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Fault detection for non-Gaussian stochastic systems via augmented Lyapunov functional approach

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Abstract: In this paper, a new fault detection (FD) scheme is studied for non-Gaussian stochastic dynamic systems using output probability density functions (PDFs). Different from the classical FD problems, the measured information is the PDFs of system output rather than its value, where the B-spline expansion technique is applied so that the considered FD problem is transformed into a nonlinear FD problem. In this context, feasible FD method is presented by combining linear matrix inequality (LMI) technique with augmented Lyapunov functional, which involves a tuning parameter and a slack variable. Furthermore, in order to improve the detection sensitivity performance, an optimal algorithm is applied to minimize the threshold by tuning the parameter. Simulation for a model in the paper-making process is given to demonstrate the efficiency of the proposed approach.

1. INTRODUCTION

The fault detection (FD) is important topics in systems engineering from the viewpoint of improving system reliability. In past two decades many significant approaches have been presented and applied to practical processes successfully (P. M. Frank, S. X. Ding [1997], W.T.Chen, M. Saif [2006]). In general, the FD results can be classified into three types. The first type is the filter-or observer-based approaches, where filters are used to generate residual signals to detect and estimate the fault in R.H.Chen et al [2003]. The second type is the identification-based FD scheme, where the identification technique is applied to estimate the model parameter changes of system in M. Basseville, I. Nikiforov [2002]. The third type is the statistic approach, where the Bayesian theory and likelihood methods can be used to evaluate and fault signals in P. Li [2001]. For the dynamic stochastic systems, the filter-based FD approach has been shown as an effective way where generally the variables are supposed to be Gaussian in R.H.Chen et al [2003] and J. Liu et al [2003]. However, in many practical processes, non-Gaussian variables exist in many stochastic systems due to nonlinearity, which may posses asymmetric and multiple-peak stochastic distributions.

On the other hand, along with the development of advanced instruments and data processing technique, the measurements for feedback can be the stochastic information (which can be described by PDFs) of the system output rather than the output itself (H. Wang [2000]). Typical examples include the retention of paper making, particle distribution, combustion process with flame grey-level distribution in H. Wang [2000]. Fig. 1 shows the combustion process, where the remote monitoring and control of combustion systems are mainly based on the image analysis of flame characteristics or temperature distribution. The flame geometry and intensity can thus be considered as stochastic processes, whose distributions are obtainable by modern infrared sensors and signal processing techniques (see H. Wang, W. Lin [2000]). The shape of the grey-level distribution of the flame image is the control and detection target, and the control can be completed through a networked system. Motivated by such typical examples, a new group of strategies that control the shape of PDFs for stochastic systems have been developed in the past a few yeas (see H. Wang [2000], M. Karny [1996]). Different from any other previous stochastic control approaches, the stochastic variables are not confined to be Gaussian and the output PDFs of the stochastic system is concerned rather than the mean or variance of the output.

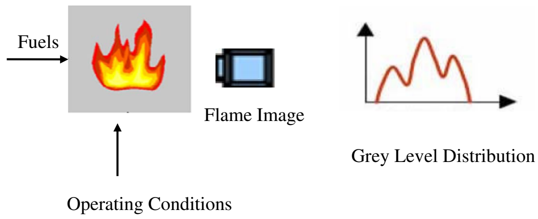

Fig. 1. Grey-level distribution of the flame image

In output PDFs shape control, B-spline expansion technique has been introduced in the output PDFs modeling in L. Guo, H. Wang [2004], L. Guo, H. Wang [2005]. The motivation of FD via the output PDFs from the retention system in papermaking was first studied in H. Wang, W. Lin [2000], where the weight dynamical system was supposed to be a precise linear model. However, linear mappings cannot change the shape of output PDFs, which implies that the fault cannot be detected through the shape change of the PDFs. To meet the requirement in complex processes, nonlinearity should be considered in the weighting dynamic behavior. Recently, a kind of observer-based FD algorithm has been established in L. Guo, H. Wang [2005], where the nonlinear weighting system was considered. However, for some cases, the fault can't be detected by using the algorithms in L. Guo, H. Wang [2005] since the threshold is larger than residual evaluation function. To improve the previous results, in this paper, firstly, square root B-spline expansion technique is applied to model the PDFs so that the concerned problem is transformed into a nonlinear FD problem. Secondly, LMI-based solution is presented such that the estimation error system is stable and the fault can be detected through a threshold. Moreover, the threshold can be minimized by introducing the tuning parameter and slack variable, which leads to less conservative FD algorithms than ones in L. Guo, H. Wang [2005]. Finally, paper-making process example is given to demonstrate the applicability of the proposed approach.

2. PROBLEM FORMULATION

In this section, firstly, we briefly review square-root B-spline approximation technique presented in L. Guo, H. Wang [2005], which is used to formulate the output PDFs with the dynamic weight and is essential in solving our FD problem.

For a dynamic stochastic system, its output PDFs is defined by $\gamma(z,u(t),F)$, where $u(t)\in R^m$ is control input, F is the fault vector to be detected, a typical example of which is an actuator fault. In L. Guo, H. Wang [2005], Y. M. Zhang et al [2006], some B-spline models have been used to approximate $\gamma(z,u(t),F)$. In this paper, we use the following square root B-spline model

$$\sqrt{\gamma(z,u(t),F)}=\sum_{i=1}^{n}v_i(u,F)b_i(z) \qquad (1)$$

where $b_i(z)(i=1,2,...,n)$ are pre-specified basis functions defined on $[a,b]$, and $v_i(u(t),F)(i=1,2,...,n)$ are the corresponding weights of such an expansion. Denote

$$B_0(z)=[b_1(z)\ \ b_2(z)\cdots\ \ b_{n-1}(z)]^\tau$$
$$V(t):=V(u(t),F)=[v_1\ \ v_2\cdots\ \ v_{n-1}]^\tau$$

and let

$$\Lambda_1=\int_a^b B_0(z)B_0^\tau(z)dz,\ \Lambda_2=\int_a^b B_0^\tau(z)b_n(z)dz,$$

$$\Lambda_3=\int_a^b b_n^2(z)dz\neq 0,\ \Lambda_0=\Lambda_1\Lambda_3-\Lambda_2\Lambda_2^\tau$$

Furthermore, it can be verified that (1) can be rewritten as (see L. Guo, H. Wang [2005] for details)

$$\sqrt{\gamma(z,u(t),F)}=B^\tau(z)V(t)+h(V(t))b_n(z) \qquad (2)$$

where

$$B^\tau(z)=B_0^\tau(z)-\frac{\Lambda_2}{\Lambda_3}b_n(z),$$

$$h(V(t))=\frac{\sqrt{\Lambda_3-V^\tau(t)\Lambda_0V^\tau(t)}}{\Lambda_3} \qquad (3)$$

Different from the linear or rational B-spline model in H. Wang [2001], the following square root B-spline model with an approximation error will be adopted

$$\sqrt{\gamma(z,u(t),F)}=B^\tau(z)V(t)+h(V(t))b_n(z)+\omega(z,u,F) \qquad (4)$$

$\omega(z,u,F)$ represents the model uncertainty or the error term on the approximation of PDFs and satisfies $|\ \omega(z,u,F)\ |\leq\delta$, for all $\{z,u(t),F\}$, where δ is assumed to be a known positive constant.

Secondly, we find the relationship between the input and the weights related to the PDFs, which corresponds to a further modeling procedure. Apart from several systems in the wet end of paper machines studied in H. Yue, H. Wang [2003], H. Wang [2001], an example is the particle size distribution control in chemical engineering, where the product quality is characterized through the PDFs of particle size. However, most published results only concerned linear precise models, while practically the relationships from control input $u(t)$ to weight vector $V(t)$ should be nonlinear dynamics and subjected to some uncertainties. As such, the following nonlinear dynamic model will be considered in this paper

$$\begin{cases}\dot{x}(t)=Ax(t)+Gg(x(t))+Hu(t)+JF(t)\\ V(t)=Ex(t)\end{cases} \qquad (5)$$

where $x(t)\in R^m$ is the unmeasured state, $F(t)$ is the fault to be detected. A, G, H, J and E represent the known parametric matrices of the dynamic part of the weight system. In fact, these matrices can be obtained either by physical modeling or the scaling estimation technique described in H. Wang [2001]. in addition, similarly to L. Guo, H. Wang [2005] and L. Guo, H. Wang [2005], the following assumptions are needed

Assumption 1. For any $x_1(t)$ and $x_2(t)$, $g(x(t))$ satisfies $g(0)=0$ and

$$\|\ g(x_1(t))-g(x_2(t))\ \|\leq\|\ U_2(x_1(t)-x_2(t))\ \| \qquad (6)$$

where U_2 is a known matrix.

Assumption 2. There is a known matrix U_1, for any $V_1(t)$ and $V_2(t)$, $h(V(t))$ denoted by (3) satisfies the followng condition

$$\|\ h(V_1(t))-h(V_2(t))\ \|\leq\|\ U_1(V_1(t)-V_2(t))\ \| \qquad (7)$$

where $\|\cdot\|$ is denoted as the Euclidean norm.

Remark 1. Inequalities (6) and (7) actually can be guaranteed by the property of functions $h(V(t))$ and $g(x(t))$ and the boundedness of $V(t)$, which is typically required in the literature on FD for nonlinear systems, e.g., B. Jiang, F. N. Chowdhury [2005], B. Jiang, F. N. Chowdhury [2005]. The assumptions condition will be help to simply the design algorithms later on.

Generally speaking, a fault-detection system consists of a residual generator, and a residual evaluator including an evaluation function and a threshold.

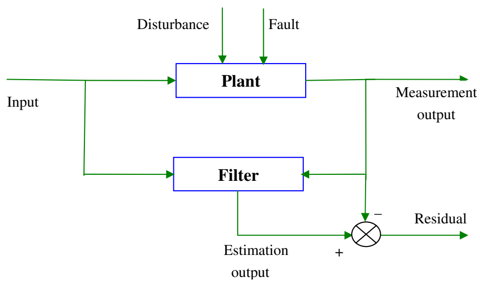

Fig. 2. The classical residual generator

2.1 residual generator

For the purpose of residual generation, we construct the following nonlinear filter

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Gg(\hat{x}(t)) + Hu(t) + L\xi(t) \\ \xi(t) = \int_a^b \sigma(z)\left(\sqrt{\gamma(z,u(t),F)} - \sqrt{\hat{\gamma}(z,u(t))}\right)dz \quad (8) \\ \sqrt{\hat{\gamma}(z,u(t))} = B^\tau(z)E\hat{x}(t) + h(E\hat{x}(t))b_n(z) \end{cases}$$

where $\hat{x}(t)$ is the estimated state, $L \in R^{m\times p}$ is the gain to be determined and $\sigma(z) \in R^{p\times 1}$ can be regarded as a pre-specified weighting vector defined on $[a,b]$.

Remark 2. The classical residual generator design methods (such as J. Stoustrup, N. N. Niemann [2002], Q. Zhao, Z. Xu [2004], M.Y. Zhong et al [2003]) is formulated in Fig. 2. Different from the classical residual generator, residual $\xi(t)$ in (8) is formulated as an integral with respect to the difference of the measured PDFs and the estimated PDFs.

By defining $e(t) = x(t)-\hat{x}(t)$, the estimation error system can be described

$$\dot{e}(t) = (A - L\Gamma_1)e(t) + G[g(x(t)) - g(\hat{x}(t))] \\ -L\Gamma_2[h(Ex(t)) - h(E\hat{x}(t))] + JF(t) - L\Delta(t) \quad (9)$$

where

$$\Gamma_1 = \int_a^b \sigma(z)B^\tau(z)Edz, \ \Gamma_2 = \int_a^b \sigma(z)b_n(z)dz,$$

$$\Delta(t) = \int_a^b \sigma(z)\omega(z,u,F)dz \quad (10)$$

It can be seen that

$$\xi(t) = \Gamma_1 e(t) + \Gamma_2(h(Ex(t)) - h(E\hat{x}(t))) + \Delta(t) \quad (11)$$

From $|\ \omega(z,u(t),F)\ | \le \delta$, it can be verified that

$$\|\ \Delta(t)\ \| = \|\int_a^b \sigma(z)\omega(z,u,F)dz\ \| \le \hat{\delta}, \hat{\delta} = \delta\int_a^b \sigma(z)dz \quad (12)$$

Thus, the problem of designing filter-based fault detection, which is one of the main objectives of this work, can be described as designing matrix L such that

- the error system (9) is asymptotically stable;
- The generated residual $\xi(t)$ is as sensitive as possible to fault $F(t)$.

2.2 residual evaluator

After designing of FD filter, the remaining important task for FD is the evaluation of the generated residual. One of the widely adopted approaches is to choose a so-called threshold $J_{th} > 0$ and, based on this, use the following logical relationship for fault detection

$$||\xi(t)|| > J_{th} \Rightarrow \text{faults} \Rightarrow \text{alarm},$$
$$||\xi(t)|| \le J_{th} \Rightarrow \text{no faults}$$

From the above logical relationship, it is clear that the fault detection sensitivity performance may be improved by minimizing the threshold J_{th}.

3. MAIN RESULT

Theorem 3. For the parameters $\lambda_i > 0 (i = 1,2)$ and ε, if there exist matrices $P_1 > 0$, P_2, R and constants $\eta_1 > 0$, $\eta_2 > 0$ satisfying

$$\Pi = \begin{bmatrix} \Pi_1 + \eta_1 I & \Pi_2 & \Pi_3 & 0 \\ * & -\varepsilon P_2^\tau - \varepsilon P_2 + \eta_2 I & 0 & \Pi_4 \\ * & * & -I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \quad (13)$$

where

$$\Pi_1 = P_2^\tau A - R\Gamma_1 + A^\tau P_2 - \Gamma_1^\tau R^\tau + \frac{1}{\lambda_1^2}E^\tau U_1^\tau U_1 E + \frac{1}{\lambda_2^2}U_2^\tau U_2$$

$$\Pi_2 = \varepsilon A^\tau P_2 - \varepsilon\Gamma_1^\tau R^\tau + P_1 - P_2^\tau$$

$$\Pi_3 = [\lambda_1 R\Gamma_2 \quad \lambda_2 P_2^\tau G], \qquad \Pi_4 = [\varepsilon\lambda_1 R\Gamma_2 \quad \varepsilon\lambda_2 P_2^\tau G]$$

then in the absence of F, the error system (9) with gain $L = P_2^{-\tau}R$ is stable and satisfies

$$||e(t)|| \le \alpha = \max\left\{||e(0)||, \ (\sqrt{\eta_1\eta_2})^{-1}(1+|\varepsilon|)||R||\hat{\delta}\right\} \quad (14)$$

$$||\xi(t)|| \le \beta = \alpha(||\Gamma_1|| + ||\Gamma_2||||U_1||||E||) + \hat{\delta} \quad (15)$$

for $t \in [0, \ \infty)$, where $\hat{\delta}$ is defined by (12).

Proof. Denote

$$\Pi_0 = \begin{bmatrix} \Pi_1 + \Pi_3^\tau\Pi_3 & \Pi_2 \\ * & -\varepsilon P_2^\tau - \varepsilon P_2 + \Pi_4^\tau\Pi_4 \end{bmatrix}$$

Using the Schur complement, it can be shown that $\Pi < 0 \Leftrightarrow \Pi_0 + diag\{\eta_1 I, \ \eta_2 I\} < 0$. Define $\tilde{g} := g(x(s)) - g(\hat{x}(s))$, $\tilde{h} := h(Ex(s)) - h(E\hat{x}(s))$ and denote the Lyapunov function candidate as follows

$$\Phi(t) = \bar{e}^\tau(t)SP\bar{e}(t) + \frac{1}{\lambda_1^2}\int_0^t [\|\ U_1 Ee(s)\ \|^2 - \|\ \tilde{h}\ \|^2]ds$$
$$+\frac{1}{\lambda_2^2}\int_0^t [\|\ U_2 e(s)\ \|^2 - \|\ \tilde{g}\ \|^2]ds \quad (16)$$

where

$$S = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \ P = \begin{bmatrix} P_1 & 0 \\ P_2 & \varepsilon P_2 \end{bmatrix}, \ \bar{e}(t) = \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix}$$

$$P_1 > 0, \ SP = P^\tau S^\tau \ge 0$$

Following (6) and (7) yields $\Phi(t) \geq 0$ for all arguments. It is noted that $\bar{e}^\tau(t)SP\bar{e}(t)$ is actually $e^\tau(t)P_1e(t)$. Hence, differentiating $\bar{e}^\tau(t)SP\bar{e}(t)$ with respect to t gives

$$\frac{d}{dt}\{\bar{e}^\tau(t)SP\bar{e}(t)\} = 2e^\tau(t)P_1\dot{e}(t) = 2\bar{e}^\tau(t)P^\tau \begin{bmatrix} \dot{e}(t) \\ 0 \end{bmatrix}$$

On the other hand, in the absence of $F(t)$, the state equation (9) ensures that

$$\alpha(t) := -\dot{e}(t) + (A - L\Gamma_1)e(t) + G\tilde{g} - L\Gamma_2\tilde{h} - L\Delta(t) = 0$$

Then along the trajectory of (9) in the absence of F, it can be shown that

$$\begin{aligned}
\dot{\Phi}(t) =& 2\bar{e}^\tau(t)P^\tau \begin{bmatrix} \dot{e}(t) \\ \alpha(t) \end{bmatrix} + \frac{1}{\lambda_1^2}[\| U_1Ee(t) \|^2 - \| \tilde{h} \|^2] \\
&+ \frac{1}{\lambda_2^2}[\| U_2e(t) \|^2 - \| \tilde{g} \|^2] \\
\leq& 2e^\tau(t)P_1\dot{e}(t) + 2e^\tau(t)(P_2^\tau A - R\Gamma_1)e(t) \\
&-2e^\tau(t)P_2^\tau\dot{e}(t) - 2e^\tau(t)R\Delta(t) \\
&-2\varepsilon\dot{e}^\tau(t)P_2^\tau\dot{e}(t) - 2\varepsilon e^\tau(t)R\Delta(t) \\
&+2\varepsilon\dot{e}^\tau(t)(P_2^\tau A - R\Gamma_1)e(t) \\
&+e^\tau(t)[\frac{1}{\lambda_2^2}U_2^\tau U_2 + \frac{1}{\lambda_1^2}E^\tau U_1^\tau U_1E]e(t) \\
&+e^\tau(t)[\lambda_2^2P_2^\tau GG^\tau P_2 + \lambda_1^2R\Gamma_2\Gamma_2^\tau R^\tau]e(t) \\
&+\dot{e}^\tau(t)[\varepsilon^2\lambda_2^2P_2^\tau GG^\tau P_2 + \varepsilon^2\lambda_1^2R\Gamma_2\Gamma_2^\tau R^\tau]\dot{e}(t) \\
\leq& \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix}^\tau \Pi_0 \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix} - 2e^\tau(t)R\Delta(t) - 2\varepsilon\dot{e}^\tau(t)R\Delta(t) \\
\leq& -\eta_1||e(t)||^2 - \eta_2||\dot{e}(t)||^2 + 2||e(t)||||R||\hat{\delta} \\
&+2|\varepsilon|||\dot{e}(t)||||R||\hat{\delta} \\
\leq& -2\sqrt{\eta_1\eta_2}||e(t)||||\dot{e}(t)|| + 2(||e(t)|| + |\varepsilon|||\dot{e}(t)||)||R||\hat{\delta}
\end{aligned}$$

where $R = P_2^\tau L$. When $||e(t)|| \geq ||\dot{e}(t)||$, if $||\dot{e}(t)|| \geq (\sqrt{\eta_1\eta_2})^{-1}(1 + |\varepsilon|)||R||\hat{\delta}$, we have $\dot{\Phi}(t) < 0$. When $|||e(t)|| \leq ||\dot{e}(t)||$, if $||e(t)|| \geq (\sqrt{\eta_1\eta_2})^{-1}(1+|\varepsilon|)||R||\hat{\delta}$, then $\dot{\Phi}(t) < 0$. Thus, if $||e(t)|| \geq (\sqrt{\eta_1\eta_2})^{-1}(1+|\varepsilon|)||R||\hat{\delta}$ occurs, then $\dot{\Phi}(t) < 0$ holds.

Furthermore, from (11), it can seen that

$$\begin{aligned}
\xi(t) \leq& \| \Gamma_1 \|\|\| e(t) \| + \| \Gamma_2 \|\|\| \tilde{h} \| + \| \Delta(t) \| \\
\leq& \| e(t) \| (\| \Gamma_1 \| + \| \Gamma_2 \|\|\| U_1 \|\|\| E \|) + \| \Delta(t) \| \\
\leq& \alpha(\| \Gamma_1 \| + \| \Gamma_2 \|\|\| U_1 \|\|\| E \|) + \hat{\delta}
\end{aligned}$$

where α is defined by (14). Therefore, it can be claimed that (14), (15) always holds and the estimation error system is stable. Q.E.D.

Remark 4. Theorems 3 is based on a newly proposed augmented Lyapunov functional of form (16), which contains a structure more general than the traditional ones as those in L. Guo, H. Wang [2005] for involving $\dot{e}(t)$ in the first term of (16). This new type of Lyapunov functional is beneficial for minimizing the threshold later on.

By Theorem 3, we can obtain the following result, which has been reported in L. Guo, H. Wang [2005] recently.

Corollary 5. For the parameters $\lambda_i > 0 (i = 1, 2)$, if there exist matrices $P > 0$, R and constant $\eta > 0$ satisfying

$$\begin{bmatrix} \Pi_1 + \eta I & \hat{\Pi}_3 \\ * & -I \end{bmatrix} < 0 \tag{17}$$

where $\hat{\Pi}_3 = [\lambda_1R\Gamma_2 \quad \lambda_2P_2^\tau G]$ then in the absence of F, the error system (9) with gain $L = P^{-1}R$ is stable and satisfies

$$||e(t)|| \leq \alpha = \max\left\{||e(0)||, \ \ 2\eta^{-1}||R||\hat{\delta}\right\} \tag{18}$$

$$||\xi(t)|| \leq \beta = \alpha(||\Gamma_1|| + ||\Gamma_2||||U_1||||E||) + \hat{\delta} \tag{19}$$

for $t \in [0, \ \infty)$, where $\hat{\delta}$ is defined by (12).

Remark 6. It is obvious that Corollary 5 is recovered by setting $\eta_1 = 2n\eta, \ \eta_2 = \frac{2}{n}\eta \ P_2 = P_1 = P, \ \varepsilon = 0$ in Theorem 3, where n is any positive constant. So, Theorem 3 in this paper is an extension of the result in L. Guo, H. Wang [2005].

In this case, we choose $||\xi(t)||$ and β as residual evaluation function and the threshold respectively. From Theorem 3, it is shown that the threshold β depends on the value of $\hat{\delta}$ and α, where α can be tuned by the parameter ε. The sensitivity of the fault detection can be improved through the following optimization problem:

$$\min_{P_1, \ P_2, \ R, \ \eta_1, \ \eta_2} \beta \qquad \textit{subjected to } (13) \tag{20}$$

Remark 7. Very recently, L. Guo, H. Wang [2005] provided a new fault detection criterion by using PDFs and nonlinear filter. Compared with the threshold in L. Guo, H. Wang [2005], the advantage in our note is that the threshold β involve the tuning parameter ε and slack variable P_2. By tuning the parameter ε and matrix P_2, the fault detection criterion in this note can provide less conservative algorithm than the Theorem 1 in L. Guo, H. Wang [2005], which can be seen from the numerical example in section 4.

4. SIMULATION

An application of paper making process is given to demonstrate the applicability of the proposed approach. The basis functions are selected similarly to Y. M. Zhang et al [2006] as follows:

$$\begin{aligned}
B(z) =& [b_1(z), \ b_2(z), ..., b_9(z)]^\tau, \\
b_i(z) =& exp(-(z - z_i)^2\sigma_i^{-2}), \ (i = 1, \ 2, ..., 10) \\
z_i =& 0.003 + 0.006(i - 1), \ \sigma_i = 0.003 \ (i = 1, \ 2, ..., 10)
\end{aligned}$$

Consider the following weighting system

$$\begin{cases} \dot{x}(t) = Ax(t) + Gg(x(t)) + Hu(t) + JF(t) \\ V(t) = x(t) \end{cases}$$

where

$$\begin{aligned}
A =& diag\{-0.83, \ -0.83, ..., -0.83\} \in R^{9\times9}, \\
G =& diag\{1, \ 1, ..., \ 1\} \in R^{9\times9}, \\
H =& [0 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01 \ 0]^\tau, \\
J =& diag\{1.18, \ 0, \ 0.56, \ 0, \ 0.56, \ 0, \ 0.56, \ 0.56, \ 0.56\}
\end{aligned}$$

$g(x(t)) = sin(x(t))$ and $u(t)$ is random number. Different from Reference H. Wang, W. Lin [2000], it is assumed that the model error exists and satisfies $||\omega(z, u, F)|| \leq 0.001$. By setting $\sigma(z) = 1$, then the following parametric matrices related to the B-spline approximation can be obtained:

$$\Lambda_0 = 10^{-5} \times \begin{bmatrix} 2.400 & 0.956 & 0.048 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.956 & 2.607 & 0.959 & 0.047 & 0 & 0 & 0 & 0 & 0 \\ 0.048 & 0.959 & 2.605 & 0.956 & 0.049 & 0.001 & 0 & 0 & 0 \\ 0 & 0.047 & 0.956 & 2.601 & 0.960 & 0.048 & 0 & 0 & 0 \\ 0 & 0 & 0.049 & 0.960 & 2.605 & 0.958 & 0.048 & 0.001 & 0 \\ 0 & 0 & 0.001 & 0.048 & 0.958 & 2.605 & 0.960 & 0.049 & 0 \\ 0 & 0 & 0 & 0 & 0.048 & 0.960 & 2.601 & 0.956 & 0.047 \\ 0 & 0 & 0 & 0 & 0.001 & 0.049 & 0.956 & 2.604 & 0.939 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.047 & 0.939 & 2.226 \end{bmatrix}$$

$$\Lambda_1 = 10^{-3} \times \begin{bmatrix} 4.9 & 2 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 5.3 & 2 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0.1 & 2 & 5.3 & 2 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 2 & 5.3 & 2 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 2 & 5.3 & 2 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 2 & 5.3 & 2 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 2 & 5.3 & 2 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 2 & 5.3 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 2 & 5.3 \end{bmatrix},$$

$$\Lambda_2 = [\, 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0.0001\ 0.0020 \,]\,, \Lambda_3 = 0.0049$$

It can be verified that $U_1 = diag\{0.05,\ 0.05, ..., 0.05\} \in R^{9\times 9}$ and it can be supposed that $U_2 = diag\{1,\ 1, ..., 1\} \in R^{9\times 9}$. Corresponding to (10), it can be calculated that $\Gamma_1 = 10^{-3} \times [\, 6.3\ 7.5\ 7.5\ 7.5\ 7.5\ 7.5\ 7.5\ 7.4\ 5.0 \,]$, $\Gamma_2 = 0.0063$, $||\Delta(t)|| \leq 0.001$. In this case, the initial value of observer (8) is selected as $\hat{x}(t) = 0 \in R^9$ for all $0 \leq t \leq +\infty$, while the initial value of (5) is selected as $x_0 = [0.3\ 0.05\ 0\ 0.1\ 0\ 0.1\ 0\ 0.01\ 0]$. The fault is supposed as

$$F(t) = \begin{cases} 0, & t < 20 \\ [0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 0.2\ 0.2], & 20 \leq t \leq 40 \\ 0, & t \geq 40 \end{cases}$$

To detect the presence of fault, theorems 3 is used where $\lambda_1 = 1,\ \lambda_2 = 1$. By using theorem 3, it can be obtained that $\eta_1 = 0.0321,\ \eta_2 = 383.1007$ and

$$P_1 = \begin{bmatrix} 2.33 & 0.42 & 0.42 & 0.42 & 0.42 & 0.42 & 0.41 & 0.42 & 0.28 \\ 0.42 & 2.48 & 0.50 & 0.50 & 0.50 & 0.50 & 0.49 & 0.50 & 0.33 \\ 0.42 & 0.50 & 2.48 & 0.50 & 0.50 & 0.50 & 0.49 & 0.50 & 0.33 \\ 0.42 & 0.50 & 0.50 & 2.48 & 0.50 & 0.50 & 0.49 & 0.50 & 0.33 \\ 0.42 & 0.50 & 0.50 & 0.50 & 2.48 & 0.50 & 0.49 & 0.50 & 0.33 \\ 0.42 & 0.50 & 0.50 & 0.50 & 0.50 & 2.48 & 0.49 & 0.50 & 0.33 \\ 0.41 & 0.49 & 0.49 & 0.49 & 0.49 & 0.49 & 2.47 & 0.49 & 0.33 \\ 0.42 & 0.50 & 0.50 & 0.50 & 0.50 & 0.50 & 0.49 & 2.48 & 0.33 \\ 0.28 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 2.20 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0.62 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & -0.01 \\ -0.02 & 0.61 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & -0.01 \\ -0.02 & -0.02 & 0.61 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & -0.01 \\ -0.02 & -0.02 & -0.02 & 0.61 & -0.02 & -0.02 & -0.02 & -0.02 & -0.01 \\ -0.02 & -0.02 & -0.02 & -0.02 & 0.61 & -0.02 & -0.02 & -0.02 & -0.01 \\ -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & 0.61 & -0.02 & -0.02 & -0.01 \\ -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & 0.61 & -0.02 & -0.01 \\ -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & -0.02 & 0.61 & -0.01 \\ -0.01 & -0.01 & -0.01 & -0.01 & -0.01 & -0.01 & -0.01 & -0.01 & 0.63 \end{bmatrix}$$

$$R = [25.95\ 30.89\ 30.89\ 30.89\ 30.89\ 30.89\ 30.48\ 30.89\ 20.59]$$

$$L = [60.65\ 72.20\ 72.20\ 72.20\ 72.20\ 72.20\ 71.24\ 72.20\ 48.13]$$

The threshold can be calculated $\beta = 0.0029$, which is obtained as 0.2586 in L. Guo, H. Wang [2005]. In Fig. 3, the three-dimensional (3-D) mesh plot shows that changes of the measured output PDFs. Fig. 4 demonstrates the residual signal, when the fault occurs from 20s to 40s. Fig. 5 shows the evolution of residual evaluation function and the threshold. From Fig. 5, it can be seen that the fault can be detected 7s after its occurrence, but it can't be detected by the result in L. Guo, H. Wang [2005] since the threshold in L. Guo, H. Wang [2005] is larger than the evaluation function $||\xi(t)||$. Thus, the less conservative FD algorithms can be obtained in this note.

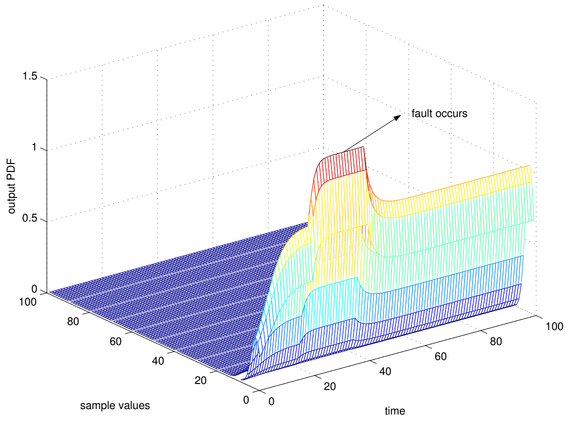

Fig. 3. 3-D mesh plot of the measured output PDFs

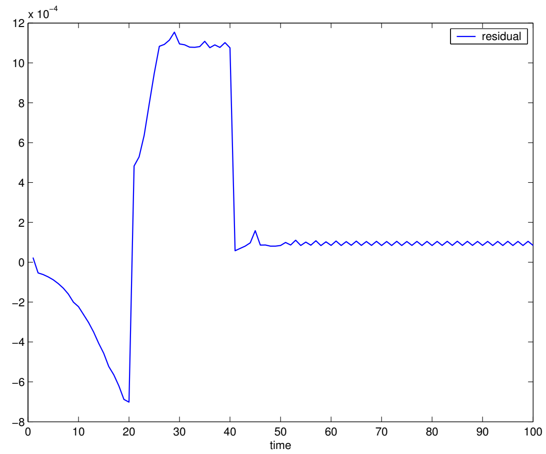

Fig. 4. The response of the residual when the fault occurs

5. CONCLUSION

In this paper, the FD problem is investigated for non-Gaussian stochastic systems using only the output PDFs and nonlinear filters, where the output PDFs can be measured rather than an output signal. Based on LMI techniques, a new criterion is obtained to detection the fault with a threshold. Moreover, by introduced the tuning parameter and slack variable, the detection sensitivity performance is improved by minimizing the threshold. Simulation is given to demonstrate the efficiency of the proposed approach.

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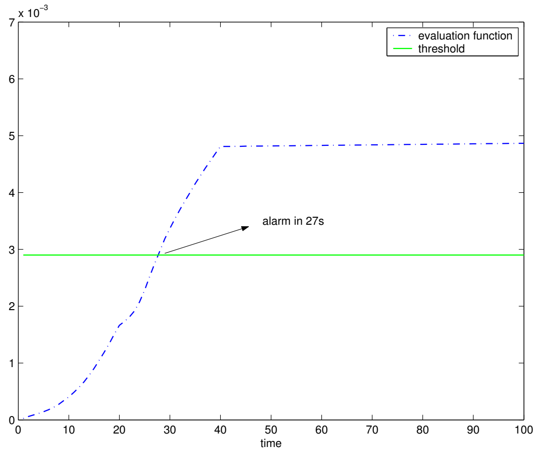

Fig. 5. The response of evaluation function and threshold

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