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On Transient Stability of Multi–Machine Power Systems: A "Globally" Convergent Controller for Structure–Preserving Models [⋆]

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Abstract: The design of excitation controllers to improve transient stabilization of power systems is a topic of renewed interest in the control community. *Existence* of a state–feedback stabilizing law for multi–machine aggregated reduced network models has recently been established. In this paper we extend this result in two directions: first, in contrast with aggregated models, we consider the more natural and widely popular structure–preserving models (SPM) that preserve the identity of the network components and allow for a more realistic treatment of the loads. Second, we *explicitly compute* a control law that, under a detectability assumption, ensures that *all trajectories* converge to the desired equilibrium point, provided that they start and remain in the region where the model makes physical sense.

1. INTRODUCTION

Classical research on transient stabilization of power systems has relied on the use of aggregated *reduced network* models that represent the system as an n–port described by a set of ordinary differential equations. Several excitation controllers that establish Lyapunov stability of the desired equilibrium (with a Lyapunov function and a well–defined domain of attraction) of these models have been reported. The nonlinear controller design techniques that have been considered include feedback linearization (Wang (1993)), damping injection (Wang (2003)), as well as, the more general, interconnection and damping assignment passivity–based control, see Sun (2000) and Ortega (2005).

In this paper, we abandon the aggregated n–port view of the network and consider the more natural *structure–preserving models*, first proposed in Bergen et al. (1981). Since these models consist of differential algebraic equations (DAE) they require the development of some suitably tailored tools for controller synthesis and stability analysis. Another original feature of the present work is that we do not aim at Lyapunov stability, but establish instead a *"global" convergence* result. [1]

In Giusto (2006) SPM were used to identify—in terms of feasibility of a linear matrix inequality—a class of power systems with nonlinear (so-called ZIP) loads and leaky lines for which a *linear time–invariant* controller renders the overall *linearized* system dissipative with a (locally) positive definite storage function, thus ensuring stability of the desired equilibrium for the nonlinear system. Unfortunately, a full–fledged *nonlinear* analysis of the problem was not possible due to the difficulty in handling the complicated interdependence of the variables appearing in the algebraic constraints of the DAEs. The Lyapunov function in that paper is obtained by adding a quadratic term in the rotor angle to the classical energy function of Varaiya (1985). This quadratic term is needed to compensate for a linear term (in rotor angle) appearing in the energy function of Varaiya (1985) and render the new storage function positive definite. To obtain our "global" convergence result we observe that removing the linear term from the energy function of Varaiya (1985) and increasing the quadratic term in bus voltages yields a function whose time derivative can be *arbitrarily assigned* with a "globally" defined static state feedback. Furthermore, although this new function is not positive definite, it is *bounded from below* and has some suitable radial unboundedness properties—features that are essential to establish boundedness of trajectories. We then select a control law that renders "globally" attractive the level set of this function that contains the desired equilibrium point. If, furthermore, the function defines a *detectable output*, then all trajectories will asymptotically converge to the equilibrium. The only critical assumption required to establish this result is that the loads are constant impedances—a condition that is implicitly assumed in all controllers derived for aggregated models.

The structure of the paper is as follows. Section 2 presents the mathematical model of the various elements comprising the power system. Then, we formulate the control problem in Section 3. Section 4 contains our main "global"

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[1] The precise meaning of the qualifier "global" will be given in the sequel. It essentially boils down to restricting to the trajectories that remain in the region where the model makes physical sense.

convergence result that relies on the aforementioned detectability assumption. In Section 5 we prove that the system is indeed detectable in the single–machine case and simulations are given in Section 6. We wrap up the paper with concluding remarks on future research in Section 7.

Caveat Due to space limitations, we have omitted the proofs that can be found in Dib et al. (2007).

Notation All vectors in the paper are *column* vectors, even the gradient of a scalar function: $\nabla_x = \frac{\partial}{\partial x}$. For any function $f : \mathbb{R}^n \to \mathbb{R}$, we define $\nabla_{z_j} f(z) := \frac{\partial f}{\partial z_j}(z)$, and for vector functions $g : \mathbb{R}^n \to \mathbb{R}^n$, we define the Jacobian $\nabla g(z) := [\nabla g_1(z), \ldots, \nabla g_n(z)]^\top \in \mathbb{R}^{n\times n}$. To simplify notation we introduce the sets

$$\mathbb{M}^n := \mathbb{S}^n \times \mathbb{R}^n \times \mathbb{R}^n_{>0} \times \mathbb{S}^n \times \mathbb{R}^n_{>0},\ n \in \bar{n} := \{1, .., n\},$$

where $\mathbb{R}^n_{>0} := \{x \in \mathbb{R}^n \mid x_i > 0\}$.

2. STRUCTURE–PRESERVING MODELLING

In this section we recall the well–known *SPM* for n–machines power systems comprised by synchronous machines and loads interconnected by transmission lines.

To simplify the presentation of our results we will assume a simplified network topology where attached to each bus there is a machine *and* a load.[2] Each bus, and their corresponding machine and load, have an associated identifier $j \in \bar{n}$. Buses are interconnected through transmission lines that are identified by the double subindex $jk \in \Omega \subset \bar{n} \times \bar{n}$, indicating that the line jk connects the bus $j \in \bar{n}$ with the bus $k \in \bar{n}$; the set avoids obvious repetitions, e.g., if $jk \in \Omega$ then $kj \notin \Omega$. We also define the set $\Omega_j := \{k \in \bar{n} \mid \exists\ jk \in \Omega\}$, that is, the set of buses that are linked to the bus j through some transmission line. All elements share as port variables the angle θ_j and the magnitude V_j of the bus voltage phasor $y_j = \mathrm{col}(\theta_j, V_j) \in \mathbb{S} \times \mathbb{R}_{>0}$. Associated to each bus are the active and reactive powers entering the machine, the load or the transmission lines, that will be denoted

$$\begin{bmatrix} P_j^M \\ Q_j^M \end{bmatrix}, \begin{bmatrix} P_l^L \\ Q_l^L \end{bmatrix}, \begin{bmatrix} P_{jk} \\ Q_{jk} \end{bmatrix} \in \mathbb{R}^2, \qquad (1)$$

respectively. Following standard convention, we take active and reactive powers as positive when entering their corresponding component.

2.1 Synchronous machines model

Each synchronous machine is described by a set of third order DAE's (Varaiya (1985))

$$\begin{aligned} \dot{\delta}_j &= \omega_j \\ M_j \dot{\omega}_j &= P_{m_j} - D_j \omega_j + P_j^M \\ \tau_j \dot{E}_j &= -\frac{x_{d_j}}{x'_{d_j}} E_j + \frac{x_{d_j} - x'_{d_j}}{x'_{d_j}} V_j \cos(\delta_j - \theta_j) + E_{F_j} \end{aligned} \qquad (2)$$

$$\begin{aligned} P_j^M &= -\frac{E_j V_j}{x'_{d_j}} \sin(\delta_j - \theta_j) - Y_{2j} V_j^2 \sin(2(\delta_j - \theta_j)) \\ Q_j^M &= (Y_{V_j} - Y_{2j} \cos(2(\delta_j - \theta_j)))V_j^2 - \frac{E_j V_j}{x'_{d_j}} \cos(\delta_j - \theta_j) \end{aligned} \qquad (3)$$

where, to simplify notation, we defined the constants

$$Y_{2j} := \frac{x'_{d_j} - x_{q_j}}{2 x_{q_j} x'_{d_j}},\ Y_{V_j} := \frac{x'_{d_j} + x_{q_j}}{2 x_{q_j} x'_{d_j}}.$$

The *state variables* $x_j := \mathrm{col}(\delta_j,\ \omega_j,\ E_j) \in \mathbb{S} \times \mathbb{R} \times \mathbb{R}_{>0}$ denote the rotor angle, the rotor speed and the quadrature axis internal e.m.f., respectively, and E_{F_j} is the field voltage. The latter is split in two terms, $E^\star_{F_j} + v_j$, the first is constant and fixes the equilibrium value, while the second one is the *control action*. The parameters are denoted as in Varaiya (1985), and are fairly standard. We will make the physically reasonable assumptions $D_j > 0$, $x_{d_j} - x'_{d_j} > 0$.

2.2 Loads model

Loads are described by the standard ZIP model,

$$\begin{aligned} P_l^L &= P_{Z_l} V_l^2 + P_{I_l} V_l + P_{0_l} \\ Q_l^L &= Q_{Z_l} V_l^2 + Q_{I_l} V_l + Q_{0_l}, \end{aligned} \qquad (4)$$

which explicitly represent the contribution of each type of load (constant impedance, current or power). As will become clear below, to state our main result we must consider a simplified model for the loads. Namely, we will assume only constant impedance loads:

$$\begin{aligned} P_l^L &= P_{Z_l} V_l^2 \\ Q_l^L &= Q_{Z_l} V_l^2 \end{aligned} \qquad (5)$$

This simplification, which is necessary to obtain the lumped parameter model used in most transient stability controller design studies, allows us to transform the algebraic constraints into a set of *linear* equations for which we can give conditions for solvability.

2.3 Transmission lines model

The transmission lines are modeled with the standard lumped Π circuit, see Anderson and Fouad (1977),

$$\begin{aligned} P_{jk} &= G_{jk} V_j^2 + B_{jk} V_j V_k \sin(\theta_j - \theta_k) - \\ &\quad G_{jk} V_j V_k \cos(\theta_j - \theta_k) \\ Q_{jk} &= (B_{jk} - B^c_{jk}) V_j^2 - B_{jk} V_j V_k \cos(\theta_j - \theta_k) - \\ &\quad G_{jk} V_j V_k \sin(\theta_j - \theta_k) \end{aligned} \qquad (6)$$

where $jk \in \Omega$. The active and reactive power entering at node k, P_{kj} and Q_{kj} can be obtained by a simple change of indexes.

Remark 1. In comparison with previous works on transient stabilization, for generality we consider lines with *capacitive* effects, a parameter that is usually small, hence reasonable to neglect.

2.4 Bus equations

From Kirchhoff's laws, at each bus we have

$$\begin{aligned} 0 &= \sum\nolimits_{k\in\Omega_j} P_{jk} + P_j^M + P_j^L \\ 0 &= \sum\nolimits_{k\in\Omega_j} Q_{jk} + Q_j^M + Q_j^L \end{aligned} \qquad (7)$$

where we recall that Ω_j is the set of buses that are linked to the bus j through some transmission line.

Remark 2. We bring to the readers attention the fact that V_j, being a magnitude of a phasor, is non–negative. Similarly, due to physical considerations, $E_j > 0$. These fundamental physical constraints of the model will be assumed for our derivations.

[2] As will become clear below the derivations are also applicable for other network topologies—at the expense of a more cluttered notation.

3. CONTROL PROBLEM AND A KEY LEMMA

To obtain the overall model we group all the algebraic constraints and write the system equations in the compact form

$$\begin{cases} \dot{x} = f(x,y) + L_v v \\ 0 = g(x,y), \end{cases} \tag{8}$$

where $(x := \mathrm{col}(x_j), y := \mathrm{col}(y_j)) \in \mathbb{M}^n$, $v := \mathrm{col}(v_j) \in \mathbb{R}^n$, $L_v := \mathrm{diag}\{\mathrm{col}(0,0,\frac{1}{\tau_j})\} \in \mathbb{R}^{3n\times n}$, and the functions $f : \mathbb{M}^n \to \mathbb{R}^{3n}$, and $g : \mathbb{M}^n \to \mathbb{R}^{2n}$ are defined by (2), and the replacement of (3), (5) and (6) into (7), respectively.

3.1 Problem formulation

Assumption A1. There exists an isolated asymptotically stable *open loop equilibrium* $(x^\star, y^\star)$ of the system (8).

Assumption A2. The matrix $\nabla_y g(x,y)$ is invertible for all $(x,y) \in \mathbb{M}^n$.

Asymptotic Convergence Problem. Consider the system (8) satisfying Assumptions A1 and A2. Find a control law $v = \hat{v}(x,y)$ such that:

$$(x(t),y(t)) \in \mathbb{M}^n,\ \forall t \geq 0 \ \Rightarrow\ \lim_{t\to\infty}(x(t),y(t)) = (x^\star, y^\star).$$

Consequently, $(x^\star, y^\star)$ is an attractive equilibrium of the closed–loop provided trajectories start, and remain, in $\mathbb{M}^n$—the set where the model is physically valid.

Remark 3. Assumption A1 is standard in transient stability studies where v is included to enlarge the domain of attraction of the operating point. Assumption A2 is needed to compute the control law. In all practical situations $\nabla_y g(x^*, y^*)$ is non–singular ensuring, via the Implicit Function Theorem (Krantz (2002)), that $\nabla_y g(x,y)$ is (locally) invertible. We have assumed that this is true throughout $\mathbb{M}^n$ to avoid cluttering notation in the main result.

Remark 4. Notice that we do not aim at proving that trajectories starting in $\mathbb{M}^n$ actually remain there, but we only assume it. In spite of that, and with obvious abuse of notation, we will say that a controller satisfying the implication above ensures a "practical" convergence that we have noted as a "global" convergence to prevent a cluttered notation.

Remark 5. Notice that the entries in the vector $x^\star$ that correspond to ω_j are equal to zero.

3.2 Proposed solution strategy

The solution to the problem stated in section 3.1 that we propose in the paper proceeds along the following steps:

(1) Give an explicit solution of the power balance equations $g(x,y) = 0$.
(2) Representation of the system dynamics as a *perturbed* port–Hamiltonian system using a Hamiltonian function with desired characteristics.
(3) Construction of a control signal that, assigning the derivative of the Hamiltonian function, ensures that trajectories will converge to the level set of the Hamiltonian that contains the equilibrium point. Trajectories will then converge to the equilibrium if the Hamiltonian function defines a detectable output.
(4) Prove that the resulting controller is well defined and convergence is guaranteed—provided the trajectories remain in $\mathbb{M}^n$.

The second and the third steps can be carried out for the model with the general ZIP loads (4). Invoking the existence of an isolated local minimum of Assumption A1, using some continuity arguments and assuming detectability we can, therefore, conclude that the proposed controller renders the equilibrium locally attractive. This kind of local results are easily obtained using linearization, and known in the power systems community as small–signal stability. In this paper we are interested in the large–signal stability problem, therefore, the last step is indispensable. To complete it, the first step is essential—unfortunately, this imposes the restrictive requirement of constant impedance loads (5).

4. MAIN RESULT

This section contains our main "global" convergence result, which is derived proceeding along the steps delineated in Subsection 3.2.

4.1 Solution of $g(x,y) = 0$

In this subsection we present an explicit solution to the algebraic constraints $g(x,y) = 0$, a result which is of interest on its own. To simplify the presentation we define, for $j \in \bar{n}$, the complex variables

$$\mathbf{V}_j := V_j e^{i\theta_j} \in \mathbb{C}, \qquad \mathbf{V} := \mathrm{col}(\mathbf{V}_j)_{j\in\bar{n}} \in \mathbb{C}^n, \tag{9}$$

and

$$E := \mathrm{col}(E_j)_{j\in\bar{n}} \in \mathbb{R}^n, \quad \delta := \mathrm{diag}\{\delta_j\}_{j\in\bar{n}} \in \mathbb{R}^{n\times n}.$$

Lemma 1. Consider the algebraic equations $g(x,y) = 0$ of the power systems model (8) defined by (3), (5), (6) and (7). If

$$\frac{1}{x'_{d_j}} + Q_{Z_j} > \sum_{k\in\Omega_j} B^c_{jk}, \quad j \in \bar{n}, \tag{10}$$

$g(x,y) = 0$ has a "globally" defined solution. That is, there exists a function $\hat{y} : \mathbb{S}^n \times \mathbb{R}^n_{>0} \to \mathbb{S}^n \times \mathbb{R}^n_{>0}$ such that $g(x, \hat{y}(x)) = 0$. Furthermore, this function can be written in the form

$$\mathbf{V} = W(\delta)E, \tag{11}$$

where $W : \mathbb{R}^{n\times n} \to \mathbb{C}^{n\times n}$ is *bounded and invertible*, with elements are rational functions of $\cos(\delta_j)$ and $\sin(\delta_j)$.

Remark 6. As indicated in Remark 1, (10) is always verified in (standard) transient stability studies, where it is assumed that $B^c_{jk} = 0$. Also, it is clear that the construction of $\hat{y}$ directly follows from (9) and (11).

4.2 Perturbed port–Hamiltonian representation

The j-th synchronous machine model dynamics (2) can be written as a perturbed port–Hamiltonian system

$$\dot{x}_j = (J_j - R_j)\nabla_{x_j} H_j(x_j, y_j) + L_{v_j} v_j + \xi_j \tag{12}$$

with the Hamiltonian functions $H_j : \mathbb{M}^1 \to \mathbb{R}$,

$$H_j := \frac{1}{2}M_j\omega_j^2 + \frac{1}{2}Y_{E_j}E_j^2 + \frac{1}{2}[\Delta_j + Y_{V_j}]V_j^2 - Y_{F_j}E_{F_j}^\star E_j - \frac{Y_{2j}}{2}\cos 2(\theta_j - \delta_j)V_j^2 - \frac{E_jV_j}{x'_{d_j}}\cos(\theta_j - \delta_j) \quad (13)$$

and we defined the matrices

$$J_j := \begin{bmatrix} 0 & \frac{1}{M_j} & 0 \\ -\frac{1}{M_j} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -J_j^\top,\ R_j := \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{D_j}{M_j^2} & 0 \\ 0 & 0 & \frac{1}{\tau_j Y_{F_j}} \end{bmatrix} \geq 0,$$

$L_{v_j} := [0, 0, \frac{1}{\tau_j}]^\top$, $\xi_j := [0, \frac{P_{mj}}{M_j}, 0]^\top$, and the constants

$$Y_{E_j} := \frac{x_{d_j}}{x'_{d_j}(x_{d_j} - x'_{d_j})}, \qquad Y_{F_j} := \frac{1}{x_{d_j} - x'_{d_j}},$$

where $\Delta_j \geq 0$ is a key *design parameter.*

One important property of H_j is that it is *quadratic* in $Z_j := \mathrm{col}(\omega_j, E_j, V_j)$ and, furthermore, *bounded from below.* (Consequently, if H_j is non–increasing, we can conclude that all signals are bounded—because Z_j will be bounded and θ_j and δ_j live in compact sets.)

Remark 7. Due to the presence of the term ξ_j in (12) it is clear that the set of open–loop equilibria and the set of minima of H_j *are disjoint.* Therefore, the new Hamiltonian cannot qualify as a Lyapunov function candidate (for the desired equilibrium).

4.3 Evaluating the time derivative of the Hamiltonian

Besides being lower bounded and quadratic (in Z_j) we will prove in the paper another fundamental property of the function H_j, namely, that the derivative of the function $H(x, y) := \sum_{j\in\bar{n}} H_j(x_j, y_j)$ can be *arbitrarily assigned* with a suitable selection of the control v.

The proof of this fact (Dib et al. (2007)) invokes Lemma 1 and requires the differentiation of a complex valued function. Let us first compute $\dot{H}$ in this subsection using standard—real domain—derivations. Towards this end, let us define

$$H(x, y) := \sum_{j\in\bar{n}} H_j(x_j, y_j) \quad (14)$$

and compute

$$\dot{H} = -\nabla_x^\top HR\nabla_x H + \tilde{\xi}(x, y) + L^\top(x, y)\tau^{-1}v + \nabla_y^\top H\dot{y}$$

where

$$R := \mathrm{diag}\{R_j\}_{j\in\bar{n}} \in \mathbb{R}^{3n\times 3n}$$
$$\tilde{\xi}(x, y) := \sum_{j\in\bar{n}} \nabla_{x_j}^\top H_j\xi_j = \sum_{j\in\bar{n}} \omega_j P_{mj} \in \mathbb{R},$$
$$L(x, y) := \mathrm{col}(\nabla_{x_j}^\top H_j L_{v_j})_{j\in\bar{n}} = \nabla_E H \in \mathbb{R}^n,$$
$$\tau := \mathrm{diag}\{\tau_j\}_{j\in\bar{n}} \in \mathbb{R}^{n\times n}.$$

To evaluate $\dot{y}$ we differentiate the algebraic constraints $g(x, y) = 0$ yielding

$$\nabla_x^\top g\dot{x} + \nabla_y^\top g\dot{y} = 0.$$

Invoking Assumption A2 we obtain

$$\dot{y} = M(x, y)\dot{x}, \quad (15)$$

where

$$M := -\nabla_y^{-\top} g\nabla_x^\top g \in \mathbb{R}^{2n\times 3n}.$$

Replacing (12) in (15) we have that $\dot{y} = F(x, y) + G(x, y)v$, where $F(x, y) \in \mathbb{R}^{2n}$ and $G(x, y) \in \mathbb{R}^{2n\times n}$. Therefore,

$$\dot{H} = -\nabla_x^\top HR\nabla_x H + \tilde{\xi}_0(x, y) + \tilde{L}^\top(x, y)\tau^{-1}v \quad (16)$$

where

$$\tilde{\xi}_0(x, y) := \tilde{\xi} + \nabla_y^\top HF \in \mathbb{R},$$
$$\tilde{L}(x, y) := L + \tau G^\top \nabla_y H \in \mathbb{R}^n. \quad (17)$$

Let us take a brief respite to analyze (16). It is clear that, wherever the vector $\tilde{L}(x, y)$ is bounded away from zero, we can easily select a control law v that assigns an arbitrary function to $\dot{H}$. In the next subsection, we will state the proposition that allows us to assign arbitrarily $\dot{H}(x, \hat{y}(x))$.

4.4 "Global" assignment of $\dot{H}(x, \hat{y}(x))$

Proposition 1. Consider the power systems model (8) with Assumptions A1 and A2 and the Hamiltonian function (13). There exists $\Delta_j^{min} > 0$ such that, for all $\Delta_j \geq \Delta_j^{min}$ we have

$$\tilde{L}^\top(x, \hat{y}(x))E > 0 \qquad \text{for all} \quad x \in \mathbb{S}^n \times \mathbb{R}^n \times \mathbb{R}^n_{>0},$$

where $\tilde{L}(x, y)$ is given in (17) and Δ_j defined in (13). Therefore, for any function $\alpha : \mathbb{M}^n \to \mathbb{R}$, the "globally" defined control law

$$v = \frac{1}{\tilde{L}^\top E}[\alpha(x, y) + \nabla_x^\top HR\nabla_x H - \tilde{\xi}_0]\tau E \quad (18)$$

ensures $\dot{H} = \alpha$.

The proof can be found in (Dib et al. (2007)).

4.5 A "globally" convergent controller

In this subsection we propose to select the function α such that, under a detectability assumption, trajectories converge to $(x^\star, y^\star)$.

Proposition 2. Consider the power systems model (8) with Assumptions A1 and A2 in closed–loop with the control (18) with

$$\alpha(x, y) = -\lambda[H(x, y) - H^\star], \quad (19)$$

where $H^\star := H(x^\star, y^\star)$, $\lambda > 0$, $\Delta_j \geq \Delta_j^{min}$, and Δ_j^{min} is as in Proposition 1.

(i) Assume $(x(t), y(t)) \in \mathbb{M}^n$, $\forall t \geq 0$. Then, trajectories are *bounded.*
(ii) If, additionally,

Assumption A3. The function $H(x, y) - H^\star$ defines a *detectable output* for the closed–loop system.

Then, $\lim_{t\to\infty}(x(t), y(t)) = (x^\star, y^\star)$.

Proof First, note that

$$\frac{d}{dt}[H(x, y) - H^\star] = -\lambda[H(x, y) - H^\star].$$

Hence H is bounded, ensuring boundedness of trajectories. Furthermore, we have that $H(x(t), y(t)) \to H^\star$. The proof is completed invoking LaSalle's Invariance Principle and the definition of detectability.

Remark 8. The controller of Proposition 1 drives the trajectories towards the level set $\{(x,y) \in \mathbb{M}^n \mid H(x,y) = H^\star\}$. The analysis of the dynamics restricted to this set is rather involved and is currently under investigation—hence the need of the detectability assumption. However, we prove in the next subsection that the assumption is verified for the classical single machine infinite bus (SMIB) system.

Remark 9. We recall that the minima of H are not equilibria of the system—hence, it is not a Lyapunov function candidate and the property $\dot{H} \leq 0$ is not sufficient to guarantee some stability/convergence properties.

5. SINGLE MACHINE SYSTEM

For the elementary case of a SMIB system neglecting the generator saliency, i.e., $n = 1$ and $Y_2 = 0$, the model (2) reduces to

$$\begin{aligned} \dot{\delta} &= \omega \\ M\dot{\omega} &= P_m - D\omega + P^M \\ \tau\dot{E} &= -\frac{x_d}{x_d'}E + \frac{x_d - x_d'}{x_d'}V\cos(\delta-\theta) + E_F^\star + v, \\ P^M &= -\frac{1}{x_d'}EV\sin(\delta-\theta) \\ Q^M &= \frac{x_d' + x_q}{2x_q x_d'}V^2 - \frac{1}{x_d'}EV\cos(\delta-\theta). \end{aligned} \tag{20}$$

The algebraic constraints imposed by the bus equations (7), assuming for simplicity $G = B^c = 0$, are

$$-\frac{EV}{x_d'}\sin(\delta-\theta) + BV\sin(\theta) + P_Z V^2 = 0 \tag{21}$$

$$(Y_V + B + Q_Z)V^2 - \frac{EV}{x_d'}\cos(\delta-\theta) - BV\cos(\theta) = 0,$$

where, following standard convention, the magnitude and the angle of the voltage phasor at the infinite bus are taken equal to 1 and 0, respectively.

To set up the notation used in the sequel, and for the sake of completeness, we give now a simplified version of Lemma 1.

Lemma 2. Assume the voltage $V(t) > 0$ for all $t \geq 0$. The algebraic constraints (21) are equivalent to

$$\begin{aligned} V\cos(\delta-\theta) = &\frac{\mathrm{Im}\{A_0\}}{|A_0|^2}B\sin(\delta) \\ &+\frac{\mathrm{Re}\{A_0\}}{|A_0|^2}\left(\frac{E}{x_d'} + B\cos(\delta)\right) \end{aligned} \tag{22}$$

$$\begin{aligned} V\sin(\delta-\theta) = &-\frac{\mathrm{Im}\{A_0\}}{|A_0|^2}\left(\frac{E}{x_d'} + B\cos(\delta)\right) \\ &+\frac{\mathrm{Re}\{A_0\}}{|A_0|^2}B\sin(\delta) =: \Psi(\delta, E), \end{aligned} \tag{23}$$

where

$$A_0 := Y_V + Q_Z + B - iP_Z \in \mathbb{C}.$$

We will now check the detectability condition (Assumption A3) for the SMIB model (20), (21) in closed–loop with the control (18), (19). Towards this end, we introduce the coordinate transformation $(\delta, \omega, E) \rightsquigarrow \eta$, where

$$\eta_1 = \delta,\ \eta_2 = \omega,\ \eta_3 = \widetilde{H}, \tag{24}$$

we defined $\widetilde{H} := H - H^\star$ and

$$\begin{aligned} H := &\frac{M}{2}\omega^2 + \frac{Y_E}{2}E^2 + \frac{1}{2}[\Delta + Y_V]V^2 - Y_F E_F^\star E \\ &-\frac{EV}{x_d'}\cos(\theta-\delta). \end{aligned}$$

Using (22) and (23) the inverse transformation for the third coordinate is obtained as $E = \Phi(\eta_1, \eta_2, \eta_3)$ where

$$\Phi := \frac{1}{2a}\left[-b(\eta_1) + \sqrt{b^2(\eta_1) - 4ac(\eta_2, \eta_3)}\right]$$

with a constant, b and c function of η_1, η_2 and η_3.

The closed–loop system, in the new coordinates, takes the form

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ M\dot{\eta}_2 &= P_m - D\eta_2 - \frac{1}{x_d'}\Phi(\eta_1, \eta_2, \eta_3)\Psi(\eta_1, \Phi) \\ \dot{\eta}_3 &= -\lambda\eta_3, \end{aligned}$$

where Ψ is defined in (23).

Establishing detectability with respect to η_3 is tantamount to proving that the equilibrium $(\delta^\star, 0)$ of the two–dimensional system

$$\begin{aligned} \dot{\tilde{\eta}}_1 &= \tilde{\eta}_2 \\ M\dot{\tilde{\eta}}_2 &= P_m - D\tilde{\eta}_2 - \frac{1}{x_d'}\Phi(\tilde{\eta}_1, \tilde{\eta}_2, 0)\Psi(\tilde{\eta}_1, \Phi) \end{aligned}$$

is asymptotically stable. For, we recall that in Proposition 2 we have already established boundedness of trajectories. Hence, recalling that trajectories in plane systems can only diverge, converge or go to a limit cycle, it suffices to prove that the latter will not occur. From Poincare–Bendixson's Theorem we know that a necessary and sufficient condition for *non–existence* of limit cycles in a system $\dot{\tilde{\eta}} = f(\tilde{\eta})$ is

$$\nabla_{\tilde{\eta}_1} f_1 + \nabla_{\tilde{\eta}_2} f_2 \neq 0.$$

Computing this expression yields

$$-\frac{D}{M} + \nabla_\Phi f_2 \nabla_{\tilde{\eta}_2}\Phi(\tilde{\eta}_1, \tilde{\eta}_2, 0) \neq 0.$$

We have numerically evaluated this function for the classical example used in the next subsection with $P_Z = Q_Z = 0.8$ and $B = 6.2112$, for which the condition above is satisfied for all $\tilde{\eta}_1$ (resp., δ) and for $\tilde{\eta}_2 \in (-5.5, 5.5)$ (resp., ω)—an interval far beyond the normal range of operation of the SMIB system.

6. SIMULATIONS

In this section, we present numerical simulations of the proposed controller for the SMIB with and without line losses. The parameters of the SMIB, taken from (Anderson and Fouad (1977)), are as follows: $x_d = 0.8958$, $x_d' = 0.8645$, $\tau = 6$, $M = 12.8$, $D = 0.25$, $P_m = 1.63$. The derivation of the equilibrium point is done with the software package PSAT (Milano (2005)).

We analyze the response of (20), (21) (*system without losses*) to a short circuit which consists of the temporary connection of a small impedance between the machine's terminal and the ground. The fault is introduced at $t = 1s$ and removed after a certain time (called the clearing time, and denoted t_{cl}), after which the system is back to its pre–disturbance topology. During the fault the trajectories make away from the equilibrium, the largest time interval "before instability", called the critical clearing time (t_{cr}), is determined via simulation.

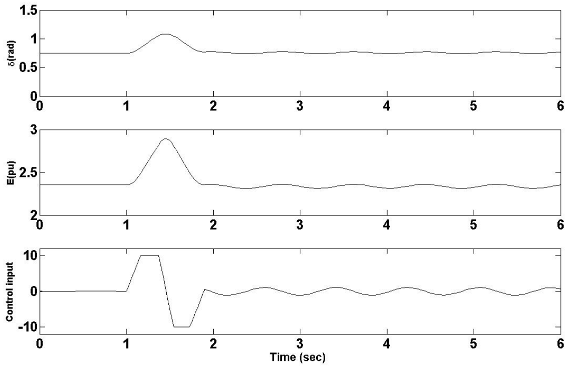

Fig. 1. SMIB with $G = 0$ and t_{cl}=0.9s, in closed loop. Behavior of load angle, internal voltage and control input.

The SMIB has a critical clearing time $t_{cr} = 0.44s$ in *open loop*. With the proposed controller, taking $\lambda = 0.001$ and $\Delta = 2000$, this time could be increased to $19.4s$—a value that is far beyond the time scale of interest in this problem. This implies that the control law has enlarged the domain of attraction of the operating point. Notice that, we can increase t_{cr} by decreasing λ but then the convergence of $\widetilde{H}$ to zero will be slower.

To tune the controller there is a compromise between the choices of Δ that, as indicated in Proposition 1, should be big enough to ensure that the denominator of the controller will stay away from zero, and λ that determines the speed of convergence to the desired level set. Indeed, Δ appears in $\widetilde{H}$ as ΔV^2 where V represents, in some way, the perturbation. Then, the bigger Δ is, the bigger $\widetilde{H}$ will be in the transient phase, and we have to decrease λ to eliminate impulsive responses in the controller during the perturbation.

We then consider the *effect of the losses* in the transmission lines setting $G = 1.1876$ S. Similarly to the lossless case, the proposed controller increases the critical clearing time from $0.36s$ to $7.2s$. Fig. 1 presents the transient behavior of the system without line losses.

7. CONCLUSIONS

We have presented in this paper an excitation controller to improve the transient stability properties of multi–machine power systems described by structure–preserving models with leaky lines including capacitive effects. Our main contribution is the explicit computation of a control law that ensures *"global" asymptotic convergence* to the desired equilibrium point of all trajectories starting and remaining in the physical domain of the system—provided a detectability assumption is satisfied. To the best of our knowledge, no equivalent result is available in the literature at this level of generality. Numerical simulations were presented for the standard SMIB system, for which the detectability assumption was numerically verified for a classical example.

Similarly to most developments reported by the control theory community on the transient stability problem, it is clear that the complexity of the proposed controller—as well as its high sensitivity to the system parameters and the assumption of full state measurement—severely stymies the practical application of this result. This kind of work pertains, however, to the realm of fundamental research where basic issues like existence of solutions are addressed. In Ortega (2005) we proved the existence of an asymptotically stabilizing controller (with a suitable Lyapunov function) for aggregated models—alas, we could only give a constructive solution for $n \leq 3$. The present paper proves that, under a detectability assumption, a solution to the "global" convergence problem for the more natural structure preserving models can indeed be *explicitly constructed.* Current research is under way to further investigate the implications of this assumption.

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