Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

An Optimal Sequential Decentralized Filter of Discrete-time Systems with Cross-Correlated Noises

Chenglin Wen*. Chuanbo Wen. Yuan Li*****

** College of automation, Hangzhou Dianzi University, Hangzhou, Zhejiang, 310018 China (Tel: 0571-86915195; e-mail: wencl@ hdu.edu.cn).*
*** Eectric Engineering School, Shanghai Dianji University, Shanghai, 200240 China (e-mail: chuanbowen@163.com)*
**** School of Information Engineering, , Shenyang Institute of Chemical Technology, Shenyang, 110142, China (e-mail: li-yuan@mail.tsinghua.edu.cn)*

Abstract: In this paper, a new sequential decentralized computational structure is developed for optimal state estimation in discrete time-varying linear stochastic control system with multiple sensors and cross-correlated noises. We uses a hierarchical structure to perform successive orthogonualizations of the measurement noises, and the Kalman filters sequentially runs based on the new constructed measurement sequency. The the estimator also can process the system with measurements delay as well as data fault because the update step is just according to the coming order of sensors in a recursive form. The precision relation between the new algorithm and the centralized multisensor fusion method is strictly proved and simulation result shows that new filter is better than other similar filters in performance.

1. INTRODUCTION

Data fusion for estimation has widespread applications since many practical problems involve data from multiple sources. An important point in these problems is to fuse the data to achieve improved accuracies and more specific inferences. Traditionally, Kalman filtering theory is the best linear recursive algorithm and has been further studied and widely applied in integrated navigation systems and process of control (Linas et al., 1990; Klein, 1993).

When multiple sensors measure the state of the same stochastic system whiles the noises in the target and sensors are uncorrelated, generally there are three different types of methods to deal with the measured data. The first method is the centralized filter processing architecture (Anderson et al., 1979), where all measured sensor data are communicated to the central site for processing, and the resulting state estimates are optimal in linear minimum variance sense. The deficiency of increasing the number of sensors on a single processor includes the need for higher computation bandwidth, and poor accuracy and stability when there is several data fault. The second technology is the distributed filter (Chong, 1979), wherein the information from local estimators can yield global state estimator and the estimate process is with lower dimensional computation (Gan et al., 2001). The third method is the sequential iterative filter (Singer, 1971), where all the data are sequentially used to update the prediction. It saves much computer time, especially for the systems with some measurement delays within one sampling period, where the update are operated without waiting all the data coming.

When the noises existing in the state and each sensor are correlated, until now, various decentralized distributed filers have been developed to solve these correlation problems. Kim (Kim, 1992) and chen et al. (Chen et al., 2003], respectively, give the multisensor optimal information fusion estimator under the assumption of normal distribution, which limits the application of this algorithm. The assumption of normal distribution is omitted in the decentralized structure of (Sun et al., 2004; Sun, 2005), which are accomplished by solving one auxiliary function with Lagrange multiplier method and develop the fusion smoother weighted by matrices (FSWM) algorithm and the optimal component fusion fixed-lag Kalman smoother (CFSWS) algorithm respectively. However, the estimate models of (Sun et al., 2004; Sun, 2005) are not accurate, which result in suboptimal estimate of both filters. Song et al (Song et al., 2007) present a distributed algorithm for correlated noises. But the drawback of this filter is that it is sensitive to data fault, because it uses the noises information of all the sensors at each step, which is similar to that of centralized filter. The use of popular distributed filtering approach seems always existing some deficiency. Instead, the sequential decentralized filtering method might be more practical because of its flexible. Up to now, to the best of the author's knowledge, the issue of sequential filtering on noises cross-correlated systems has not been fully investigated.

This paper presents a sequential filtering for discrete time-varying linear stochastic system with cross-correlated state noise and measurement noises. An optimal sequential decentralized information fusion algorithm is given without the assumption of normal distribution. New algorithm is different with the centralized filter or the distributed filter mentioned in the above paragraph, and it is suitable for processing the system with data delay and also be adaptive for the situation with some data fault.

The rest is organized as follows. In Section 2, the problem formulation is presented. A new sequential Kalman filtering fusion formula with cross-correlated noises is proposed in Section 3 and the performance comparison with the centralized Kalman filter is proved in Section 4. In Section 5, the simulation example in a tracking system with three-sensor is shown. Finally, we provide a conclusion in Section 6.

2. PROBLEM FORMULATION

Consider the discrete time-varying linear stochastic control system with N sensors

$$x(k+1)=\Phi(k)x(k)+\Gamma(k)u(k)+w(k) \tag{1}$$

$$z_i(k)=H_i(k)\,x(k)+v_i(k)\;\; i=1,2,\cdots,\;N \tag{2}$$

where k represents the discrete-time index, $x(k)\in R^{n\times 1}$ is the state of the system to be estimated, $\Phi(k)\in R^{n\times n}$ and $\Gamma(k)\in R^{n\times p}$ are system matrices; $u(k)\in R^{p\times 1}$ is a known input vector, $w(k)\in R^{n\times 1}$ is a zero-mean white noise series, $z_i(k)\in R^{m_i\times 1}$ is the measurement vector from i -th sensor, $H_i(k)\in R^{m_i\times n}$ is the measurement matrix and $v_i(k)\in R^{m_i\times 1}$ is measurement noise, which satisfies $E\{v_i(k)\}=0$

Assumption 1: Process noise $w(k)$ is independent of measurement noises $v_i(k)$, $i=1,2,\cdots,\;N$, which satisfies

$$E\left\{\begin{pmatrix} w(k)\\ v_i(k)\end{pmatrix}\begin{pmatrix} w^T(l) & v_j^T(l)\end{pmatrix}\right\}=\begin{bmatrix} Q(k) & 0\\ 0 & R_i(k)\end{bmatrix}\delta_{ij}\delta_{kl} \tag{3}$$

where the superscript E and T denote the exception and the transpose, respectively, and δ_{ij} , δ_{kl} are Kronecker delta function.

Assumption 2: Process noise $w(k)$ is correlated with measurement noises $v_i(k)$, $i=1,2,\cdots,\;N$, but $v_i(k)$ is independent with each other, which satisfies

$$E\left\{\begin{pmatrix} w(k)\\ v_i(k)\end{pmatrix}\begin{pmatrix} w^T(l) & v_j^T(l)\end{pmatrix}\right\}=\begin{bmatrix} Q(k) & \Omega_i(k)\\ \Omega_j^T(k) & R_i(k)\delta_{ij}\end{bmatrix}\delta_{kl} \tag{4}$$

Assumption 3: Process noise $w(k)$ and measurement noises $v_i(k)$, $i=1,2,\cdots,\;N$,at the same time index are correlated with each other, with following statistic property

$$E\left\{\begin{pmatrix} w(k)\\ v_i(k)\end{pmatrix}\begin{pmatrix} w^T(l) & v_j^T(l)\end{pmatrix}\right\}=\begin{bmatrix} Q(k) & \Omega_i(k)\\ \Omega_j^T(k) & S_{ij}(k)\end{bmatrix}\delta_{kl} \tag{5}$$

with $R_i(k)=S_{ii}(k)$ and $R_i(k)$ is a positive matrix.

Assumption 4: The initial state $x(0)$ is independent of $w(k)$ and $v_i(k)$, $i=1,2,\cdots,\;N$ and

$$E\{x(0)\}=x_0\,,\;E\left\{[x(0)-x_0][x(0)-x_0]^T\right\}=P_0 \tag{6}$$

Our aim is to develop a new sequential decentralized filter in the sense of minimum the state estimate error covariance under assumptions 3 and 4.

3. THE SEQUENTIAL DECENTRALIZED INFORMATION FUSION ALGORITHM

For the dynamic system described in (1) and (2), this section is devoted to the development of a sequential decentralized Kalman filter (SDKF) under the assumptions 3 and 4, whose update process is depend on the coming order of sensors. The prediction results of the standard Kalman filter at the k -th moment are summarized below, with minor modification (Hashemipour et al., 1988).

$$\begin{aligned}\hat{x}(k|k-1)&=E\{x(k)|Z(1),Z(2),\cdots,Z(k-1)\}\\ &=\Phi(k-1)\hat{x}(k-1|k-1)+\Gamma(k-1)u(k-1)\\ &\quad+\Omega(k-1)R^{-1}(k-1)[Z(k-1)-H(k-1)\hat{x}(k-1|k-1)]\\ &=\overline{\Phi}(k-1)\hat{x}(k-1|k-1)+\overline{\Gamma}(k-1)\overline{u}(k-1)\end{aligned} \tag{7}$$

and

$$P(k|k-1)=\overline{\Phi}(k-1)P(k-1|k-1)\overline{\Phi}^T(k-1)+\overline{Q}(k-1) \tag{8}$$

where

$$Z^T(k-1)=\left[z_1^T(k-1),z_2^T(k-1),\cdots,z_N^T(k-1)\right]$$

$$V^T(k-1)=\left[v_1^T(k-1),v_2^T(k-1),\cdots,v_N^T(k-1)\right]$$

$$H^T(k-1)=\left[H_1^T(k-1),H_2^T(k-1),\cdots,H_N^T(k-1)\right]$$

$$R(k-1)=E\{V(k-1)V^T(k-1)\}$$

$$\overline{\Phi}(k-1)=\Phi(k-1)-\Omega(k-1)R^{-1}(k-1)H(k-1),$$

$$\overline{\Gamma}(k-1)=\left[\Gamma(k-1)\quad \Omega(k-1)R^{-1}(k-1)\right]$$

$$\overline{u}(k-1)=\begin{bmatrix} u(k-1)\\ Z(k-1)\end{bmatrix}$$

$$\overline{Q}(k-1)=Q(k-1)-\Omega(k-1)R^{-1}(k-1)\Omega^T(k-1)$$

With the prediction (7), the measurements $Z(k)$ will be used to update which in a recursive form.

3.1 Decorrelated the noises

From assumption 3, we know that the systems are correlated, which is hard to develop the sequential Kalman filter directly. These noises should be decorrelated and the Knowledge of covariance from (5) allows us to modify the decorrelation process for the multiple sensors case. Without lose of generality, the coming order of data from sensors at k -th moment is assumed to be $z_1,z_2,\cdots,z_N$.

For the technical convenience, let us first define the following additional notations.

$$Z_i^T(k) := \left[z_1^T(k), z_2^T(k)\cdots, z_i^T(k)\right],\ i = 1,2,\cdots,\ N \tag{9}$$

$$V_i^T(k) := \left[v_1^T(k), v_2^T(k)\cdots, v_i^T(k)\right],\ i = 1,2,\cdots,\ N \tag{10}$$

$$\Psi_i^T(k) := \left[H_1^T(k), H_2^T(k),\cdots, H_i^T(k)\right],\ i = 1,2,\cdots,\ N \tag{11}$$

$$\begin{aligned}\overline{S}_i(k) &:= E\{v_i(k)V_{i-1}^T(k)\} = E\{v_i(k)\left[v_1^T(k), v_2^T(k)\cdots, v_{i-1}^T(k)\right]\} \\ &= \left[S_{i1}(k), S_{i2}(k),\cdots, S_{i\,i-1}(k)\right],\quad i = 2,3,\cdots,N\end{aligned} \tag{12}$$

$$\begin{aligned}\Lambda_i(k) &:= E\{V_i(k)V_i^T(k)\} \\ &= E\{\left[v_1^T(k), v_2^T(k)\cdots, v_i^T(k)\right]^T\left[v_1^T(k), v_2^T(k)\cdots, v_i^T(k)\right]\} \\ &= \begin{bmatrix} R_1(k) & S_{12}(k) & \cdots & S_{1i}(k) \\ S_{21}(k) & R_2(k) & \cdots & S_{2i}(k) \\ \vdots & \vdots & \ddots & \vdots \\ S_{i1}(k) & S_{i2}(k) & \cdots & R_i(k) \end{bmatrix}\end{aligned} \tag{13}$$

$$G_i(k) := \overline{S}_i(k)\Lambda_{i-1}^{-1}(k),\ i = 2,3,\cdots,\ N \tag{14}$$

Using (2), the extend measurement equation of first i sensors is

$$Z_i(k) = \Psi_i(k)x(k) + V_i(k),\ i = 1,2,\cdots,\ N \tag{15}$$

For each local sensor, with (2), we have

$$z_1(k) = H_1(k)x(k) + v_1(k) \tag{16a}$$

$$\begin{aligned}z_i(k) &= H_i(k)x(k) + v_i(k) \\ &= H_i(k)x(k) + v_i(k) - G_i(k)V_{i-1}(k) + G_i(k)V_{i-1}(k) \\ &\qquad i = 2,3,\cdots,N\end{aligned} \tag{16b}$$

Combing (15) and (16), for $i = 2,3,\cdots,N$, we obtain

$$z_i(k) = H_i(k)x(k) + v_i(k) - G_i(k)V_{i-1}(k) + G_i(k)\left[Z_{i-1}(k) - \Psi_{i-1}(k)x(k)\right] \tag{17}$$

Let

$$\overline{z}_i(k) = \begin{cases} z_1(k), & i = 1 \\ z_i(k) - G_i(k)Z_{i-1}(k), & i = 2,3,\cdots,N \end{cases} \tag{18}$$

$$\overline{v}_i(k) = \begin{cases} v_1(k), & i = 1 \\ v_i(k) - G_i(k)V_{i-1}(k), & i = 2,3,\cdots,N \end{cases} \tag{19}$$

$$\overline{H}_i(k) = \begin{cases} H_1(k), & i = 1 \\ H_i(k) - G_i(k)\Psi_{i-1}(k), & i = 2,3,\cdots,N \end{cases} \tag{20}$$

and

$$\overline{R}_i(k) = E\{\overline{v}_i(k)\overline{v}_i^T(k)\} \tag{21}$$

(16) and (17) will be rewritten as

$$\overline{z}_i(k) = \overline{H}_i(k)x(k) + \overline{v}_i(k),\ i = 1,2,\cdots,N \tag{22}$$

which is the new measurement equation.

Theorem 1: The new constructed measurement noises $\overline{v}_i(k)$ in (22), for $i = 1,2,\cdots,N$, is independent with each other.

Proof: Let us consider $\overline{v}_i(k)$ and $\overline{v}_j(k)$ for $i \neq j$. Without lose of generality, we assume $1 \le i < j$. The covariance of $\overline{v}_j(k)$ and $V_{j-1}(k)$ is

$$\begin{aligned}\Delta_j(k) &= E\{\overline{v}_j(k)V_{j-1}^T(k)\} \\ &= E\{\left[v_j(k) - G_j(k)V_{j-1}(k)\right]V_{j-1}^T(k)\} \\ &= \overline{S}_j(k) - G_j(k)\Lambda_{j-1}(k) \\ &= 0\end{aligned} \tag{23}$$

or

$$\begin{aligned}\Delta_j(k) &= E\{\overline{v}_j(k)V_{j-1}^T(k)\} \\ &= E\{\overline{v}_j(k)\left[v_1^T(k), v_2^T(k)\cdots, v_{j-1}^T(k)\right]\} \\ &= \left[E\{\overline{v}_j(k)v_1^T(k)\}, E\{\overline{v}_j(k)v_2^T(k)\},\cdots, E\{\overline{v}_j(k)v_{j-1}^T(k)\}\right]\end{aligned} \tag{24}$$

We have

$$E\{\overline{v}_j(k)v_l^T(k)\} = 0 \qquad l = 1,2,\cdots, j-1$$

If $i = 1$, we obtain

$$E\{\overline{v}_j(k)\overline{v}_1^T(k)\} = E\{\overline{v}_j(k)v_1^T(k)\} = 0 \tag{25}$$

else

$$\begin{aligned}E\{\overline{v}_j(k)\overline{v}_i^T(k)\} &= E\{\overline{v}_j(k)\left[v_i(k) - G_i(k)V_{i-1}(k)\right]^T\} \\ &= E\left\{\overline{v}_j(k)\left[v_i(k) - \sum_{t=1}^{i-1} G_i(\cdot,t,k)v_t(k)\right]^T\right\} \\ &= E\{\overline{v}_j(k)v_i^T(k)\} - \sum_{t=1}^{i-1} E\{\overline{v}_j(k)v_t^T(k)\}G_i^T(\cdot,t,k)\end{aligned} \tag{26}$$

where $G_i(\cdot,t,k)$ is the matrix composed by the $\sum_{l=1}^{t-1} m_l + 1$-th to $\sum_{l=1}^{t} m_l$-th columns of $G_i(k)$.

With (25) and (26), we make the conclusion that

$$E\{\overline{v}_j(k)\overline{v}_i^T(k)\} = 0 \tag{27}$$

Remark 1: The decorrelated method adapted above is similar to the Gram-Schmidt orthogonalization process (Lax, Peter D., 1996). When the measurement noises are uncorrelated, $\overline{S}_i(k) = 0$, $\overline{\Lambda}_i(k) = 0$ and the new constructed measurement equation (22) is just to be (2).

3.2 Derivation and construction

This section is to develop the SDKF under assumptions 3 and 4 based on the construction given in section 3.1. The sequential decentralized fusion algorithm (denoted as α) based on $\bar{z}_1(k), \bar{z}_2(k), \cdots, \bar{z}_N(k)$ runs a separate Kalman filter for each sensor, and the recursive structure is

$$\begin{aligned}\hat{x}_1^{(\alpha)}(k|k) &= E\{x(k)|\bar{z}_1(k)\} \\ &= \hat{x}(k|k-1) + K_1^{(\alpha)}(k)[\bar{z}_1(k) - \bar{H}_1(k)\hat{x}(k|k-1)]\end{aligned} \tag{28a}$$

$$\begin{aligned}\hat{x}_i^{(\alpha)}(k|k) &= E\{x(k)|\bar{z}_1(k), \bar{z}_2(k), \cdots, \bar{z}_i(k)\} \\ &= \hat{x}_{i-1}^{(\alpha)}(k|k) + K_i^{(\alpha)}(k)[\bar{z}_i(k) - \bar{H}_i(k)\hat{x}_{i-1}^{(\alpha)}(k|k)]\end{aligned} \tag{28b}$$

where

$$K_1^{(\alpha)}(k) = P(k|k-1)\bar{H}_1^T(k)\left[\bar{H}_1(k)P(k|k-1)\bar{H}_1^T(k) + \bar{R}_1(k)\right]^{-1} \tag{29a}$$

$$K_i^{(\alpha)}(k) = P_{i-1}^{(\alpha)}(k|k)\bar{H}_i^T(k)\left[\bar{H}_i(k)P_{i-1}^{(\alpha)}(k|k)\bar{H}_i^T(k) + \bar{R}_i(k)\right]^{-1} \tag{29b}$$

The corresponding variance matrix of the optimal information fusion estimator is computed by

$$P_1^{(\alpha)}(k|k) = [I - K_1^{(\alpha)}(k)\bar{H}_1(k)]P(k|k-1) \tag{30a}$$

$$P_i^{(\alpha)}(k|k) = [I - K_i^{(\alpha)}(k)\bar{H}_i(k)]P_{i-1}^{(\alpha)}(k|k), \; i = 2,3,\cdots,N \tag{30b}$$

Once $\bar{z}_N(k)$ is coming and used to update the prediction, the sequential update at this moment is end, i.e. $\hat{x}^{(\alpha)}(k|k) := \hat{x}_N^{(\alpha)}(k|k)$, and $P^{(\alpha)}(k|k) := P_N^{(\alpha)}(k|k)$ are obtained, and the entire process is replaced for the new set of measurements at next time step.

The idea of the sequential decentralized fusion algorithm under the assumptions 3 and 4 are illustrated in figure 1.

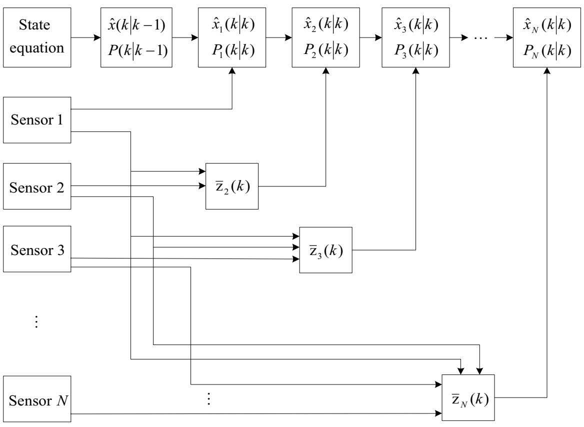

Fig.1. The fusion structure of SDKF

Remark 2: As shown in Fig.1, the observations update the prediction sequentially, so it can be used to process the system with observe time-delay within one period according to their coming order. Also because new algorithm does not require all the data obtained, it can also operated with some data fault or missing.

4. PERFORMANCE COMPARISON

In this section, the performance of SDKF is presented to compare with that of the centralized Kalman filter.

Lemma 1(Singer, 1971)***:*** For system (1) and (2) with assumptions 1 and 4, the sequential filter has the same estimate accuracy as the centralized Kalman filter.

Lemma 2: For system (1) and (2) with assumptions 2 and 4, the sequential filter has the same estimate accuracy as the centralized Kalman filter.

Proof: The proof of Lemma 2 is similar to that of Lemma 1, which is omitted here.

Theorem 2: Under the assumptions 3 and 4, the estimate accuracy of sequential decentralized Kalman filter based on $\bar{z}_1(k), \bar{z}_2(k), \cdots, \bar{z}_N(k)$ (denoted as α) and that of centralized Kalman filter based on $Z(k)$ (denoted as β) are the same, i.e. $\hat{x}^{(\alpha)}(k|k) = \hat{x}^{(\beta)}(k|k)$, $P^{(\alpha)}(k|k) = P^{(\beta)}(k|k)$.

Proof: Before giving the discussion process, let us introduce the centralized Kalman filter based on $\bar{Z}(k)$ (denoted as γ), which is a bridge of connecting the performance of these two algorithms.

For technique convenience, some new notations are defined again

$$\bar{Z}^T(k) = \left[\bar{z}_1^T(k), \bar{z}_2^T(k) \cdots, \bar{z}_N^T(k)\right]$$

$$\bar{V}^T(k) = \left[\bar{v}_1^T(k), \bar{v}_2^T(k) \cdots, \bar{v}_N^T(k)\right]$$

$$\bar{H}^T(k) = \left[\bar{H}_1^T(k), \bar{H}_2^T(k), \cdots, \bar{H}_N^T(k)\right]$$

$$\bar{R}(k) = E\left\{\bar{V}(k)\bar{V}^T(k)\right\} = diag\left[\bar{R}_1(k), \bar{R}_2(k) \cdots, \bar{R}_N(k)\right],$$

The measurement (22) can be rewritten with extended dimension form as

$$\bar{Z}(k) = \bar{H}(k)x(k) + \bar{V}(k) \tag{31}$$

which is used to update (7), and the update equation is

$$\hat{x}^{(\gamma)}(k|k) = \hat{x}(k|k-1) + K^{(\gamma)}(k)[\bar{Z}(k) - \bar{H}(k)\hat{x}(k|k-1)] \tag{32}$$

where

$$K^{(\gamma)}(k) = P(k|k-1)\bar{H}^T(k)\left[\bar{H}(k)P(k|k-1)\bar{H}^T(k) + \bar{R}(k)\right]^{-1} \tag{33}$$

and the estimate error covariance is

$$P^{(\gamma)}(k|k) = \left[I - K^{(\gamma)}(k)\bar{H}(k)\right]P(k|k-1) \tag{34}$$

Also, the centralized Kalman filter based on $Z(k)$ is

$$\hat{x}^{(\beta)}(k|k) = \hat{x}(k|k-1) + K^{(\beta)}(k)[Z(k) - H(k)\hat{x}(k|k-1)] \tag{35}$$

where

$$K^{(\beta)}(k) = P(k|k-1)H^T(k)[H(k)P(k|k-1)H^T(k) + R(k)]^{-1} \tag{36}$$

and

$$P^{(\beta)}(k \mid k) = [I - K^{(\beta)}(k)H(k)]P(k \mid k-1) \tag{37}$$

Next, we will take two steps to prove this theorem. What we first to do is to illustrated that the two different centralized Kalman filters β and γ have identical estimate accuracy and then also prove the performance of SDKF and that of the centralized Kalman filter based on $\overline{Z}(k)$ are the same.

A. The performance of the two centralized Kalman filter are the same, i.e. $\hat{x}^{(\gamma)}(k|k) = \hat{x}^{(\beta)}(k|k)$, $P^{(\gamma)}(k|k) = P^{(\beta)}(k|k)$

According to (11), (20) and the definition of $\overline{H}(k)$, we have

$$\overline{H}(k) = \begin{bmatrix} \overline{H}_1(k) \\ \overline{H}_2(k) \\ \vdots \\ \overline{H}_N(k) \end{bmatrix} = \begin{bmatrix} H_1(k) \\ H_2(k) - G_2(k)\Psi_1(k) \\ \vdots \\ H_N(k) - G_N(k)\Psi_{N-1}(k) \end{bmatrix} = \begin{bmatrix} H_1(k) \\ H_2(k) - G_2(k)H_1(k) \\ \vdots \\ H_N(k) - \sum_{i=1}^{N-1} G_N(.,i,k)H_i(k) \end{bmatrix} \tag{38}$$
$$= \mathrm{M}(k)H(k)$$

where

$$\mathrm{M}(k) = \begin{bmatrix} 1 & 0 & 0 & & 0 \\ -G_2(k) & 1 & & 0 & 0 \\ -G_3(\cdot,1,k) & -G_3(\cdot,2,k) & \ddots & \vdots & \vdots \\ \vdots & \vdots & & 1 & 0 \\ -G_N(\cdot,1,k) & -G_N(\cdot,2,k) & & -G_N(\cdot,N-1,k) & 1 \end{bmatrix} \tag{39}$$

which is invertible.

Also because

$$\overline{V}(k) = \begin{bmatrix} \overline{v}_1(k) \\ \overline{v}_2(k) \\ \vdots \\ \overline{v}_N(k) \end{bmatrix} = \begin{bmatrix} v_1(k) \\ v_2(k) - G_2(k)V_1(k) \\ \vdots \\ v_N(k) - G_N(k)V_{N-1}(k) \end{bmatrix} = \mathrm{M}(k)V(k) \tag{40}$$

We have

$$\overline{R}(k) = E\{\overline{V}(k)\overline{V}^T(k)\} = \mathrm{M}(k)R(k)\mathrm{M}^T(k) \tag{41}$$

which is inserted in (33) to obtain

$$\begin{aligned} K^{(\gamma)}(k) &= P(k|k-1)\overline{H}^T(k)\left[\overline{H}(k)P(k|k-1)\overline{H}^T(k) + \overline{R}(k)\right]^{-1} \\ &= P(k|k-1)H^T(k)M^T(k) \\ &\quad \times\left[M(k)H(k)P(k|k-1)H^T(k)M^T(k) + M(k)R(k)M^T(k)\right]^{-1} \\ &= K^{(\beta)}(k)\mathrm{M}^{-1}(k) \end{aligned} \tag{42}$$

Combining (32), (34), (35) and (37), we have

$$\begin{aligned} \hat{x}^{(\gamma)}(k \mid k) &= \hat{x}(k \mid k-1) + K^{(\beta)}(k)\left[\overline{Z}(k) - \overline{H}(k)\hat{x}(k \mid k-1)\right] \\ &= \hat{x}(k \mid k-1) + K^{(\gamma)}(k)\mathrm{M}^{-1}(k) \\ &\quad \times\left[\mathrm{M}(k)Z(k) - \mathrm{M}(k)H(k)\hat{x}(k \mid k-1)\right] \\ &= \hat{x}^{(\beta)}(k \mid k) \end{aligned} \tag{43a}$$

and

$$\begin{aligned} P^{(\gamma)}(k|k) &= \left[I - K^{(\gamma)}(k)\overline{H}(k)\right]P^{(\gamma)}(k|k-1) \\ &= \left[I - K^{(\beta)}(k)H(k)\right]P(k|k-1) \\ &= P^{(\beta)}(k|k) \end{aligned} \tag{43b}$$

So, the two centralized Kalman filter have same estimate value and estimate error covariance, which is to say the two centralized Kalman filter have the same performance.

B. The performance of SDKF and that of the centralized Kalman filter based on $\overline{Z}(k)$ are the same, i.e.

$\hat{x}^{(\alpha)}(k|k) = \hat{x}^{(\gamma)}(k|k)$, $P^{(\alpha)}(k|k) = P^{(\gamma)}(k|k)$.

The measurement noises in (22) are white. With lemma 2, we can make the conclusion that the sequential algorithm based on $\overline{z}_1(k), \overline{z}_2(k), \cdots, \overline{z}_N(k)$ and the centralized algorithm based on $\overline{Z}(k)$ have same performance. i.e.

$$\hat{x}^{(\alpha)}(k \mid k) = \hat{x}^{(\gamma)}(k \mid k) \tag{44a}$$

$$P^{(\alpha)}(k|k) = P^{(\gamma)}(k|k) \tag{44b}$$

From (42) and (43), we have

$$\hat{x}^{(\alpha)}(k \mid k) = \hat{x}^{(\beta)}(k \mid k) \tag{45a}$$

$$P^{(\alpha)}(k|k) = P^{(\beta)}(k|k) \tag{45b}$$

The results in Theorem 2 show that the sequential decentralized Kalman filter is also the optimal linear estimator in the sense of minimum estimate error covariance.

5. SIMULATION EXAMPLE

Consider a discrete time-varying dynamic linear system with three sensors

$$x(k+1) = \begin{bmatrix} 1 & 0.2\sin(\pi k/L) \\ 0 & 0.5 \end{bmatrix} x(k) + \begin{bmatrix} 0.6 & 0 \\ 0 & 0.5\sin(\pi k/L) \end{bmatrix} u(k) + w(k) \tag{46}$$

$$z_i(k) = H_i(k)x(k) + v_i(k),\ v_i(k) = c_i(k)w(k) + \mu_i(k),\ i = 1,2,3. \tag{47}$$

where $x(k) = \begin{bmatrix} \xi(k) \\ \zeta(k) \end{bmatrix}$ is the state, $u(k) = [u_1(k), u_2(k)]^T$ is the known two-dimensional (2-D) input that satisfies $u_1(k) = 1$, $u_2(k) = -1$ if $0 < k \le L/2$ and $u_1(k) = -1$, $u_2(k) = 1$ if $L/2 < k \le L$. $z_i(k)$, $i = 1,2,3$ are measurement signals of three sensors with time-varying measurement matrices $H_1 = \begin{bmatrix} 1 & 0 \\ 0.6\cos(\pi k/L) & 0.4 \end{bmatrix}$, $H_2 = \begin{bmatrix} 1 & -0.7\sin(\pi k/L) \\ 0.6 & 0.5 \end{bmatrix}$ and $H_3 = \begin{bmatrix} 0.6 & 0.1 \\ 0 & 0.2\sin(\pi k/L) \end{bmatrix}$. $w(k)$ is a zero mean system noise with variance $Q(k)$. $v_i(k)$, $i = 1,2,3$, are, respectively, the measurement noises of three sensors, which are consist of two parts and are correlated with the input system noise $w(k)$. $\mu_i(k), i=1,2,3$, are 2-D Gaussian white noises with zero mean and variance matrices $Q_{\mu_i}(k)$. Our aim is to present the sequential decentralized Kalman filter and illustrate its advantage in performance.

In the simulation, setting $Q(k)=1.5I_2$, $Q_{\mu_1}(k)=0.4I_2$, $Q_{\mu_2}(k)=0.2I_2$, $Q_{\mu_3}(k)=0.1I_2$; $c_1=0.9$, $c_2=0.5$, $c_3=0.7$, the initial state is $x_0=\begin{bmatrix}0\\10\end{bmatrix}$ and $P_0=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, we will take $L=100$ sampling data. To compare with the CFSWS and FSWM, the variance of which are also shown in Fig.2

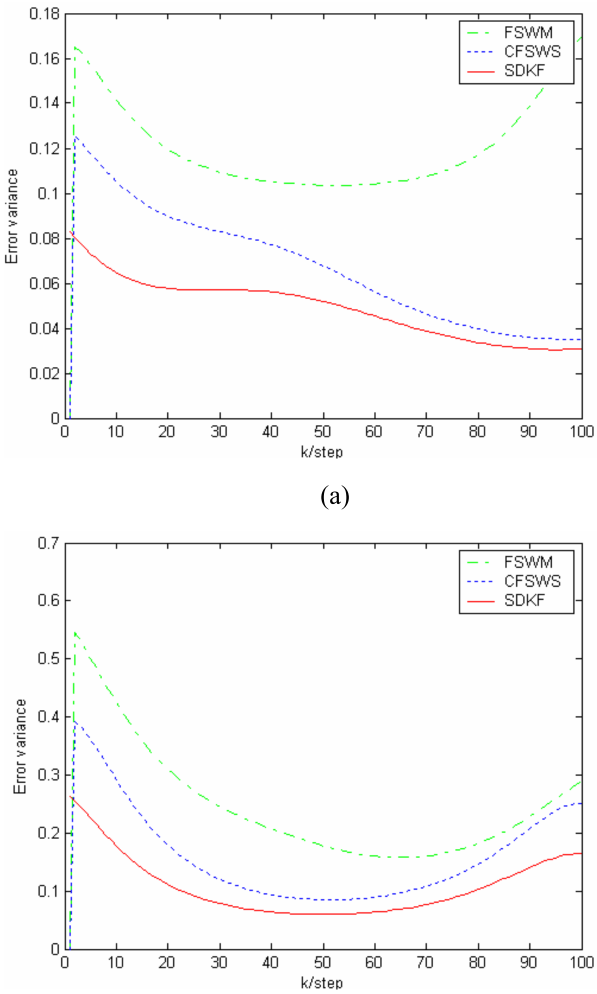

Fig. 2. Comparison of precision for three decentralized fusion filters. (a) Estimating error variance of component $\xi(k)$; (b) Estimating error covariance of component $\zeta(k)$.

The dash-dot curves denote the variances of two components of CFSWS, the dashed curves denote the variances of two components of FSWM, and the solid curves denote the variances of two components of SDKF. From Fig.2, we see that SDKF have better precision than both FSWM and CFSWS do, also the precision of CFSWS is higher than that of FSWM algorithm.

ACKNOWLEDGEMENTS

This work was supported by the NSFC (60434020)

6. CONCLUSION

For discrete time-varying linear stochastic control systems with multiple sensors and correlated noises, an optimal sequential decentralized fusion estimator is given by using a serious of successive orthogonalizations on measurement noises subspace for each sensor. We prove that its performance is equivalent to that of the standard centralized Kalman filter in estimate accuracy. When each processor implements its filtering algorithm in a sequential manner, the next coming new constructed measurement will be used to update it. Because the estimators process the data in a sequential way, some data delay which is less than a sampling period is allowed. Simulation results show the advantage of new algorithm by comparing its estimate precision with other two decentralized algorithms.

REFERENCES

Linas, J. and Waltz, E. (1990), *Multisensor Data fusion*. Boston, MA: Artech House, 1990

Klein, L. A. (1993), *Sensor and Data Fusion Concepts and Applications*. SPIE Opt. Engineering Press, Tutorial Texts, vol. 14, 1993.

Anderson, B. D. O. and Moore (1979), J. B., *Optimal filtering*. Englewood Cliffs, NJ: Prentice-Hall.

Chong, C. Y. (1979), Hierarchical estimation, in *Proc. 2nd MIT/ONR C3 Workshop*, Monterey, CA.

Singer, R. A., and Sea, R. G.(1971), Increasing the computational efficiency of discrete Kalman filters, *IEEE Tran. Auto. Cont., 16*:254-257.

Kim, K. H.(1994). Development of track to track fusion algorithm, *in Proc. American Control Conf.,* 1037-1041.

Chen, H., Kirubarajan, T. and Bar-Shalom, Y. (2003), Performance limits of track-to-track fusion vs. centralized estimation: Theory and application, *IEEE Trans. Aerosp. Electron. Syst., 39*:386-398.

Sun, S. L. and Deng, Z. L. (2004). Multi-sensor Optimal Information Fusion Kalman filter. *Automatic. 40*(6):1017-1023.

Sun, S. L. (2005). Distributed optimal component fusion weighted by scalars for fixed-lag Kalman smoother. *Automatic*, *41*(12): 2153-2159.

Song E. B.(2007), Y. M. Zhu, J. Zhou and Z. S. You. Optimal Kalman filtering fusion with cross-correlated sensor noises. *Automatic* 43: 1450-1456.

Gan, Q. and Harris, C. J. (2001). Comparison of two measurement fusion methods for the Kalman-filter-based multisensor data fusion. *IEEE Tran. Aerosp. Electron. Syst., 37*(1): 273-279.

Hashemipour, H. R., Roy, S. and Laub, A. J. (1988), Decentralized structures for parallel Kalman filtering. *IEEE Trans. Auto. Cont., 33*(1): 88-94.

Lax, Peter D. (1996). Lax. *Linear Algebra*. New York: Wiley.