the procedure given in Table 1, can also be used to search for higher-order Lyapunov functions.

The bold dot-dash line in figure 4 shows the pitch angle response of the closed loop system with the optimal full order NDI-AW compensation scheme. The performance of the nominal closed loop system has been recovered while the saturation nonlinearity is active. The delay in tracking the signal is only 1.3 seconds and thus the optimal NDI-AW scheme is 94% faster compared to the IMC anti-windup scheme. In the transient region, the optimal NDI-AW scheme shows no oscillatory response, resembling the nominal control performance much more closely than the IMC anti-windup scheme. This suggests that the NDI-AW using the $\mathscr{L}_2$ performance optimisation is advantageous over the baseline IMC scheme.

5. CONCLUSIONS AND FUTURE WORK

In this paper, a recently developed NDI-AW scheme has been applied to a realistic aerospace application. It is shown that the NDI-AW scheme performs better than the useful, but limited, IMC anti-windup scheme. To the authors' knowledge, this is the first realistic aerospace application of a systematic anti-windup compensation design scheme which may be applied to the industrially favoured NDI approach.

An objective for future work is to assess handling quality requirements for the NDI controller with anti-windup. In addition, the impact of different performance weighting matrices C_z will be considered. Indeed, the real-time wind tunnel testing of the nonlinear dynamic inversion anti-windup scheme will be completed in the near future.

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