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Linear least-squares estimation based on covariances from multiple correlated uncertain observations ⋆

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Abstract:
In this paper, the linear least-squares estimation problem of signals from correlated uncertain observations coming from multiple sensors is addressed. It is assumed that, at each sensor, the signal is measured in the presence of additive white noise and that the uncertainty in the observations is characterized by a set of Bernoulli random variables which are only correlated at consecutive time instants. Assuming that the probability and correlation of such variables are not necessarily the same for all the sensors, a recursive filtering and fixed-point smoothing algorithm is proposed. The derivation of such algorithm does not require the knowledge of the signal state-space model, but only the covariance functions of the processes involved in the observation equation of each sensor, as well as the probability and correlation of the Bernoulli variables modeling the uncertainty. Recursive expressions for the estimation error covariance matrices are also provided, and the performance of the estimators is illustrated by a numerical simulation example wherein a signal is estimated from correlated uncertain observations coming from two sensors with different uncertainty characteristics.

1. INTRODUCTION

There are many practical situations concerning the estimation of signals from measurements coming from different multiple sensors. For example, in engineering applications involving communication networks with a heavy network traffic, the measurements available may not be up-to-date and so, the signal is estimated from delay observations coming from multiple sensors (see e.g. Matveev and Savkin [2003] and Hounkpevi and Yaz [2007], among others).

There is also a large class of real-world problems where the signal appears in the observation in a random manner, such as problems where there are intermittent failures in the observation mechanism, fading phenomena in propagation channels, target tracking, accidental loss of some measurements or data inaccessibility during certain times. These situations are characterized by the fact that the signal is not always present in the observations but there exists a positive probability that the observations do not contain the signal. This consideration is modeled including in the observation equation not only an additive noise, but also a multiplicative noise consisting of a sequence of Bernoulli random variables taking the value one if the observation contains signal plus noise, or the value zero if it is only noise (*uncertain observations*). Assuming that the state-space model is completely known, the least-squares (LS) estimation problem in this kind of systems has been studied by several authors under different hypotheses on the processes involved in the state-space model (see Hermoso and Linares [1995] and NaNacara and Yaz [1997], among others). On the other hand, considering that the state-space model of the signal is not available, but only the covariance functions of the processes involved in the observation equation and the probability that the signal exists in the observations are known, the LS estimation problem has been also addressed (see e.g. Nakamori et al. [2003] and Nakamori et al. [2004]). In the above papers, the variables modeling the uncertainty in the observations are assumed to be independent, so the distribution of the multiplicative noise is fully determined by the probability that each particular observation contains the signal. More general situations, in which such independence assumption is not valid since the variables modeling the uncertainty are correlated at consecutive instants, have been previously considered by Jackson and Murthy [1976] who, using also a state-space approach, derived a LS linear filtering algorithm which provides the estimator at any time from those in the two preceding instants. This situation has been also treated in Nakamori et al. [2005a] and Nakamori

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et al. [2005b] under a covariance approach and filtering and fixed-point smoothing algorithms have been derived for this uncertain observation model.

Taking into account the model studied by Hounkpevi and Yaz [2007] about the estimation of signals considering multiple delayed sensors with different delay characteristics, the aim of this paper is to approach the estimation problem of signals from correlated uncertain observations coming from multiple sensors with different uncertainty characteristics. Recursive algorithms for the filtering and fixed-point smoothing estimators are proposed together with recursive formulas to calculate the corresponding estimation error covariance matrices (as a measure of the estimation accuracy). To derive them, it is assumed that the state-space model of the signal to be estimated is not known, but only the covariance functions of the signal and noise processes involved in the observation equations of each sensor, as well as the probability and correlation of the Bernoulli variables modeling the uncertainty are available.

2. PROBLEM STATEMENT

In this section, the least-squares (LS) linear estimation problem of an n-dimensional discrete-time random signal, z_k, which cannot be directly observed but only through observations coming from m different sensors, is formulated. Suppose that, at each sensor, there exist intermittent failures in the measure mechanism which cause that, at each sampling time, the signal may be present or not in the observation in a random way. First, the observation model for these measurements and the hypotheses about the signal and noise processes are described.

2.1 Observation model

Consider m sensors which at any time $k \geq 1$ provide scalar measurements of an n-dimensional signal, z_k, perturbed by additive noises and assume that each measurement can be only noise with a known probability. So, if y_k^i, $i = 1, \ldots, m$ denote the available observations, v_k^i is the noise perturbing the signal transmitted by the i-th sensor, and $1-p_k^i$ is the *false alarm probability* (that is, the probability that only noise is observed or, equivalently, that y_k^i does not contain the transmitted signal), the observations are specified as

$$y_k^i = \begin{cases} H_k^i z_k + v_k^i, & \text{with probability } \; p_k^i \\ v_k^i, & \text{with probability } \; 1 - p_k^i. \end{cases}$$

Therefore, if $\{\gamma_k^i;\ k \geq 1\}$, $i = 1, \ldots, m$ denote sequences of Bernoulli random variables (binary switching sequences taking the values 0 or 1) with $P[\gamma_k^i = 1] = p_k^i$, the observations of the signal can be described by

$$y_k^i = \gamma_k^i H_k^i z_k + v_k^i, \quad k \geq 1, \quad i = 1, \ldots, m. \qquad (1)$$

Thus, at the i-th sensor, the observations $\{y_k^i;\ k \geq 1\}$ are perturbed by a multiplicative noise, $\{\gamma_k^i;\ k \geq 1\}$, which models the uncertainty about the signal being present in each observation (this is called the *uncertainty* of the observation).

The estimation problem will be carried out by assuming that the signal to be estimated, $\{z_k;\ k \geq 1\}$, is a zero-mean process with covariance function admitting a semi-degenerate kernel form, $E[z_k z_s^T] = A_k B_s^T$, $s \leq k$, where A and B are known $n \times M$ matrix functions, and the additive noises, $\{v_k^i;\ k \geq 1\}$, $i = 1, \ldots, m$, are zero-mean white processes with known variances $Var[v_k^i] = R_k^i$, for all $k \geq 1$. Moreover, the signal process, $\{z_k;\ k \geq 1\}$, and the noise processes, $\{v_k^i;\ k \geq 1\}$ and $\{\gamma_k^i;\ k \geq 1\}$, for $i = 1, \ldots, m$, are mutually independent.

It is also assumed that, at each sensor, the uncertainty in the observation at time k depends on the uncertainty at time $k-1$, but it is independent of uncertainties at times previous to $k-1$; this is formulated by imposing the stochastic independence of the Bernoulli variables γ_k^i and γ_s^i when $|k-s| \geq 2$.

To simplify the notation, (1) is rewritten in a compact form as follows:

$$Y_k = D_k^\gamma H_k z_k + V_k, \quad k \geq 1, \qquad (2)$$

where $Y_k = \left(y_k^1, \ldots, y_k^m\right)^T$, $H_k = \left((H_k^1)^T, \ldots, (H_k^m)^T\right)^T$, $D_k^\gamma = Diag\left(\gamma_k^1, \ldots, \gamma_k^m\right)$, $V_k = \left(v_k^1, \ldots, v_k^m\right)^T$ and, from the model hypotheses, the following properties hold:

- (i) The m-dimensional process $\{V_k;\ k \geq 1\}$ is a zero-mean white sequence with covariance matrix function $E[V_k V_k^T] = R_k = Diag\left(R_k^1, \ldots, R_k^m\right)$.
- (ii) The random matrices D_k^γ and D_s^γ are independent for $|k-s| \geq 2$. The mean of D_k^γ is $D_k^p = Diag\left(p_k^1, \ldots, p_k^m\right)$ and the covariance of D_k^γ and D_{k-1}^γ, which can be a nonzero matrix, is $K_{k,k-1}^\gamma = Diag\left(E[\gamma_k^1\gamma_{k-1}^1] - p_k^1 p_{k-1}^1, \ldots, E[\gamma_k^m \gamma_{k-1}^m] - p_k^m p_{k-1}^m\right)$.
- (iii) $\{z_k;\ k \geq 1\}$, $\{V_k;\ k \geq 1\}$ and $\{D_k^\gamma;\ k \geq 1\}$ are mutually independent.

2.2 Linear LS estimation problem

Given the uncertain observations $\{Y_1, \ldots, Y_L\}$, $L \geq k$, the aim is to find a recursive LS linear estimator, $\widehat{z}_{k/L}$, of the signal z_k. As known, this estimator is the orthogonal projection of the vector z_k onto $\mathcal{L}(Y_1, \ldots, Y_L)$, the linear space spanned by $\{Y_1, \ldots, Y_L\}$. Since the observations are generally nonorthogonal vectors, we use an innovation approach, based on an orthogonalization procedure wherein we transform the observation process $\{Y_k;\ k \geq 1\}$ to an equivalent one (*innovation process*) of orthogonal vectors $\{\nu_k;\ k \geq 1\}$, equivalent in the sense that each set $\{\nu_1, \ldots, \nu_L\}$ spans the same linear subspace as $\{Y_1, \ldots, Y_L\}$.

The vector ν_k, named innovation at time k, is defined as $\nu_k = Y_k - \widehat{Y}_{k/k-1}$ where $\widehat{Y}_{1/0} = 0$ and, for $k \geq 2$, $\widehat{Y}_{k/k-1}$, the one-stage linear predictor of Y_k, is the projection of Y_k onto $\mathcal{L}(\nu_1, \ldots, \nu_{k-1})$; the orthogonality property allows us to find this projection by separately projecting onto each of the previous orthogonal vectors; that is,

$$\widehat{Y}_{k/k-1} = \sum_{j=1}^{k-1} E[Y_k \nu_j^T](E[\nu_j \nu_j^T])^{-1} \nu_j, \quad k \geq 2. \qquad (3)$$

In a similar way, the replacement of the observation process by the innovation one leads to the following expression for the LS linear estimator of the signal

$$\widehat{z}_{k/L} = \sum_{j=1}^{L} S_{k,j}\Pi_j^{-1}\nu_j, \qquad (4)$$

where $S_{k,j} = E[z_k\nu_j^T]$ and $\Pi_j = E[\nu_j\nu_j^T]$. In view of expression (4), we will start by obtaining an explicit formula for the innovations. Afterwards, we will derive recursive formulas for the filter, $\widehat{z}_{k/k}$, and for the fixed-point smoother, $\widehat{z}_{k/L},\ L > k$.

3. INNOVATION PROCESS

When, for each $i = 1, \ldots, m$, the Bernoulli variables $\{\gamma_k^i;\ k \geq 1\}$ modeling the uncertainty at the i-th sensor are independent, all the information prior to time k which is required to estimate y_k^i is provided by the one-stage predictor $\widehat{z}_{k/k-1}$ (see Nakamori et al. [2003] and Nakamori et al. [2004]). However, for the problem at hand, the correlation between γ_{k-1}^i and γ_k^i, which must be considered for such estimation, is not contained in $\widehat{z}_{k/k-1}$. Therefore, to obtain the current innovation ν_k, it is necessary to find the new expression for the one-stage predictors of y_k^i, or equivalently for the one-stage predictor (3).

Taking into account the model hypotheses,

$$E[Y_k\nu_i^T] = D_k^p H_k E[z_k\nu_i^T], \qquad i \leq k-2$$

and

$$\begin{aligned} E[Y_k\nu_{k-1}^T] = &\ E[C_k^\gamma C_{k-1}^{\gamma T}] \circ (H_kA_kB_{k-1}^TH_{k-1}^T) \\ &-D_k^pH_kE[z_k\widehat{Y}_{k-1/k-2}^T], \quad k \geq 2, \end{aligned}$$

where $C_k^\gamma = Col(\gamma_k^1, \ldots, \gamma_k^m)$ and $\circ$ denotes the Hadamard product ($[A \circ B]_{ij} = A_{ij}B_{ij}$). Substituting these expressions in (3), we obtain

$$\begin{aligned} \widehat{Y}_{k/k-1} = &\ D_k^pH_k\sum_{i=1}^{k-1}E[z_k\nu_i^T]\Pi_i^{-1}\nu_i \\ &+E[C_k^\gamma C_{k-1}^{\gamma T}] \circ (H_kA_kB_{k-1}^TH_{k-1}^T)\Pi_{k-1}^{-1}\nu_{k-1} \\ &-D_k^pH_kE[z_kY_{k-1}^T]\Pi_{k-1}^{-1}\nu_{k-1}, \quad k \geq 2. \end{aligned}$$

Since $E[z_kY_{k-1}^T] = A_kB_{k-1}^TH_{k-1}^TD_{k-1}^p$, we have

$$D_k^pH_kE[z_kY_{k-1}^T] = (E[C_k^\gamma]E[C_{k-1}^{\gamma T}]) \circ (H_kA_kB_{k-1}^TH_{k-1}^T)$$

and from $E[C_k^\gamma C_{k-1}^{\gamma T}] - E[C_k^\gamma]E[C_{k-1}^{\gamma T}] = K_{k,k-1}^\gamma$ it is concluded that

$$\widehat{Y}_{k/k-1} = D_k^pH_k\widehat{z}_{k/k-1} + \Xi_{k,k-1}\Pi_{k-1}^{-1}\nu_{k-1}, \quad k \geq 2 \qquad (5)$$

where

$$\Xi_{k,k-1} = K_{k,k-1}^\gamma \circ (H_kA_kB_{k-1}^TH_{k-1}^T). \qquad (6)$$

From (5), the innovation is obtained as a linear combination of the new observation, the predictor of the signal and the previous innovation; namely

$$\begin{aligned} &\nu_k = Y_k - D_k^pH_k\widehat{z}_{k/k-1} - \Xi_{k,k-1}\Pi_{k-1}^{-1}\nu_{k-1}, \quad k \geq 2; \\ &\nu_1 = Y_1. \end{aligned} \qquad (7)$$

Next, a recursive expression is derived for the one-stage predictor of the signal which, from (4), is given by

$$\widehat{z}_{k/k-1} = \sum_{i=1}^{k-1} S_{k,i}\Pi_i^{-1}\nu_i. \qquad (8)$$

To calculate $S_{k,i} = E[z_k\nu_i^T]$, expression (7) for ν_i is substituted in (8), obtaining

$$S_{k,i} = E[z_kY_i^T] - E[z_k\widehat{z}_{i/i-1}^T]H_i^TD_i^p - E[z_k\nu_{i-1}^T]\Pi_{i-1}^{-1}\Xi_{i,i-1}^T$$

and then, taking into account the hypotheses on the model for $E\left[z_kY_i^T\right]$ and (8) for $E[z_k\widehat{z}_{i/i-1}^T]$, we have that

$$\begin{aligned} S_{k,i} = &\ A_kB_i^TH_i^TD_i^p - \sum_{j=1}^{i-1}S_{k,j}\Pi_j^{-1}S_{i,j}^TH_i^TD_i^p \\ &-S_{k,i-1}\Pi_{i-1}^{-1}\Xi_{i,i-1}^T, \quad 2 \leq i \leq k; \\ S_{k,1} = &\ A_kB_1^TH_1^TD_1^p. \end{aligned}$$

This expression for $S_{k,i}$ guarantees that

$$S_{k,i} = A_kJ_i, \quad i \leq k \qquad (9)$$

where J is a function satisfying

$$\begin{aligned} J_i = &\ B_i^TH_i^TD_i^p - \sum_{j=1}^{i-1}J_j\Pi_j^{-1}S_{i,j}^TH_i^TD_i^p \\ &-J_{i-1}\Pi_{i-1}^{-1}\Xi_{i,i-1}^T, \quad 2 \leq i \leq k; \\ J_1 = &\ B_1^TH_1^TD_1^p. \end{aligned} \qquad (10)$$

So, if we denote

$$O_k = \sum_{i=1}^{k} J_i\Pi_i^{-1}\nu_i, \quad k \geq 1; \quad O_0 = 0 \qquad (11)$$

it is clear, from (8) and (9), that the one-stage predictor of the signal is given by

$$\widehat{z}_{k/k-1} = A_kO_{k-1} \qquad (12)$$

where, from (11), the vector O_{k-1} is obtained from the recursive relation

$$O_k = O_{k-1} + J_k\Pi_k^{-1}\nu_k, \quad k \geq 1; \quad O_0 = 0.$$

Next, we proceed establishing an expression for J_k. By putting $i = k$ in (10) and taking into account (9), we obtain

$$\begin{aligned} J_k = &\ B_k^TH_k^TD_k^p - \sum_{i=1}^{k-1}J_i\Pi_i^{-1}J_i^TA_k^TH_k^TD_k^p \\ &-J_{k-1}\Pi_{k-1}^{-1}\Xi_{k,k-1}^T. \end{aligned}$$

Then, by denoting

$$r_k = E\left[O_kO_k^T\right] = \sum_{i=1}^{k} J_i\Pi_i^{-1}J_i^T, \quad k \geq 1; \quad r_0 = 0, \qquad (13)$$

we have

$$J_k = \left[B_k^T - r_{k-1}A_k^T\right]H_k^TD_k^p - J_{k-1}\Pi_{k-1}^{-1}\Xi_{k,k-1}^T$$

where, from (13), the matrix function r satisfy the following recursive relation

$$r_k = r_{k-1} + J_k\Pi_k^{-1}J_k^T, \quad k \geq 1; \quad r_0 = 0.$$

Finally, an expression is obtained for the covariance matrix of the innovation ν_k,

$$\Pi_k = E\left[Y_kY_k^T\right] - E\left[\widehat{Y}_{k/k-1}\widehat{Y}_{k/k-1}^T\right].$$

From expressions (5) and (12) for the predictors $\widehat{Y}_{k/k-1}$ and $\widehat{z}_{k/k-1}$, respectively, the hypotheses on the model together with (13) lead to

$$\begin{aligned}\Pi_k = & \; P_k \circ (H_k A_k B_k^T H_k^T) + R_k \\ & -D_k^p H_k A_k r_{k-1} A_k^T H_k^T D_k^p - \Xi_{k,k-1}\Pi_{k-1}^{-1}\Xi_{k,k-1}^T \\ & -D_k^p H_k A_k E[O_{k-1}\nu_{k-1}^T]\Pi_{k-1}^{-1}\Xi_{k,k-1}^T \\ & -\Xi_{k,k-1}\Pi_{k-1}^{-1}E[\nu_{k-1}O_{k-1}^T]A_k^T H_k^T D_k^p, \quad k \geq 2,\end{aligned}$$

where

$$P_k = \begin{pmatrix} p_k^1 & \cdots & p_k^1 p_k^m \\ \vdots & \ddots & \vdots \\ p_k^m p_k^1 & \cdots & p_k^m \end{pmatrix}. \tag{14}$$

The expression of Π_k is then obtained by substituting $E[O_{k-1}\nu_{k-1}^T] = J_{k-1}$, which follows from the recursive relation for O_{k-1}, taking into account that the vector O_{k-2} is orthogonal to ν_{k-1}.

All these results are summarized in the following theorem.

Theorem 1. The innovation process associated with the observations given in (2) satisfies

$$\begin{aligned}\nu_k &= Y_k - D_k^p H_k A_k O_{k-1} - \Xi_{k,k-1}\Pi_{k-1}^{-1}\nu_{k-1}, \quad k \geq 2; \\ \nu_1 &= Y_1\end{aligned} \tag{15}$$

where $\Xi_{k,k-1}$ is given in (6), and the vectors O_k are recursively calculated from

$$O_k = O_{k-1} + J_k \Pi_k^{-1}\nu_k, \quad k \geq 1; \quad O_0 = 0 \tag{16}$$

with

$$\begin{aligned}J_k &= \left[B_k^T - r_{k-1}A_k^T\right] H_k^T D_k^p - J_{k-1}\Pi_{k-1}^{-1}\Xi_{k,k-1}^T, \quad k \geq 2; \\ J_1 &= B_1^T H_1^T D_1^p\end{aligned} \tag{17}$$

and

$$\begin{aligned}\Pi_k = & \; P_k \circ (H_k A_k B_k^T H_k^T) + R_k \\ & -D_k^p H_k A_k r_{k-1} A_k^T H_k^T D_k^p - \Xi_{k,k-1}\Pi_{k-1}^{-1}\Xi_{k,k-1}^T \\ & -D_k^p H_k A_k J_{k-1}\Pi_{k-1}^{-1}\Xi_{k,k-1}^T \\ & -\Xi_{k,k-1}\Pi_{k-1}^{-1}J_{k-1}^T A_k^T H_k^T D_k^p, \quad k \geq 2; \\ \Pi_1 = & \; P_1 \circ (H_1 A_1 B_1^T H_1^T) + R_1,\end{aligned} \tag{18}$$

where P_k is given in (14) and r_k verifies

$$r_k = r_{k-1} + J_k \Pi_k^{-1} J_k^T, \quad k \geq 1; \quad r_0 = 0. \tag{19}$$

4. FIXED-POINT SMOOTHING ALGORITHM

In the following theorem we present the recursive formulas for the filter $\widehat{z}_{k/k}$ and the fixed-point smoother $\widehat{z}_{k/L}$, for $L > k$.

Theorem 2. The filtering and fixed-point smoothing estimates of the signal z_k verify

$$\begin{aligned}\widehat{z}_{k/L} &= \widehat{z}_{k/L-1} + S_{k,L}\Pi_L^{-1}\nu_L, \qquad L > k; \\ \widehat{z}_{k/k} &= A_k O_k\end{aligned} \tag{20}$$

where the innovation ν_L and the vector O_k are both given in Theorem 1.

The matrices $S_{k,L}$ are calculated from

$$\begin{aligned}S_{k,L} &= [B_k - E_{k,L-1}]\, A_L^T H_L^T D_L^p - S_{k,L-1}\Pi_{L-1}^{-1}\Xi_{L,L-1}^T, \; L > k; \\ S_{k,k} &= A_k J_k\end{aligned} \tag{21}$$

where $E_{k,L}$ satisfy

$$\begin{aligned}E_{k,L} &= E_{k,L-1} + S_{k,L}\Pi_L^{-1}J_L^T, \quad L > k; \\ E_{k,k} &= A_k r_k.\end{aligned} \tag{22}$$

Proof. The recursive relation for the fixed-point smoother, $\widehat{z}_{k/L}$, $L > k$, is immediate from the general expression of the linear estimators (4). This expression for $L = k$, together with (9) and (11), provides also the formula for the filter, which constitutes the initial condition for the smoothing algorithm. Then, we only need to prove (21) for $S_{k,L} = E[z_k \nu_L^T]$ and (22) for $E_{k,L}$.

Using (15) for ν_L, and since $E[z_k Y_L^T] = B_k A_L^T H_L^T D_L^p$, we obtain

$$\begin{aligned}S_{k,L} = & \left[B_k - E[z_k O_{L-1}^T]\right] A_L^T H_L^T D_L^p \\ & -E[z_k \nu_{L-1}^T]\Pi_{L-1}^{-1}\Xi_{L,L-1}^T.\end{aligned}$$

This expression leads to (21), just by denoting $E_{k,L} = E[z_k O_L^T]$. From (9), the initial condition in (21) is immediate.

Finally, the recursive relation (22) is obtained from (16). Its initial condition is derived taking into account that, from the orthogonality, $E[z_k O_k^T] = E[\widehat{z}_{k/k} O_k^T]$, and using the expression of the filter and (13). □

The performance of the filtering and fixed-point smoothing estimates can be measured by the estimation errors $z_k - \widehat{z}_{k/L}$, $L \geq k$ and, more specifically, by the covariance matrices of these errors,

$$\Sigma_{k/L} = E\left[(z_k - \widehat{z}_{k/L})(z_k - \widehat{z}_{k/L})^T\right].$$

Next, a recursive formula to obtain $\Sigma_{k/L}$ is derived from the filtering and fixed-point smoothing algorithm proposed in Theorem 2.

Since the error $z_k - \widehat{z}_{k/L}$ is orthogonal to the estimator $\widehat{z}_{k/L}$, it is easy to verify that

$$\Sigma_{k/L} = E[z_k z_k^T] - E\left[\widehat{z}_{k/L}\widehat{z}_{k/L}^T\right]$$

and, taking into account that ν_L and $\widehat{z}_{k,L-1}$ are orthogonal, equation (20) for $\widehat{z}_{k/L}$ leads to

$$\Sigma_{k/L} = \Sigma_{k/L-1} - S_{k,L}\Pi_L^{-1}S_{k,L}^T, \; L > k. \tag{23}$$

The initial condition for this equation is immediately derived from the filter expression,

$$\Sigma_{k/k} = A_k\left[B_k^T - r_k A_k^T\right]. \tag{24}$$

5. EXAMPLE: SCALAR SIGNAL ESTIMATION

This section shows a numerical simulation example to illustrate the application of the recursive algorithm proposed in the current paper. To show the effectiveness of the proposed estimators, a program in MATLAB has been run, simulating at each iteration the signal and the observed values and providing the filtering and fixed-point smoothing estimates, as well as the corresponding error covariance matrices.

Consider a zero-mean scalar signal $\{z_k;\; k \geq 1\}$ with autocovariance function given by

$$E[z_k z_s^T] = 1.025641 \times 0.95^{k-s}, \quad s \leq k,$$

which is factorizable according to model hypothesis taking

$$A_k = 1.025641 \times 0.95^k, \qquad B_k = 0.95^{-k}.$$

For the simulation, the signal is supposed to be generated by the following first-order autoregressive model

$$z_{k+1} = 0.95 z_k + w_k$$

where $\{w_k;\ k \geq 1\}$ is a zero-mean white Gaussian noise with $Var\,[w_k] = 0.1$, for all k.

Consider two sensors whose uncertain measurements,

$$\widetilde{y}_k^i = \gamma_k^i z_k + v_k^i, \quad i = 1,\ 2$$

are perturbed by independent additive zero-mean white Gaussian noises, $\left\{v_k^i;\ k \geq 1\right\}$, with constant variances for all k, $Var\left[v_k^1\right] = 0.5$ and $Var\left[v_k^2\right] = 0.9$.

Now, in accordance with our theoretical study, it is assumed that, at any time instant $k \geq 1$, the uncertainty of each sensor at time k is correlated only with the uncertainty at time $k-1$. To model this uncertainty we can consider, for example, two independent sequences $\{\theta_k^i;\ k \geq 1\}$, $i = 1,\ 2$, of independent Bernoulli random variables with constant probabilities, $P\left[\theta_k^i = 1\right] = \theta_i$, $\forall k \geq 1$, and define

$$\gamma_k^i = \theta_{k+1}^i (1 - \theta_k^i), \quad k \geq 1.$$

So, the variables γ_k^i are also Bernoulli random variables and, since θ_k^i and θ_s^i are independent, γ_k^i and γ_s^i are also independent for $|k - s| \geq 2$. The common mean of these variables is $p^i = \theta_i(1 - \theta_i)$ and its covariance function is given by

$$E[(\gamma_k^i - p^i)(\gamma_s^i - p^i)] = \begin{cases} 0, & |k - s| \geq 2 \\ -(p^i)^2, & |k - s| = 1. \end{cases}$$

Since the mean and covariance functions of the variables γ_k^i are the same if the value $1 - \theta_i$ is considered instead of θ_i, only the case $\theta_i \leq 0.5$ is examined here. Next, we show and compare the results obtained, using different values of the parameters θ_i.

First, to compare the effectiveness of the proposed filtering and fixed-point smoothing estimators, one hundred iterations of the respective algorithms have been performed considering different values of θ_1 and θ_2, which lead to different values of the false alarm probabilities $1 - p^i = 1 - \theta_i(1 - \theta_i)$, $i = 1, 2$; on the one hand, we consider $\theta_1 = 0.1$, $\theta_2 = 0.3$ and, on the other, $\theta_1 = 0.3$, $\theta_2 = 0.5$. Also, for these values, the error variances of the filtering and fixed-point smoothing estimators have been calculated.

Fig. 1 displays the filtering error variances, $\Sigma_{k/k}$, and the fixed-point smoothing error variances, $\Sigma_{k/k+2}$ and $\Sigma_{k/k+10}$. This figure shows, on the one hand, that the error variances corresponding to the fixed-point smoothers are less than the filtering ones and, on the other, that as the values of θ_1 and θ_2 increase (and, hence the false alarm probabilities decrease in both sensors) the error variances are smaller and consequently, the performance of the estimators is better. From this figure, it is also deduced that the accuracy of the smoother at each fixed-point k is better as the number of available observations increases.

Next, we study the filtering error variances, $\Sigma_{k/k}$, and the fixed-point smoothing error variances $\Sigma_{k/k+10}$ for a constant value $\theta_1 = 0.5$ and θ_2 varying from 0.1 to 0.5; the results are given in Fig. 2. From this figure it is gathered that, when the false alarm probability of one of the sensors is fixed (θ_1 fixed) and the false alarm probability of the other increases (θ_2 decreases), the error variances

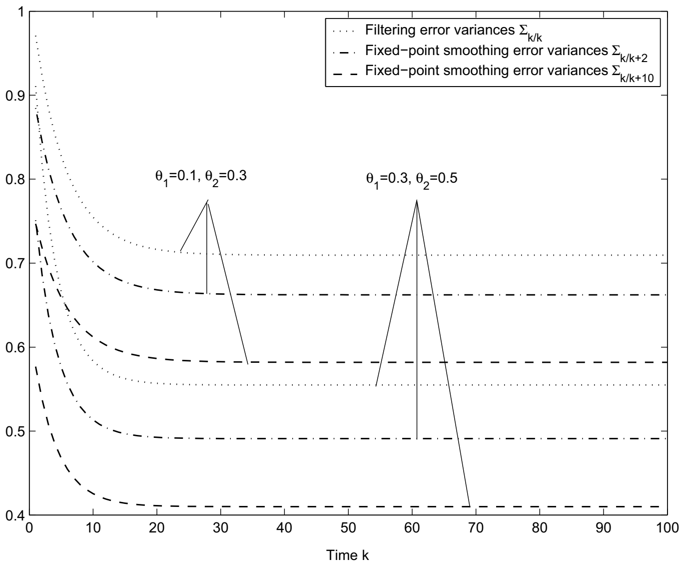

Fig. 1. Filtering and fixed-point smoothing error variances for $\theta_1 = 0.1,\ \theta_2 = 0.3$ and $\theta_1 = 0.3,\ \theta_2 = 0.5$

become greater and, consequently, the performance of the estimators is worse.

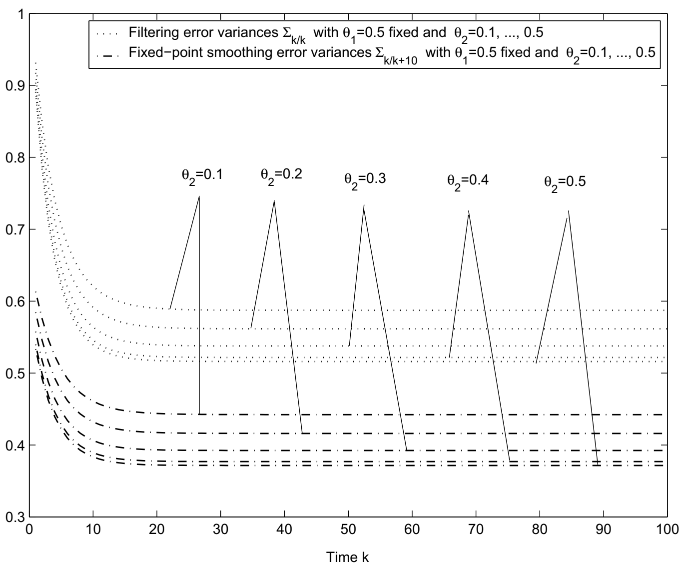

Fig. 2. Filtering and fixed-point smoothing error variances for constant $\theta_1 = 0.5$ and θ_2 varying from 0.1 to 0.5

Finally, we study the filtering error variances, $\Sigma_{k/k}$, when both θ_1 and θ_2 are varied from 0.1 to 0.5. It must be noted that such error variances stabilize around a constant value; for this reason, Fig. 3 displays the filtering error variances $\Sigma_{100/100}$ versus θ_1 (for constant values of θ_2) and Fig. 4 shows these variances versus θ_2 (for constant values of θ_1). From these figures it is gathered again that, as the false alarm probability of both sensors increases, the filtering error variances become greater and, consequently, the performance of the estimators is worse.

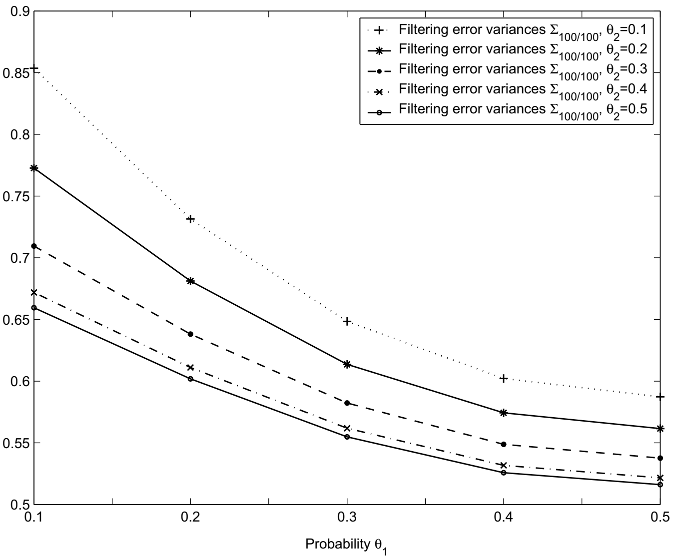

Fig. 3. Filtering error variances $\Sigma_{100/100}$ versus θ_1 with θ_2 varying from 0.1 to 0.5

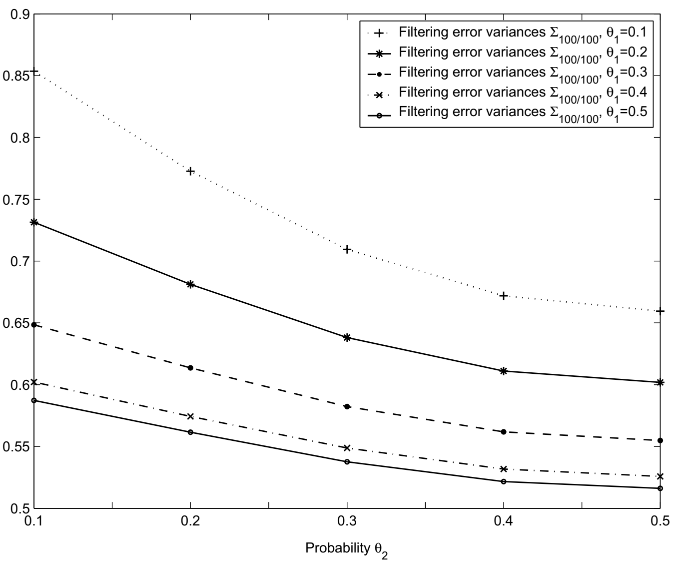

Fig. 4. Filtering error variances $\Sigma_{100/100}$ versus θ_2 with θ_1 varying from 0.1 to 0.5

6. CONCLUSION

In this paper, a least-squares linear filtering and fixed-point smoothing algorithm is proposed to estimate signals from correlated uncertain observations coming from multiple sensors with different uncertainty characteristics. This is a realistic assumption in situations concerning sensor data that are transmitted over communication networks where, generally, multiple sensors with different properties are involved.

The uncertainty in each sensor is modeled by a sequence of Bernoulli random variables which are correlated at consecutive time instants. A real application of such observation model arises for example in signal transmission problems where the sensors may fail and, consequently, there is no signal to transmit; however, at the transmitting end, this failure is immediately detected and the old sensor is replaced, so the signal cannot be missing in two successive observations.

Using an innovation approach, the estimation algorithms are derived without requiring the knowledge of the signal state-space model, but only the covariance functions of the processes involved in the observation equation, as well as the probability and correlation of the Bernoulli variables modeling the uncertainty in each sensor. To measure the performance of the estimators, the filtering and smoothing error covariance matrices are also provided.

To illustrate the theoretical results established in this paper, a simulation example is presented, in which the proposed algorithm is applied to estimate a signal from correlated uncertain observations coming from two sensors with different uncertainty characteristics.

A natural extension of the observation model studied in this paper is to consider that the uncertainty in the observation at time k depends on the uncertainty at times $k-1, \ldots, k-r$, where r is an arbitrary positive number, but it is independent of the uncertainties at times previous to $k-r$; this is modeled by assuming that the Bernoulli random variables γ_k^i and γ_s^i are correlated when $|k-s| \leq r$ and independent when $|k-s| > r$.

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