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Nonlinear Model Predictive Spread Acceleration Guidance with Impact Angle Constraint for Stationary Targets

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Abstract: A new technique named as model predictive spread acceleration guidance (MPSAG) is proposed in this paper. It combines nonlinear model predictive control and spread acceleration guidance philosophies. This technique is then used to design a nonlinear suboptimal guidance law for a constant speed missile against stationary target with impact angle constraint. MPSAG technique can be applied to a class of nonlinear problems, which leads to a closed form solution of the lateral acceleration (latax) history update. Guidance command assumed is the lateral acceleration (latax), applied normal to the velocity vector. The new guidance law is validated by considering the nonlinear kinematics with both lag-free as well as first order autopilot delay. The simulation results show that the proposed technique is quite promising to come up with a nonlinear guidance law that leads to both very small miss distance as well as the desired impact angle.

1. INTRODUCTION

Guidance provides maneuvering commands to a flight vehicle to follow a desired path. A popular classical guidance law is the "proportional navigation (PN)" (Zarchan (1997)) (and its variants), which has been both well studied and successfully implemented in many flight vehicles. It is also well-known that the PN guidance law serves as an "optimal guidance law" (Bryson & Ho (1975)). However, these optimality results come with certain assumptions like linearized engagement geometry, proper choice of the navigation constant etc. Modern techniques, on the other hand, formulate the problem in the framework of optimal control theory and attempt to come up with more powerful guidance laws. For example, in a recent paper (Ryoo et al. (2006)), the authors have come up with an optimal guidance law that not only minimizes the miss distance, but also leads to a desired impact angle (an impact angle constrained guidance law leads to enhancement of warhead effect for anti-ship or anti-tank missiles).

In reference (Ryoo et al. (2006)) (as well as in many other literatures), the optimal guidance law available relies on linearized engagement geometry, which facilitates the usage of "linear optimal control theory" (Bryson & Ho (1975)). Linear optimal guidance laws, however, either have limited application domain or degraded performance (because of the linearized engagement geometry). Both to make the application domain wider as well as better performance, it is desirable that the nonlinear engagement kinematics is considered in developing the guidance law itself. A variation of nonlinear optimal control theory which is widely used in practice (especially in chemical engineering problems) is model predictive control (Cannon (2004)), where the idea is to use output dynamics instead of state dynamics (to reduce numerical complexity). Here the output is predicted for a finite time window using a guess control and based on the error in the output the control history is updated by minimizing a performance index.

Reverting back to the PN guidance law, it is a fact that when the engagement is far away from the collision triangle, the latax demand is substantial. However, when the collision triangle condition is achieved (i.e. the line-of-sight rate is zero), then the latax demand is also zero. Whereas zero latax is a very much desirable feature, it may not be feasible for the control system to meet the initial high latax demand (leading to latax saturation, and hence, performance degradation). One way to address this issue is the idea of "spread acceleration guidance" (Ghose et al. (1989)). This technique uses a predictive guidance mechanism which projects the position of the target and then attempts to spread the missile latax (rather uniformly) over the entire range of time-to-go.

In this paper we attempt to combine the philosophies of model predictive control and spread acceleration guidance to present a model predictive spread acceleration guidance (MPSAG) law. The control variable (latax) is first parameterized as a linear function of the time-to-go. Because of the parametrization of the latax requirement and closed form update of these parameter values, the computational requirements are much reduced and the algorithm can be implemented online. Using this newly-developed MPSAG technique, the impact angle constrained guidance law design problem addressed in (Ryoo et al. (2006)) is revisited here. However, in contrast to (Ryoo et al. (2006)), the guidance law is derived here uses the nonlinear engagement kinematics as such (i.e.without linearizing it). The simulation results show that the MPSAG technique is quite promising to address the problem preserving the nonlinear kinematics of engagement. It is observed that the new nonlinear guidance law proposed in the paper leads to the following advantages as compared to the existing linearized optimal guidance law :(i)Latax calculation is carried out using nonlinear dynamics and (ii) singularity at the end of guidance command for first order delay system is completely eliminated.

2. THE ENGAGEMENT DYNAMICS

Fig. 1 shows the engagement geometry for a stationary and slowly moving target. The missile is moving with a constant speed and the target under consideration is assumed to be stationary. The guidance command is the lateral acceleration (latax), which is the acceleration normal to the velocity vector (Ryoo et al. (2005)). The equation of motion for this homing missile problem is formulated both for lag free and first order autopilot delay system
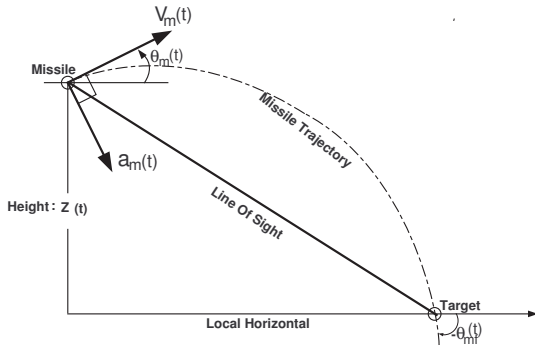

Fig. 1. Engagement Geometry

2.1 Equation of motion for Lag free Autopilot

Impact angle, for a stationary target is the missile flight angle at the time of interception (Ryoo et al. (2005)). The terms z, a_m, V_m, θ_m and θ_{mf} are defined as the missile height, the acceleration applied normal to the velocity vector, the missile velocity, the flight path angle and the predetermined impact angle respectively. Then the equation of motion of lag -free autopilot is given by (Ryoo et al. (2005))

$$\dot{z} = V_m(t)\sin\theta_m(t) \tag{1}$$

$$V_m(t)\dot{\theta}_m(t) = a_m(t) \tag{2}$$

System dynamics in normalized form can be written as

$$\dot{z_N} = \frac{V_m \sin(\theta_m^* \theta_{m_N})}{z^*} \tag{3}$$

$$\dot{\theta}_{m_N} = \frac{a_m}{V_m \theta_m^*}$$

where $z_N \triangleq z/z^*$ and $\theta_{m_N} \triangleq \theta_m/\theta_m^*$ are the normalized state variables. The symbols z^* and θ_m^* represent the normalizing variables.

2.2 Equation of motion for First order Autopilot

The dynamics of first order autopilot can be written as

$$(\dot{a_m} - \dot{a_c}) + \frac{1}{\tau}(a_m - a_c) = 0 \tag{4}$$

The closed form solution of (4) is given as

$$a_m = a_c - e^{\frac{-t}{\tau}}(a_{c_0} - a_{m_0}) \tag{5}$$

where a_m is the achieved missile latax and a_c is the commanded missile latax. So the latax for first order is replaced by the closed form solution given by (5). Now the system model can be written as (Ryoo et al. (2005))

$$\dot{z} = V_m(t)\sin\theta_m(t) \tag{6}$$

$$V_m(t)\dot{\theta}_m(t) = a_c - e^{\frac{-t}{\tau}}(a_{c_0} - a_{m_0}) \tag{7}$$

System dynamics in normalized form can be written as

$$\dot{z}_N = \frac{V_m \sin(\theta_m^* \theta_{m_N})}{z^*} \tag{8}$$

$$\dot{\theta}_{m_N} = \frac{a_c}{V_m \theta_m^*} - \frac{e^{\frac{-t}{\tau}}(a_{c_0} - a_{m_0})}{V_m \theta_m^*}$$

where $z_N \triangleq z/z^*$ and $\theta_{mN} \triangleq \theta_m/\theta_m^*$ are the normalized state variables.

2.3 Problem Objectives

Main objective of the problem is to design a nonlinear suboptimal missile guidance law with terminal impact angle constraint using the newly developed MPSAG technique. The technique combines model predictive static programming and spread acceleration guidance. Mathematical formulation of MPSAG technique is given in detail in the next section in fair detail.

3. MPSAG DESIGN:MATHEMATICAL FORMULATION

In this section, we present the mathematical details of the new Model Predictive Spread Acceleration (MPSAG) design. In this design, we consider general nonlinear systems in discrete form, the state and output dynamics of which are given by

$$X_{k+1} = F_k(X_k, U_k) \tag{9}$$

$$Y_k = h(X_k) \tag{10}$$

where $X \in \Re^n$, $U \in \Re^m$, $Y \in \Re^p$ and $k = 1, 2, \ldots, N$ are the time steps. The primary objective is to come up with a suitable control history U_k, $k = 1, 2, \ldots, N-1$, so that the output at the final time step Y_N goes to a desired value Y_N^*, i.e. $Y_N \to Y_N^*$. In addition, we aim to achieve this task with minimum control effort.

For the MPSAG technique presented here, one needs to start from a "guess history" of the control solution. With the application of such a guess history, obviously the objective is not expected to be met, and hence, there is a need to improve this solution. In this section, we present a way to compute an error history of the control variable, which needs to be subtracted from the previous history to get an improved control history. This iteration continues until the objective is met (i.e. the algorithm converges), i.e. $Y_N \to Y_N^*$. Note that the technique presented here comes up with an error history in closed form, and hence, the computational requirement is less and the algorithm can be used online.

To meet the objective $Y_N \to Y_N^*$, first we define the error in the output as $\delta Y_N \triangleq Y_N - Y_N^*$. Next, using small error approximation we write

$$\delta Y_N \cong dY_N = \left[\frac{\partial Y_N}{\partial X_N}\right] dX_N \tag{11}$$

However from (9), we can write the error in state at time step $(k+1)$ as

$$dX_{k+1} = \left[\frac{\partial F_k}{\partial X_k}\right] dX_k + \left[\frac{\partial F_k}{\partial U_k}\right] dU_k \qquad (12)$$

where dX_k and dU_k are the error of state and control at time step k respectively. Expanding dX_N as in (12) and substituting it in (11), we get

$$dY_N = \left[\frac{\partial Y_N}{\partial X_N}\right]\left(\left[\frac{\partial F_{N-1}}{\partial X_{N-1}}\right] dX_{N-1} + \left[\frac{\partial F_{N-1}}{\partial U_{N-1}}\right] dU_{N-1}\right) (13)$$

Similarly the error in state at time step $(N-1)$, dX_{N-1}, can be expanded in terms of the errors in state and control at time step $(N-2)$ and (13) can be re-written as

$$dY_N = \left[\frac{\partial Y_N}{\partial X_N}\right]\left[\frac{\partial F_{N-1}}{\partial X_{N-1}}\right]\left(\left[\frac{\partial F_{N-2}}{\partial X_{N-2}}\right] dX_{N-2} + \left[\frac{\partial F_{N-2}}{\partial U_{N-2}}\right] dU_{N-2}\right)$$
$$+ \left[\frac{\partial Y_N}{\partial X_N}\right]\left[\frac{\partial F_{N-1}}{\partial U_{N-1}}\right] dU_{N-1}$$

Next, dX_{N-2} can be expanded in terms of dX_{N-3} and dU_{N-3} and so on. Continuing the process $k = 1$, we can write

$$dY_N = A\, dX_1 + B_1 dU_1 + B_2 dU_2 + \ldots + B_{N-1} dU_{N-1} \qquad (14)$$

$$\text{where } A = \left[\frac{\partial Y_N}{\partial X_N}\right]\left[\frac{\partial F_{N-1}}{\partial X_{N-1}}\right] \cdots \left[\frac{\partial F_1}{\partial X_1}\right] \qquad (15)$$

$$B_k = \left[\frac{\partial Y_N}{\partial X_N}\right]\left[\frac{\partial F_{N-1}}{\partial X_{N-1}}\right] \cdots \left[\frac{\partial F_{k+1}}{\partial X_{k+1}}\right]\left[\frac{\partial F_k}{\partial U_k}\right],$$

$$k = 1,\ldots,N-1$$

Since the initial condition is assumed to be specified, there is no error in the first term; which means $dX_1 = 0$. With this (14) reduces to

$$dY_N = B_1 dU_1 + B_2 dU_2 + \ldots + B_{N-1} dU_{N-1} = \sum_{k=1}^{N-1} B_k dU_k (16)$$

Note that while deriving (16), we have assumed that the control (decision) variables at each time steps are independent of the previous values of states or control. At this point, we would like to point out that if one evaluates each of the B_k, $k = 1,\ldots,(N-1)$ as in (15), it will be a computationally intensive tasks (especially when N is high). However, fortunately it is possible to compute them recursively. For doing this, first we define B_{N-1}^0 as follows

$$B_{N-1}^0 = \left[\frac{\partial Y_N}{\partial X_N}\right] \qquad (17)$$

Next we compute B_k^0, $k = (N-2),(N-3),\ldots,1$ as

$$B_k^0 = B_{k+1}^0 \left[\frac{\partial F_{k+1}}{\partial X_{k+1}}\right] \qquad (18)$$

Finally, B_k, $k = (N-2),(N-3),\ldots,1$ can be computed as

$$B_k = B_k^0 \left[\frac{\partial F_k}{\partial U_k}\right] \qquad (19)$$

Equation (17)-(19) provides a recursive way of computing B_k^0, $k = (N-1),(N-2),\ldots,1$, and hence, it leads to saving of

computational time enormously. With respect to this equation for error in output (16) the formulation can be extended to linear parameterizations. In this formulation the control is considered to be linear function of $t_{g_o} \triangleq t_f - t$

$$U_k = a t_{go_k} + b \qquad (20)$$
$$U_k = U_k^0 - dU_k \qquad (21)$$

Therefore the error in control can be given as

$$dU_k = U_k^0 - U_k \qquad (22)$$
$$= (a_0 t_{go_k} + b_0) - (a t_{go_k} + b)$$
$$= (a_0 - a)t_{go_k} + (b_0 - b)$$
$$\qquad (23)$$

Substituting for dU_k for $k = 1,\ldots,N-1$ in (16) we get

$$B_\lambda - \left(\sum_{k=1}^{N-1} B_k t_{go_k}\right) a - \left(\sum_{k=1}^{N-1} B_k\right) b = dY_N$$

$$C_y a + D_y b = B_\lambda - dY_N \qquad (24)$$

where

$$B_\lambda \triangleq \left(B_1 U_1^0 + B_2 U_2^0 + ..B_{N-1} U_{N-1}^0\right) \qquad (25)$$

$$C_y \triangleq \left(\sum_{k=1}^{N-1} B_k t_{go_k}\right) \qquad (26)$$

$$D_y \triangleq \left(\sum_{k=1}^{N-1} B_k\right) \qquad (27)$$

At this point, we would like to point out that we have used "small error approximation" in deriving the closed form control update. The approximation may not hold good in general. Hence the process needs to be repeated in an iterative manner before one arrives at the converged (optimal) solution, which is define as the solution when $Y_N \to Y_N^*$. However to minimize computational time, only one iteration may be carried out at each instant of time following the principle of "iteration unfolding" (McHenry et al. (1979)).

3.1 Mathematical Formulation of MPSAG Applied to Lag Free System

The system dynamics (3) can be discretized using Euler integration (Atkinson (2001)) as

$$z_N(k+1) = z_N(k) + \frac{\Delta t V_m sin(\theta_{m_N}^* \theta_{m_N})}{z^*} \qquad (28)$$

$$\theta_{m_N}(k+1) = \theta_{m_N}(k) + \frac{\Delta t a_m}{V_m \theta_{m_N}^*} \qquad (29)$$

The required expressions to compute dx_{k+1} from (12) are

$$\frac{\partial F_k}{\partial X_k} = \begin{bmatrix} 1 & \frac{\Delta t V_m \theta_{m_N}^* cos(\theta_{m_N}^* \theta_{m_N})}{z^*} \\ 0 & 1 \end{bmatrix}, \frac{\partial F_k}{\partial U_k} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (30)$$

Table 1. Parameter Values

Symbol	Parameter Name	Values
x_{m_0}, z_{m_0}	Missile position	0m 0m
x_{t_0}, z_{t_0}	Target Position	0m , 4km
θ_{m_0}	Initial heading angle	90^0

3.2 Mathematical Formulation of MPSAG Applied to First Order Autopilot

The system dynamics (8) can be discretized making use of Euler integration (Atkinson (2001))

$$z_N(k+1) = z_N(k) + \frac{\Delta t V_m sin(\theta_m^* \theta_{m_N})}{z^*}$$

$$\theta_{m_N}(k+1) = \theta_{m_N}(k) + \Delta t \left(\frac{a_m}{V_m} - \frac{e^{\frac{-t}{\tau}}(a_{m_0} - a_{c_0})}{V_m} \right)$$

The required expressions to compute dx_{k+1} from (12) are $\frac{\partial F_k}{\partial X_k}$ and $\frac{\partial F_k}{\partial U_k}$ are written as

$$\frac{\partial F_k}{\partial X_k} = \begin{bmatrix} 1 & \dfrac{\Delta t V_m \theta_m^* cos(\theta_m^* \theta_{mN})}{z^*} \\ 0 & 1 \end{bmatrix}$$

$$\frac{\partial F_k}{\partial U_k} = \begin{bmatrix} 0 \\ \dfrac{\Delta t}{V_m} \end{bmatrix}$$

The main goal of the problem is to make $Y_N \rightarrow Y_N^*$. For this particular problem $Y_N = [\, z \;\; \theta_m \,]$ and $Y_N^* = [\, 0 \;\; \theta_{mf} \,]$

4. SIMULATION RESULTS

The main assumptions behind the simulations are

- Missile velocity is assumed as constant
- Target is assumed to be stationary
- The missile trajectory is assumed as a third order polynomial for time-to-go estimation
- Drag and gravity components are neglected

The missile velocity is taken as a constant value of $300m/sec$ and the impact angle varies from -90 to 90. The various initial values used for simulation are tabulated as in Table.1

Accurate estimation of time-to-go is very important because poor estimation of time-to-go severely degrades the guidance performance (Ryoo et al. (2006)-Ryoo et al. (2005)). The conventional method used for calculating time-to-go is the range over closing velocity. This method is a good estimate for PNG type guidance law. But for impact angle based guidance law, this method is not accurate because missile trajectory is curved in general. One can refer the details of time-to-go estimation available in the literature (Ryoo et al. (2006)-Song & Shin (1999)), which is also included as an appendix in this paper. Time-to-go estimation given in appendix is used for obtaining the missile optimal trajectories. The time constant(τ) for first order system is taken as 0.2 sec. The simulation results are obtained as shown in Fig. 2 and Fig. 3respectively. The results shows the comparative plot between linearized system and MPSAG technique for first order delay system at different impact angles and at a range of $5km$ (In all comparative plots solid line is used to represent MPSAG technique and dotted
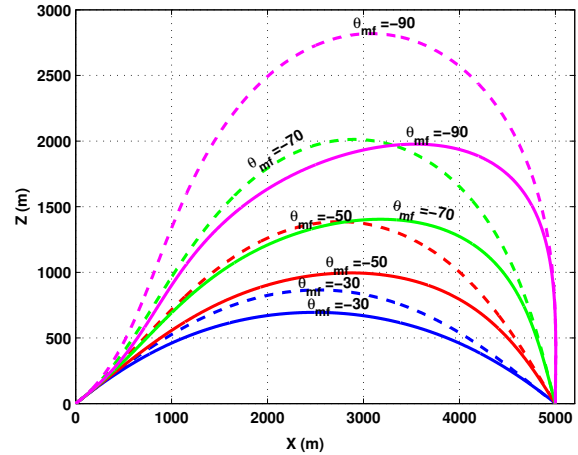

Fig. 2. Comparison of optimal trajectories between linearized method and MPSAG approach for different impact angle of the first order delay system for a range of 5km
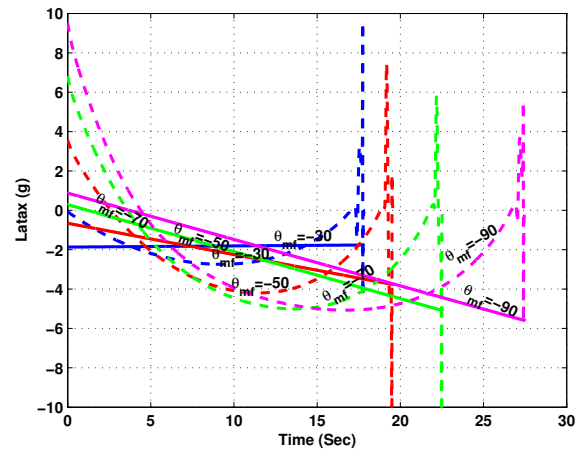

Fig. 3. Comparison between commanded Latax for linearized method and MPSAG Approach for different impact angle of first order delay system for a range of 5km

line for linearized method). The respective plots are drawn for different impact angles $-30^0, -50^0, -70^0, -90^0$. The results in Fig. 6 represents the comparison plot between linearized system and the system using MPSAG technique for different ranges of $5km, 6km$ and$7km$. These are generated taking an impact angle of $-90degrees$. The results in Fig. 7 represents the comparison plot of guidance command between linearized system and system using MPSAG technique for different ranges of $5km, 6km$ and $7km$. The singularity at the end of guidance command is completely avoided by using MPSAG technique. As it is evident from the latax comparative plot that positive-negative correction required for guidance command is less for MPSAG technique compared to the linearized method. The state responses and engagement responses are as shown in Fig. 4 and Fig. 5 respectively. The figures are generated for different initial conditions such as $\theta_{m_0} = 90$ and $\theta_{m_0} = 30$. It is clear from the subplot that the goals like miss distance and impact angle are satisfied and the missile is always hitting the target at the desired range of 5km, whatever be the heading angle. Fig. 8 represents the comparison of commanded and achieved latax for different ranges.
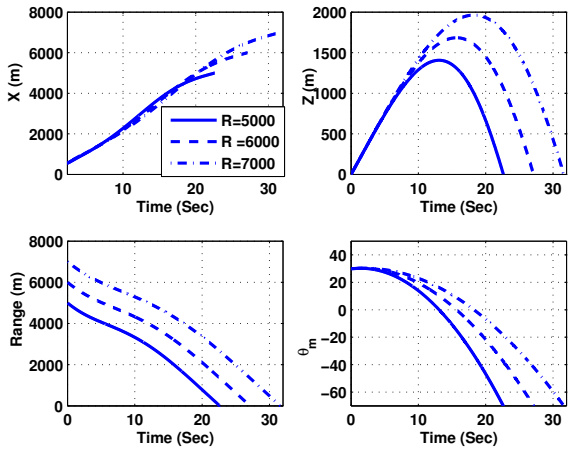
Fig. 4. subplot of responses for a first order system for $\theta_{m_0} = 30$, $\theta_{mf} = -70$ a)X Vs Time b)Z Vs Time c)R Vs Time d) θ_m Vs Time
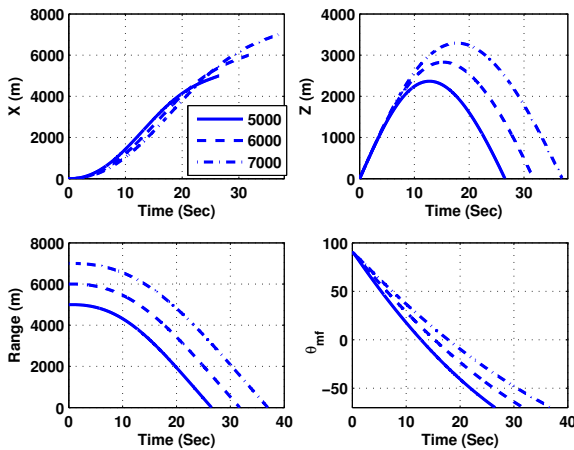

Fig. 5. subplot of responses for a first order system for $\theta_{m_0} = 90$, $\theta_{mf} = -70$ a) X Vs Time b)Z Vs Time c)R Vs Time d)θ_m Vs Time
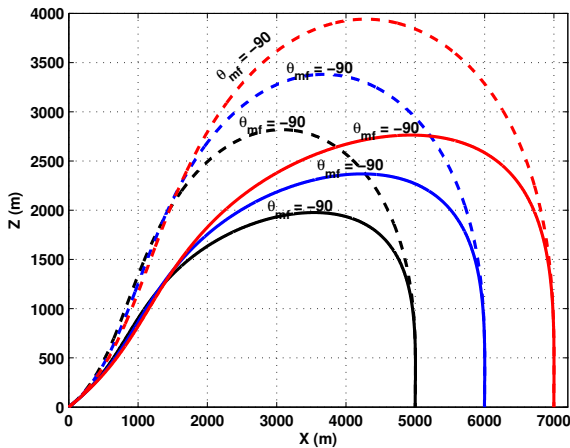

Fig. 6. Comparison of optimal trajectories between linearized method and MPSAG approach for different ranges
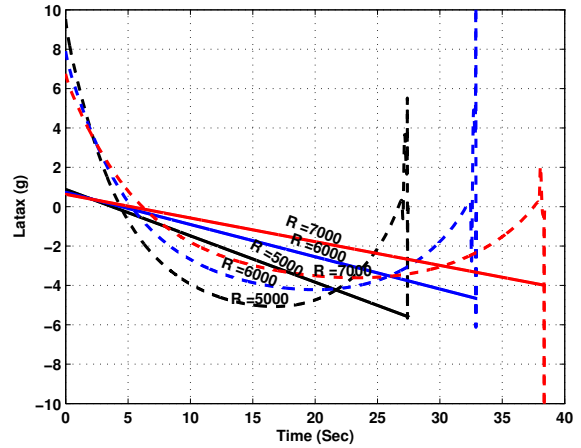

Fig. 7. Comparison between commanded Latax for linearized method and MPSAG Approach for different ranges
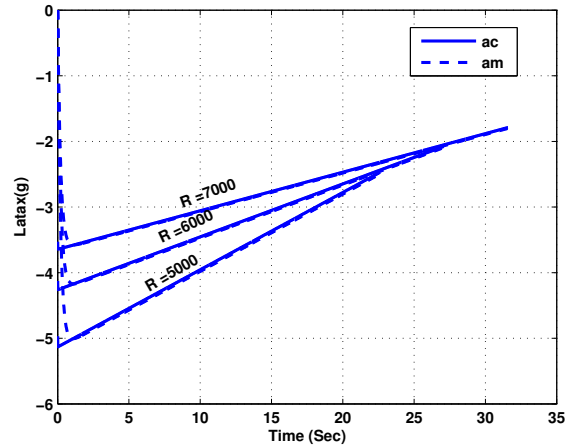

Fig. 8. Comparison between commanded Missile Latax and Achieved Missile Latax

5. CONCLUSIONS

The paper discusses about the nonlinear optimal guidance law using MPSAG technique, which is a new technique combining model predictive control theory and spread acceleration guidance. The paper discusses about stationary target however the basic philosophy presented here can possibly be extended to moving target as well. The technique leads to a rapid update of guidance history, and hence can be implemented online. Optimal guidance law using linearized dynamics is already present in the literature, where the system dynamics is linearized and the latax is obtained using the linearized dynamics. However the guidance law presented here using optimal control theory without linearizing the system dynamics. The guidance law is tested by assuming first order autopilot. In the design the guidance law is able to hit the target at the desired range of 5 km. It is important to note that the approach presented leads to an approximate closed form sub optimal solution of the guidance history update for the given nonlinear problem. Hence, the computational requirements are low and it can be implemented online. The technique does not demand very high latax, and it is able to meet the required criteria of minimum miss distance.

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6. APPENDIX -REVIEW OF LINEARIZED METHOD

The linearized method proposes an optimal guidance law with impact angle constraint with linearized dynamics (Ryoo et al. (2005), Ryoo et al. (2006)). It gives a closed form solutions for controlling the impact angle as well as terminal miss distance. It also proposes a generalized formulation of energy minimization optimal guidance law for constant speed missiles with an arbitrary system order for controlling both impact angle and miss distance. Optimal guidance law for first order delay system are derived in this paper for completeness.

6.1 Guidance Law for First order delay System

System dynamics for first order delay system is given by Ryoo et al. (2005)

$$\dot{z} = V_m(t)\sin(\theta_m(t))$$
$$V_m(t)\dot{\theta}_m(t) = -a_m(t)$$
$$\dot{a}_m = \frac{u - a_c}{\tau}$$

Optimal guidance law in this case is given as

$$u = -V_m[-t_{go}w_1z + w_2\theta_m + w_3\theta_{mf}] + w_4a_m \qquad (31)$$

The following definitions are used in the optimal guidance law formulation for a first order delay system

$$w_1 \triangleq (1/\Delta)(s_1K_4D_1 - V_mS_2K_3D_2)$$
$$w_2 \triangleq (1/\Delta)(s_1D_1(t_{go}K_4 - K_2) + V_ms_2D_2(K_1 - t_{go}K_3))$$
$$w_3 \triangleq (1/\Delta)(s_1K_2D_1 - V_mS_2K_1D_2)$$
$$w_4 \triangleq (1/\alpha\Delta)(s_1D_1(K_2D_2 - K_4D_1) + V_ms_2D_2(K_3D_1 - K_1D_2))$$
$$\Delta \triangleq K_1K_4 - K_2K_3$$
$$K_1 \triangleq 1 + s_1(1/2(\alpha^3) + t_{go}/(\alpha^2) - t_{go}^2/\alpha + t_{go}^3/3$$
$$\quad -2t_{go}e^{-\alpha t_{go}}/\alpha^2 - e^{-2\alpha t_{go}}/2\alpha^3)$$
$$K_2 \triangleq V_ms_2(1/2\alpha^2 - t_{go}/\alpha + t_{go}^2/\alpha + t_{go}^3/3 - 2t_{go}e^{-2\alpha t_{go}}/2\alpha^2)$$
$$K_3 \triangleq s_1K_2/V_ms_2$$
$$K_4 \triangleq 1 + V_ms_2(t_{go} + 2e^{-\alpha t_{go}}/\alpha - 3/2\alpha - e^{-2\alpha t_{go}}/2\alpha$$
$$D_1 \triangleq t_{go} - 1/\alpha + e^{-\alpha t_{go}}/\alpha$$
$$D_2 \triangleq (1/V_m)(1 - e^{-\alpha t_{go}})$$
$$\alpha \triangleq 1/\tau$$

Note that for the first order lag system the guidance command goes to zero as $t_{go} \to 0$. One can refer (Ryoo et al. (2005)) for more details.

6.2 Time-to-go Estimation

The method of determining time-to-go is to find the range over average velocity, which denotes the projected velocity on the line-of-sight (Ryoo et al. (2005)). This method is superior compared to the other methods available in literatures. The projected velocity on the line-of-sight is calculated as $\bar{V}_m\cos\bar{\theta}_m$. Expanding $\cos\bar{\theta}_m$ using binomial expansion, the expressions for $\bar{V}_m$ and t_{go} are

$$\bar{V}_m = \frac{1}{R}\int_0^R V_m\cos\bar{\theta}_m dx \qquad (32)$$

$$= V_m\left[1 - \frac{\bar{\theta}_m^2 + \bar{\theta}_{mf}^2}{15} + \frac{\bar{\theta}_m\bar{\theta}_{mf}f}{30} + \frac{\bar{\theta}_m^4 + \bar{\theta}_{mf}^4}{420}\right.$$
$$\left. - \frac{\bar{\theta}_m\bar{\theta}_{mf}(\bar{\theta}_m^2 + \bar{\theta}_{mf}^2 - \bar{\theta}_m\bar{\theta}_{mf})}{840}\right]$$

$$t_{go} = \frac{R}{\bar{V}_m} \qquad (33)$$

where $\bar{\theta}_m$ and $\bar{\theta}_{mf}$ are defined as $\bar{\theta}_m \triangleq \theta_m + \theta$ and $\bar{\theta}_{mf} \triangleq \theta_{mf} + \theta$, respectively