Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

An Approach to Unknown Input Observation for Non-Input-Affine Nonlinear MIMO Systems

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Abstract: This paper presents the application of a linear state and input observation approach to a non-input-affine nonlinear MIMO system. To detect and identify disturbances on the system's input, the system is transformed into an input-affine form by dynamic compensation and then linearized. The linearization carried out is not exact due to the unknown disturbance. It is shown that the linearization error can be represented as an equivalent input disturbance to the exact linear system representation and the linear observation approach therefore holds. The usefulness of the proposed method is demonstrated by a practical example. The system under consideration is a car-like mobile platform subject to steering angle errors. A control approach to compensate for the input disturbance based on the observation results is presented.

Keywords: observers for linear systems, nonlinear system control, application of nonlinear analysis and design, tracking, mobile robots, fault detection and diagnosis

1. INTRODUCTION

The detection and reconstruction of unknown system inputs is of great importance in diagnosis as well as in control of uncertain and/or disturbed systems. In general, two different basic problems can be distinguished: the estimation of system uncertainties that can be presented as additional system inputs and the diagnosis of unknown system inputs due to e.g. actuator failures. Both problems result in similar observation approaches.

A solution to the problem of simultaneous unknown input (UI) and state estimation for uncertain continuous-time systems was presented by Corless and Tu (1998). The employed system model, consisting of a linear, autonomous part and a time- and state-dependent input part, allows the utilization of the presented results for uncertain systems as well as systems with input disturbances. Xiong and Saif (2003) generalized the approach, incorporating the adaption approach presented by Wang and Daley (1996). In Edwards (2004), a comparison of two different observation approaches is carried out: sliding mode and unknown input, emphasizing the interconnection of them. The considered observer's application is limited to linear systems. An approach to ease the necessary condition for the existence of UI observers through delay was presented by Sundaram and Hadjicostis (2007). It is applicable to linear, discrete-time systems. The notion of input observability, without the necessity of state observability, was first introduced by Hou and Patton (1998). Also, the interconnection of system invertibility and input observability was pointed out. Inversion approaches to reconstructing unknown inputs of nonlinear systems were presented by Devasia (1999) and Edelmayer et al. (2004), the latter presenting the geometric viewpoint and a residual generation to detect multiple faults. Both concepts are limited to input-affine systems.

Farza et al. (2005) presented a design method for high gain UI observers for a class of nonlinear MIMO systems. The developed observer was evaluated by Cheviron et al. (2007) in a stochastic setting under the assumption of all states being available. Witczak et al. (2007) presented an UI and state observer for a class of nonlinear discrete-time systems.

The problem discussed in this paper is that of observing the unknown, i.e. disturbed, input of a class of non-input-affine nonlinear MIMO systems, not fitting into any of the system classes considered in the works cited above. The main idea to solve the observation problem is to first transform the system into an input-affine form by dynamic compensation, carry out an input-output linearization along a predefined trajectory and then design an UI observer based on the procedures presented in Corless and Tu (1998) and Xiong and Saif (2003). The crux is to show that the linear observation approach holds: The performed linearization is influenced by the unknown input leading to a deviation from the predefined trajectory. Therefore the central point is to derive a transformation procedure to represent the flaw in linearization as an equivalent unknown input and identify the relation of it to the originally wanted input disturbance. As a side-effect, the presented approach shows that, for a certain class of nonlinear systems, uncertainties leading to inexact linearization results can be dealt with as presented here.

The paper is organized as follows: In section 2 the observation problem under consideration is stated. The transformation of the problem into an equivalent UI observation problem of the linearized system is carried out and discussed in section 3. The following section 4 presents the adopted UI observer. Throughout the paper a step-by-step example illustrating the presented results is given. A decoupled trajectory control approach based on the observation result is presented for the example system in section 5. Simulation results are shown in section 6. The paper closes with a conclusion.

2. PROBLEM STATEMENT AND PRELIMINARIES

We consider a class of stable nonlinear MIMO systems with uncertain inputs described by

$$\dot{\boldsymbol{x}} = f_n(\boldsymbol{x}, \boldsymbol{u}_{eff}), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{1a}$$

$$\boldsymbol{y} = h(\boldsymbol{u}_{eff}) \tag{1b}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the system state, $\boldsymbol{y}(t) \in \mathbb{R}^m$ is the system output, and $\boldsymbol{u}_{eff} \in \mathbb{R}^m$ is the effective physical input

$$\boldsymbol{u}_{eff} = \boldsymbol{u} + \boldsymbol{\epsilon} \tag{2}$$

consisting of the nominal system input $\boldsymbol{u}$ and the unknown disturbance $\boldsymbol{\epsilon}$. The number of inputs is assumed to be the same as the number of outputs which often holds for physical systems. The output is assumed to be an algebraic function of the system's input which holds for a major class of mechatronic systems with motors as actuators and encoders or tachometers as sensors. We further make the following assumption on the system's output, which will be required in the course of the linearization process.

Assumption 1. The Jacobian of the output function

$$\boldsymbol{J}_h = \begin{pmatrix} \frac{\partial h_1}{\partial u_1} & \cdots & \frac{\partial h_1}{\partial u_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_m}{\partial u_1} & \cdots & \frac{\partial h_m}{\partial u_m} \end{pmatrix} \tag{3}$$

is nonsingular. The Jacobian being nonsingular can be interpreted as some kind of "differential input observability".

An algebraic calculation of the effective input values is theoretically possible, though highly inadvisable due to the unreliability of such open-loop calculations. Therefore an observer approach with inherent feedback structure is sought.

Throughout the presented approach, the following example illustrating the results and their applicability will be considered:

Example 2. The system under consideration is a car-like mobile autonomous platform with two physical inputs: the velocity v of the rear axis and the steering angle δ. The velocity is measurable, the steering angle is not. It is subject to actuator faults as well as external disturbances. To solve the tracking problem for the mobile platform, the steering angle needs to be estimated. The available sensor signals are only the speeds v_r an v_l of the right and left rear wheels. This leads to the system output being a direct function of the system input as assumed in the problem statement.

The dynamics of the mobile platform is given by (cf. Mitschke and Wallentowitz (2004)):

$$\dot{x} = v\cos(\psi + \beta) \tag{4a}$$

$$\dot{y} = v\sin(\psi + \beta) \tag{4b}$$

$$\dot{\psi} = \dot{\psi} \tag{4c}$$

$$\ddot{\psi} = -\frac{C}{v}\dot{\psi} + D\delta \tag{4d}$$

$$\dot{\beta} = -\frac{A}{v}\beta + \frac{B}{v}\delta - \dot{\psi}, \tag{4e}$$

where x and y are the coordinates of the mobile platform's position w.r.t. a global coordinate frame, ψ is the orientation of the platform, and β is the attitude angle. The velocities v_r and v_l of the right an left rear wheel are

$$y_{1,2} = v \pm vE\tan\delta = h(v, \delta). \tag{5}$$

A, B, C, D, and E are positive constants determined by the physical properties of the platform.

Remark: The platform's dynamic model is defined for velocities unequal zero, which, in our case, does not represent any constraint, because the platform is considered during motion only.

For control purposes, a concept to monitor the (measurable) velocity and the (uncertain) steering angle of the platform will be developed as an illustrative example accompanying the presented theoretical results.

3. TRANSFORMATION AND LINEARIZATION

To transform the system into a form to which a linear observation approach is applicable, the system is first transformed into an input-affine form by dynamic compensation and then linearized along a trajectory.

3.1 Transformation to linear form

Dynamic compensation. By choosing the new system inputs to be

$$\boldsymbol{v} = \dot{\boldsymbol{u}} \tag{6}$$

the following augmented system is obtained:

$$\dot{\tilde{\boldsymbol{x}}} = \begin{pmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{u}} \end{pmatrix} = \underbrace{\begin{pmatrix} f_n(\boldsymbol{x}, \boldsymbol{u}) \\ \boldsymbol{0} \end{pmatrix}}_{f(\tilde{\boldsymbol{x}})} + \underbrace{\begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{pmatrix}}_{g} \boldsymbol{v} \tag{7a}$$

$$\boldsymbol{y} = h(\boldsymbol{u}) = h(\tilde{\boldsymbol{x}}) \tag{7b}$$

with $\boldsymbol{I}$ being an $m \times m$ identity matrix. The order of the augmented system is given by $\tilde{n} = n + m$.

Remark: The reference values of $\boldsymbol{u}$ are assumed to be generated a priori by a corresponding algorithm, therefore the use of their time derivatives does not impose any problem.

Example 3. Applying the dynamic compensation to the system presented in example 2 leads to the following equation

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} \dot{v} \\ \dot{\delta} \end{pmatrix}, \tag{8}$$

the resulting augmented system is given by

$$\dot{\widetilde{\boldsymbol{x}}} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \ddot{\psi} \\ \dot{\beta} \\ \dot{v} \\ \dot{\delta} \end{pmatrix} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \end{pmatrix} = \underbrace{\begin{pmatrix} v\cos(\psi+\beta) \\ v\sin\psi+\beta \\ \dot{\psi} \\ -\frac{C}{v}\dot{\psi}+D\delta \\ -\frac{A}{v}\beta+\frac{B}{v}\delta-\dot{\psi} \\ 0 \\ 0 \end{pmatrix}}_{f(\widetilde{\boldsymbol{x}})} + \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}}_{g} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \tag{9}$$

with the output equation remaining unchanged.

IO linearization. The following theorems show that

- IO linearization (cf. Isidori (1989)) can always be carried out
- the linearization leads to an IO behavior resembling m decoupled integrators

for systems of the form (1) fulfilling assumption 1.

Theorem 4. The compensated system of the form given by (7a) and (7b) always has a vector relative degree

$$\boldsymbol{r} = (r_1 \ \cdots \ r_m) = (1 \ \cdots \ 1)$$

if Assumption 1 holds.

Proof. The two conditions on $\boldsymbol{r}$ are

(i) $L_{gj}L_f^k h_i(\widetilde{\boldsymbol{x}}) = 0$
for all: $i \le j \le m, \quad 1 \le i \le m, \quad k < r_i - 1$

(ii) $|\boldsymbol{A}| = \begin{vmatrix} L_{g1}L_f^{r_1-1}h_1(\widetilde{\boldsymbol{x}}) & \cdots & L_{gm}L_f^{r_1-1}h_1(\widetilde{\boldsymbol{x}}) \\ \vdots & \ddots & \vdots \\ L_{g1}L_f^{r_m-1}h_m(\widetilde{\boldsymbol{x}}) & \cdots & L_{gm}L_f^{r_m-1}h_m(\widetilde{\boldsymbol{x}}) \end{vmatrix} \neq 0.$

It can be readily seen that the Jacobian given by (3) is identical to the transformation matrix

$$\boldsymbol{A} = \begin{pmatrix} L_{g1}h_1(\widetilde{\boldsymbol{x}}) & \cdots & L_{gm}h_1(\widetilde{\boldsymbol{x}}) \\ \vdots & \ddots & \vdots \\ L_{g1}h_m(\widetilde{\boldsymbol{x}}) & \cdots & L_{gm}h_m(\widetilde{\boldsymbol{x}}) \end{pmatrix}. \tag{10}$$

Its nonsingularity therefore implies $\boldsymbol{r} = (1 \ \cdots \ 1)$. □

Theorem 5. A linearizing input transformation for the compensated system of the form given by (7a) and (7b) is given by

$$\boldsymbol{w} = \boldsymbol{A}\boldsymbol{v}. \tag{11}$$

Proof. Differentiation of the system output yields

$$y_i^{(r_i)} = L_f^{r_i}h_i + \sum_{j=1}^{m} L_{gj}L_f^{r_i-1}h_i u_j$$

with $r_i = 1$ and

$$L_f h_i = 0 \ \forall i \qquad \square$$

The implications of theorem 4 and 5 together with the fact that the distribution

$$G = \operatorname{span}\{g_1, \ldots, g_m\} \tag{12}$$

is always involutive for the compensated system lead to a transformation matrix of the form

$$\boldsymbol{T} = \begin{pmatrix} h_1(\widetilde{\boldsymbol{x}}) \\ \vdots \\ h_m(\widetilde{\boldsymbol{x}}) \\ \phi_{m+1} \\ \vdots \\ \phi_{\tilde{n}} \end{pmatrix}, \tag{13}$$

transforming the compensated system into m single integrators as the resulting IO relation with internal dynamics given by

$$\begin{pmatrix} z_{m+1} \\ \vdots \\ z_{\tilde{n}} \end{pmatrix} = \begin{pmatrix} \phi_{m+1} \\ \vdots \\ \phi_{\tilde{n}} \end{pmatrix}. \tag{14}$$

By choosing

$$\begin{pmatrix} z_{m+1} \\ \vdots \\ z_{\tilde{n}} \end{pmatrix} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \tag{15}$$

the original system's stable dynamics is recovered in the internal dynamics of the compensated, linearized system.

The linearization is carried out along a predefined nominal trajectory. The linearizing input transformation $\boldsymbol{A}$ is a function of the system's physical input $\boldsymbol{u}$ subject to unknown disturbances $\boldsymbol{\epsilon}$ not accounted for in the calculation of the linearizing input transformation. This leads to a flaw in the linearization. To still observe the system with a linear approach, the equivalence of the observation problems has to be investigated.

Example 6. The compensated system of example 3 has a relative vector degree $r = (1 \ 1)$. The linearizing state feedback is given by the matrix

$$\boldsymbol{A} = \begin{pmatrix} L_{g1}h_1 & L_{g2}h_1 \\ L_{g1}h_2 & L_{g2}h_2 \end{pmatrix} = \begin{pmatrix} 1 + E\tan\delta & \frac{Ev}{\cos^2\delta} \\ 1 - E\tan\delta & -\frac{Ev}{\cos^2\delta} \end{pmatrix} \tag{16}$$

which is nonsingular $\forall \boldsymbol{x}$ with $x_6 = v \neq 0$. By choosing the transformation matrix

$$T(\boldsymbol{x}) = \begin{pmatrix} x_6 + x_6 E\tan x_7 \\ x_6 - x_6 E\tan x_7 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \end{pmatrix} \tag{17}$$

to be

$$T(\boldsymbol{x}) = \begin{pmatrix} x_6 + x_6 E\tan x_7 \\ x_6 - x_6 E\tan x_7 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} v + vE\tan\delta \\ v - vE\tan\delta \\ x \\ y \\ \psi \\ \dot{\psi} \\ \beta \end{pmatrix} \tag{18}$$

we obtain the following normal form

$$\dot{z}_1 = w_1 = u_1(1 + E\tan\delta) + u_2\frac{Ev}{\cos^2\delta} \quad (19a)$$
$$\dot{z}_2 = w_2 = u_1(1 - E\tan\delta) - u_2\frac{Ev}{\cos^2\delta} \quad (19b)$$
$$\dot{z}_3 = \dot{x}_1 \quad (19c)$$
$$\dot{z}_4 = \dot{x}_2 \quad (19d)$$
$$\dot{z}_5 = \dot{x}_3 \quad (19e)$$
$$\dot{z}_6 = \dot{x}_4 \quad (19f)$$
$$\dot{z}_7 = \dot{x}_5 \quad (19g)$$

The resulting linear system comprises two uncoupled integrators, the internal dynamics ((19c) to (19g)) is given by the original system's dynamics, which is open-loop stable.

3.2 Equivalence of system representations

To investigate the equivalence of observing a disturbance $\boldsymbol{\epsilon}$ of the physical system input (see Fig. 1(a)) to observing a disturbance $\boldsymbol{e}$ at the input of the linearized system (see Fig. 1(b)) we take advantage of the decoupling effect of the linearization carried out. A disturbance ϵ_i affecting the i^{th} input of the physical system will have an effect on certain components of the system output determined by (1b). The following procedure leads to a direct relation between ϵ_i and $\boldsymbol{e}$

(1) Determine the outputs y_j that are not influenced by ϵ_i using (1b).
(2) Choose the elements e_j of the equivalent input disturbance vector $\boldsymbol{e}$ to be zero if the j^{th} output is not influenced by ϵ_i.
(3) Calculate
$$\boldsymbol{v}_{eff} = \boldsymbol{A}^{-1}(\boldsymbol{w} + \boldsymbol{e}). \quad (20)$$
(4) Determine the relation of the two input disturbances by comparing the components of $\boldsymbol{v}_{eff} = \dot{\boldsymbol{u}} + \dot{\boldsymbol{\epsilon}}$ to $\boldsymbol{u}_{eff}$.

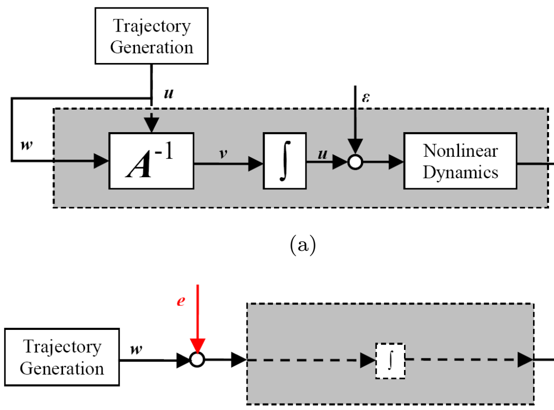


Fig. 1. (a) System with unexact feedforward linearization, (b) Equivalent linear system with input disturbance

The sketched procedure leads to a direct relation between ϵ_i and $\boldsymbol{e}$ and can therefore be utilized to transform an observed input disturbance of the linearized equivalent system back into the originally wanted physical disturbance.

Example 7. Following the procedure sketched above, we first consider the influence of ϵ on the two output components v_r and v_l of the system linearized in example 6.

$$\begin{pmatrix} v_r \\ v_l \end{pmatrix} = \begin{pmatrix} v + vE\tan\delta \\ v - vE\tan\delta \end{pmatrix}.$$

Both inputs are influenced, therefore we choose

$$\begin{pmatrix} w_{1eff} \\ w_{2eff} \end{pmatrix} = \begin{pmatrix} w_1 + e_1 \\ w_2 + e_2 \end{pmatrix}.$$

Evaluating (20) leads to

$$v_{1eff} = \frac{1}{2}(w_1 + w_2) + \frac{1}{2}(e_1 + e_2) \quad (21)$$
$$v_{2eff} = \frac{\cos^2\delta - E\cos^2\delta\tan\delta}{2vE}w_1 - \frac{\cos^2\delta + E\cos^2\delta\tan\delta}{2vE}w_2 + \frac{\cos^2\delta}{vE} \quad (22)$$

Comparing this result to

$$\begin{pmatrix} u_{1eff} \\ u_{2eff} \end{pmatrix} = \begin{pmatrix} v \\ \delta + \epsilon \end{pmatrix}$$

and obeying (6) we obtain

$$e_1 = -e_2 = e \quad (23)$$
$$\epsilon = \int e\frac{\cos^2\delta}{vE}\mathrm{d}t \quad (24)$$

through comparison to the nominal case (11).

4. UI OBSERVATION

For the resulting linearized system given by

$$\begin{pmatrix} \dot{z}_1 \\ \vdots \\ \dot{z}_m \\ \dot{z}_{m+1} \\ \vdots \\ \dot{z}_{\tilde{n}} \end{pmatrix} = \begin{pmatrix} w_1 \\ \vdots \\ w_m \\ \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{pmatrix} \quad (25)$$
$$\begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} = \begin{pmatrix} z_1 \\ \vdots \\ z_m \end{pmatrix} \quad (26)$$

an UI observer as presented by Corless and Tu (1998) and Xiong and Saif (2003) is now used to determine the effective input. The observer structure is shown in Fig. 2.

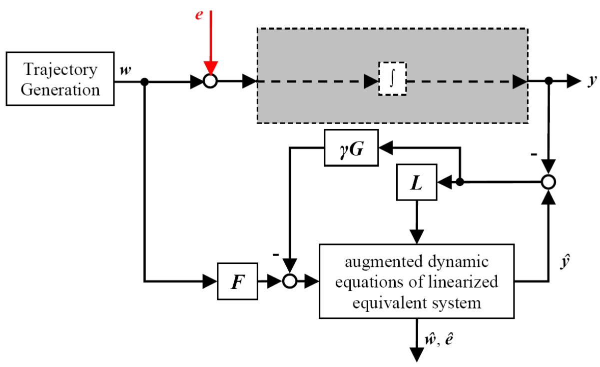


Fig. 2. Structure of the UI observer for the equivalent system

The matrix $\boldsymbol{F}$ representing the influence of the nominal inputs $\boldsymbol{w}$ is given by

$$\boldsymbol{F} = \begin{pmatrix} \boldsymbol{I} \\ \boldsymbol{0} \end{pmatrix} \quad (27)$$

with $\boldsymbol{I}$ being an $m \times m$ itdentity matrix and the $\boldsymbol{0}$ matrix having the dimension $l \times m$ where $l \leq m$ is the number of equivalent input disturbances.

Within the observer, the input matrix $\boldsymbol{B}$ is augmented to account for the influence of $\boldsymbol{e}$:

$$\boldsymbol{B} = (\boldsymbol{I} \;\; \boldsymbol{D}) \tag{28}$$

with $\boldsymbol{I}$ being an $m \times m$ identity matrix and $\boldsymbol{D}$ having the dimension $m \times l$. $\boldsymbol{D}$ is readily determined by following the procedure described in subsection 3.2.

The resulting observer equations are:

$$\dot{\hat{\boldsymbol{y}}} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} f_o + \boldsymbol{L}\,[\boldsymbol{I}\hat{\boldsymbol{y}} - \boldsymbol{y}] \tag{29}$$

$$f_o = \boldsymbol{F}\boldsymbol{w} - \gamma \boldsymbol{G}\,[\boldsymbol{I}\hat{\boldsymbol{y}} - \boldsymbol{y}] \tag{30}$$

The observer matrices $\boldsymbol{L}$ and $\boldsymbol{G}$ are calculated as depicted in the workings of Corless and Tu (1998) and Xiong and Saif (2003).

Example 8. The observer matrices for the dynamic platform of examples 2 to 7 are

$$\boldsymbol{F} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}. \tag{31}$$

The input matrix $\boldsymbol{B}$ is augmented to account for the opposing influence of e:

$$\boldsymbol{B} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}. \tag{32}$$

The resulting observer reliably determines the input disturbance $\boldsymbol{e}$ from which the disturbance ϵ on the steering can be calculated. Simulation results for different disturbance scenarios are shown and discussed in section 6. The utilization of the achieved results for trajectory control purposes is shown in section 5.

5. TRAJECTORY CONTROL – A PRACTICAL EXAMPLE

Based on the observation of the unknown equivalent input e of the system of examples 2 to 8, a closed-loop structure as shown in Fig. 3 is now used to compensate for deviations of the steering angle δ from the reference value.

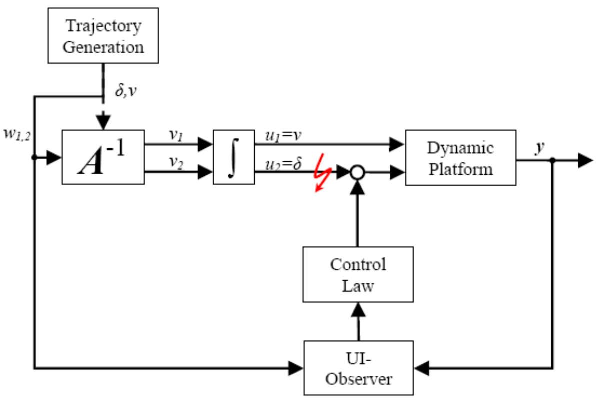


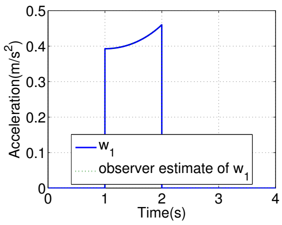
Fig. 3. Structure of closed-loop system

The measurable platform velocity is controlled by a decoupled outer control loop not depicted in the figure. To utilize the observer estimate $\hat{\boldsymbol{e}}$ for the steering angle control, relation (24) is incorporated in the control law. A linear fast I-type controller with the reconstructed ϵ as input is sufficient for the sought compensation.

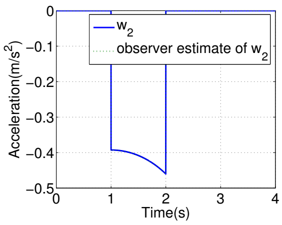
The presented control approach leads to a decoupled velocity and steering angle control for trajectory tracking based on very limited sensor information.

6. SIMULATION RESULTS

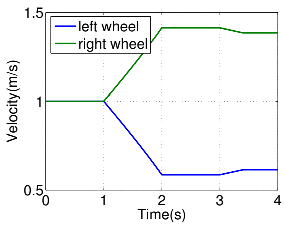
Figures 4 and 5 show simulation results for two different disturbance scenarios. In the first case, the platform is initially driving straight with constant velocity v_0. From $t = 1\,\text{s}$ to $t = 2\,\text{s}$ the steering angle δ is linearly increased to its final reference value $\frac{\pi}{8}$ rad, causing the platform to pursue a circular path. At $t = 3\,\text{s}$, an external disturbance causes a gradual reduction of the steering angle by $\frac{\pi}{128}$ rad. In the second case, the reference trajectory of the platform is the same as in the first case but now the consecutive loss of motor steps, leading to less steering action as wanted, is considered. From $t = 1.75\,\text{s}$ on the steering angle does not change any more.



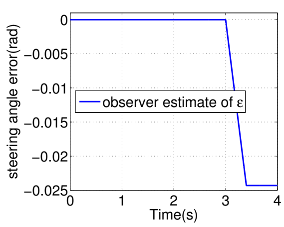
(a) input 1



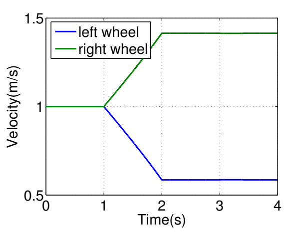
(b) input 2



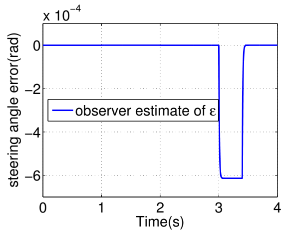
(c) wheel velocities



(d) steering angle error



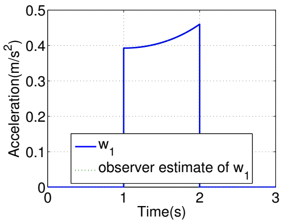
(e) wheel velocities, controlled case



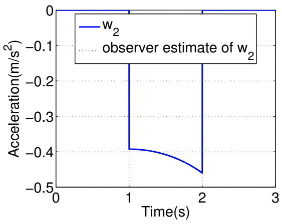
(f) steering angle error, controlled case

Fig. 4. Simulation results, circular driving with external disturbance

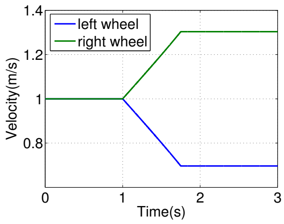
The plots are arranged the same way in both figures: (a) and (b) show the reference end estimated values of w_1 and w_2. (c) shows the resulting system output v_r and v_l. The estimated steering angle disturbance is shown in (d). (e) and (f) show the velocities and error in steering angle for the closed loop structure presented in section 5 respectively.



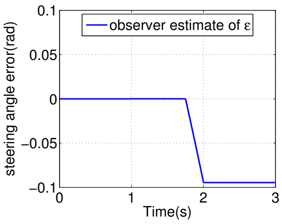
(a) input 1



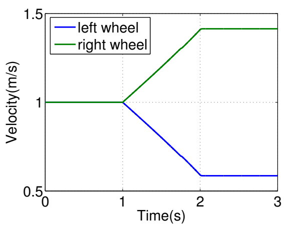
(b) input 2



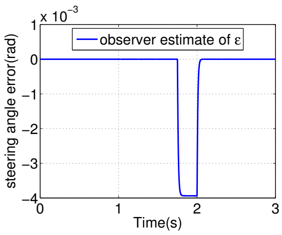
(c) wheel velocities



(d) steering angle error



(e) wheel velocities, controlled case



(f) steering angle error, controlled case

Fig. 5. Simulation results, consecutive step losses during steering action

Clearly, in both cases the observation and compensation of the input disturbance are successful. The chosen scenarios show that the presented approach works for disturbances occurring during steady state operation of the platform as well as for disturbances interfering with the transient behavior.

7. CONLUSION

The reconstruction of unknown inputs of nonlinear MIMO systems by a linearization approach is presented in this paper. The class of systems considered is characterized by having general, nonlinear state equations and nonlinear algebraic output equations depending on the system inputs only.

To achieve the reconstruction aim the system is first transformed into an augmented input-affine system and then linearized along a trajectory by exact IO linearization. The unknown inputs of the original system cause this linearization to be inaccurate because they lead to deviations from the reference trajectory used for linearization. To still achieve a correct observation result, a method of transforming the linearization inaccuracies into an equivalent input disturbance of the nominal linearized system is presented. Afterwards, a linear UI observer for the resulting nominally linear system with input disturbance is designed.

A step-by-step practical example is given to illustrate the developed procedure and demonstrate the observation results. For the example system, a car-like mobile platform with steering angle errors, a trajectory control approach based on the observation results is shown as a sample application.

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