**Proceedings of the 17th World Congress**
**The International Federation of Automatic Control**
**Seoul, Korea, July 6-11, 2008**

# An anti-windup based approach to the control of manufacturing systems

**W.A.P. van den Bremer, R.A. van den Berg, A.Y. Pogromsky, J.E. Rooda**

*Department of Mechanical Engineering*
*Eindhoven University of Technology*
*P.O.Box 513, 5600 MB Eindhoven, The Netherlands*
*W.A.P.v.d.Bremer@student.tue.nl,*
*{R.A.v.d.Berg, A.Pogromsky, J.E.Rooda}@tue.nl*

**Abstract:** This paper focusses on the problem of controlling the production rate of a discrete-event manufacturing system such that the total production meets a certain reference demand. There is a need for a simple and structured approach to design controllers for manufacturing systems. Therefore, we choose for a continuous approximation model of a manufacturing machine, which is controlled using a PI controller with anti-windup. Convergent systems theory and a nonlinear extension of frequency response functions are used to evaluate the performance of this continuous approximation model with the proposed controller. Next, the controller is implemented on the discrete-event system and performance is evaluated using discrete-event simulations. Simulation results of the manufacturing system with an anti-windup controller agree with the observations that were made during the frequency domain performance analysis of the continuous approximation.

Keywords: Manufacturing Systems, Discrete-event systems, Feedback control, Anti-windup, Harmonic response characteristics

## 1. INTRODUCTION

Today's manufacturing systems have become highly dynamic and complex. In order to stay competitive, manufacturing systems must use a good control strategy to rapidly respond to demand fluctuations. Simple discrete-event manufacturing systems can be controlled by policies such as PUSH, CONWIP or Kanban (see e.g. Hopp and Spearman [2000]). However, as manufacturing systems become more complex, these policies become less effective.

Another control approach is based on the use of ordinary differential equations (ODE's) to model a manufacturing system (see e.g. Alvarez-Vargaz et al. [1994], Boukas [2006]). Such ODE models are a continuous approximation of the discrete-event system and as a result the control problem is much simpler. Moreover, control theory for ODE's is widely available, which makes it attractive to work with such models.

In such ODE models, a manufacturing machine is usually interpreted as an integrator, where the cumulative number of finished product is the integral of the production rate. There are, however, some restrictions on the production rate that should be taken into account in the model in order to maintain a good representation of the actual manufacturing system. A machine cannot produce products at any rate: the production rate must be nonnegative and cannot exceed some maximum production rate, due to the limited capacity of the machine. These restrictions, which in fact render the system nonlinear, can be interpreted as saturation of the production rate, something that in practice is common to all actuators.

A well-accepted control strategy that uses flow models and that is able to account for the limited capacity of the system is Model (based) Predictive Control (MPC). Examples of MPC for reentrant manufacturing systems can be found in Vargas-Villamil et al. [2003] and references therein. Such control strategies can become complex and computationally expensive. Moreover, the performance of these strategies depends on the predictions of the future demand. Such predictions can be hard to make and are often inaccurate.

Therefore, there is need for a simple, straightforward control strategy for manufacturing systems, that does not rely on predictions of the future demand. We try to derive such a strategy by using feedback control of continuous systems. In this paper, we choose to use a simple PI controller to set the production rate in the ODE model of a manufacturing system such that the production meets a certain demand. The use of an integral action in the controller, combined with input saturation gives rise to certain problems, such as integrator windup and appropriate performance evaluation for this nonlinear system. These problems will be discussed in this paper and a suitable solution will be provided.

The combination of input saturation and the integrator in the PI controller leads to a phenomenon called integrator windup. When the actuator saturates, the effective control signal cannot exceed some value, which affects the system behavior and therefore again the control signal. As a result of this, the closed-loop performance of the system can deteriorate, and in some situations the system can even become unstable. By adding a so-called anti-windup controller to the system, this loss of performance can be counteracted by "turning off" the integrator in the controller when the machine saturates. In the past, many anti-windup controllers have been proposed in literature. In this article we use the anti-windup design as presented in van den Berg et al. [2006]. Since this design is based on convergent systems, the performance evaluation of the system simplifies.

In Pavlov et al. [2007], nonlinear equivalents of the magnitude of the linear sensitivity functions are given for uniformly convergent systems. These are the so-called generalized sensitivity functions, which show to be useful performance measures for the considered anti-windup manufacturing system. We use the method as presented in van den Berg et al. [2007], based on harmonic linearization to efficiently and accurately calculate bounds on these generalized sensitivity functions.

The presented control strategy is first evaluated in the frequency domain by means of the generalized sensitivity functions. Subsequently, the controller is implemented in the discrete-event domain and the results are evaluated.

The outline of this paper is as follows. In Section 2 the investigated manufacturing system is described. It is shown how this system can be modeled by a continuous model and how it can be controlled. Section 3 discusses the conditions for uniform convergency and the frequency domain analysis of the anti-windup controlled manufacturing system. In Section 4 it is illustrated how bounds on the generalized sensitivity functions can be efficiently calculated for the manufacturing system. Furthermore, the controller is implemented in the discrete-event domain and it is shown by means of discrete-event simulations that anti-windup control can be successfully applied to control a discrete-event manufacturing system. Finally, Section 5 concludes the paper.

## 2. CONTROL OF A MANUFACTURING MACHINE

This section introduces the control problem of a discrete-event manufacturing machine that is considered throughout this paper. Here the manufacturing machine is approximated by a continuous model and it will be shown that the closed loop system with PI control can be formulated as a system for which convergency has been proven in van den Berg et al. [2006].

Consider a simple manufacturing system, as depicted in Fig. 1, consisting of one machine producing lots from an infinite capacity buffer. It is assumed that the supply of raw materials to the buffer is always sufficient, such that the machine never starves.

The machine processes lots from the buffer with a process rate $u(t)$, which can be interpreted as the velocity at which the machine operates. The relation for the cumulative number of products that has been processed by the machine, $y(t)$, is given as

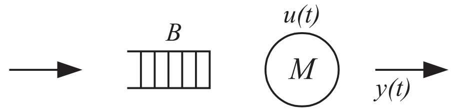


Fig. 1. Manufacturing system with buffer $B$ and machine $M$.

$$\dot{y}(t) = u(t). \tag{1}$$

The machine can be interpreted as a pure integrator. By using a feedback controller to set the production rate $u(t)$ one can control the cumulative output $y(t)$ such that the machine can track a given desired production $y_d(t)$.

A possible reference production is given by

$$y_d(t) = u_d t + y_{d0} + r(t), \tag{2}$$

which has a part that is linear, where $u_d$ is the desired production rate and $y_{d0}$ is the desired production at $t = 0$ and a bounded term $r(t)$, which can be interpreted as a – for instance seasonal – fluctuation of the demand.

It can be argued by means of the final value theorem from linear control theory (see for instance Franklin et al. [2001]) that for $r(t) = 0$ a controller with integral action should be used to track the error $e(t) = y_d(t) - y(t)$ to zero.

Therefore, we also choose to use an integral action for the case where $r(t) \neq 0$. The simplest controller with integral action is a PI controller for which the controller output at time $t$ is given by:

$$y_c(t) = k_P e(t) + k_I \int_0^t e(\tau) d\tau, \tag{3}$$

with $k_P$ and $k_I$ the controller parameters. Using the Routh-Hurwitz stability criterion, it can be concluded that the closed loop system is stable if $k_P, k_I > 0$. This is a necessary and sufficient condition for *stability.* A specific choice of these parameters has to be made based on *performance* criteria, for instance certain demands for the sensitivity and complementary sensitivity.

In practice all physical actuators saturate at some level. For the machine in our simple manufacturing system we can state that the production rate cannot become negative and that it cannot exceed some maximum rate $u_{max}$:

$$0 \leq u(t) \leq u_{max}. \tag{4}$$

If we take $k = \frac{1}{2} u_{max}$ we can rewrite this constraint as

$$-k \leq u(t) - k \leq k.$$

Using the fact that $u(t) - k$ equals the saturated value of the controller output $y_c$ between $-k$ and $+k$, this can be written as

$$u(t) = \text{sat}_k(y_c) + k, \tag{5}$$

where $\text{sat}_k(\cdot)$ is the saturation function

$$\text{sat}_k(x) = \text{sign}(x)\min(k, |x|).$$

Instead of working with unbounded input $y_d$ and unbounded output $y$, we rather work with bounded signals. We are interested in how the disturbance $r(t)$ is propagated through the system, so we no longer want to have $y_d(t)$ as reference and $y(t)$ as output. In order to have $r(t)$ as the input to the system, we subtract the linear part $u_d \cdot t + y_{d0}$ from the desired output $y_d(t)$, which leaves the bounded term $r(t)$.

The linear term $u_d t + y_{d0}$ also needs to be subtracted from the output $y(t)$, which yields

$$y(t) - u_d t - y_{d0} = -e(t) + r(t) \stackrel{\text{def}}{=} z(t),$$

where $z(t)$ is defined to be the normalized output. Differentiating $z(t)$ with respect to time and using (5) results in

$$\dot{z}(t) = u - u_d = \text{sat}_k(y_c) + k - u_d. \qquad (6)$$

The constant $d$ is defined to be $d = k - u_d$. In the remainder of this paper, it is assumed that $k = u_d$, which means that we have an average utilization of 50% and $d = 0$. Note that the initial conditions for $y(t)$ and $z(t)$ need to be equivalent:

$$y(t = 0) = 0 \quad \Rightarrow \quad z(t = 0) = -y_{d0}.$$

The resulting system is depicted in Fig. 2, which is equivalent to the system with original coordinates, but we now have bounded reference $r(t)$ and normalized output $z(t)$. This system can be used to investigate how the bounded disturbance $r(t)$ is propagated by the system by looking at the normalized output $z(t)$.

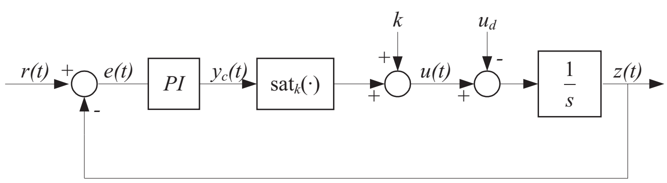


Fig. 2. Normalized system with saturation.

The combination of actuator saturation and the integral action in the PI-controller leads to a phenomenon called "integrator windup" (see for instance Franklin et al. [2001]. When the actuator saturates, the effective control signal cannot exceed some value, which means that it takes longer for the control action to take effect. This means that for a certain period there remains a tracking error, which the integrator keeps integrating. This is called "integrator windup" and it causes the controller output to grow. This increase in the controller signal has no effect, because the actuator is already at its saturation limit. A considerable negative error is required to bring the integrator output back within the proportional band where the control action is not saturated.

The solution to this problem is the so-called integrator anti-windup, which "turns off" the integral action in the controller when the actuator saturates. One possible way of implementing anti-windup is depicted in Fig. 3. The value for the static anti-windup gain $k_A$ needs to be chosen, based on performance criteria.

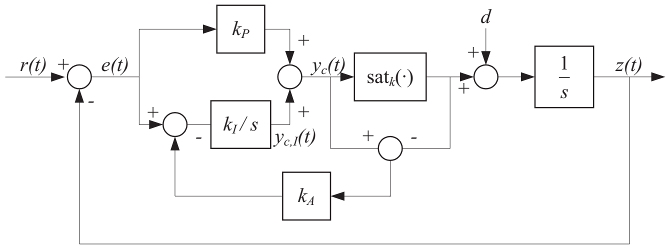


Fig. 3. Implementation of anti-windup for the normalized system.

The closed loop dynamics of the system in Fig. 3 can be written in the following Lur'e form, where we leave $d$ out of the equations as $d = 0$:

$$\begin{aligned} \dot{x} &= Ax - B\phi(y_c) + Fu \\ y_c &= Cx + Du \\ z &= Hx + Eu, \end{aligned} \qquad (7)$$

where $x = \begin{bmatrix} z \\ y_{c,I} \end{bmatrix}$, with $z$ being the normalized output and $y_{c,I}$ the integrator part of controller (3), $u(t) = r(t)$ and $\phi(\cdot) = \text{sat}_k(\cdot)$ with

$$A = \begin{bmatrix} 0 & 0 \\ -k_I + k_I k_A k_P & -k_I k_A \end{bmatrix}, \; B = \begin{bmatrix} -1 \\ -k_I k_A \end{bmatrix},$$

$$F = \begin{bmatrix} 0 \\ k_I - k_I k_A k_P \end{bmatrix}, \qquad C = [\,-k_P \;\; 1\,],$$

$$D = [\,k_P\,], \; H = [\,1 \;\; 0\,], \; E = [\,0\,].$$

An important observation is the fact that the introduction of saturation has rendered the system nonlinear. It is known that for a stable *linear* time-invariant (LTI) system with a given harmonic input, the solution will converge to a unique harmonic steady-state solution. This harmonic steady-state solution only depends on the input signal and is independent of the initial conditions. In other words, for a given input signal, solutions with different initial conditions will converge to the same unique steady-state solution.

This is in general not true for nonlinear systems as multiple solutions might coexist. There is a, however, a class of nonlinear systems for which a *unique* bounded steady-state solution exists, which is independent of the initial conditions, and only depends on the input signal. Roughly speaking, this class of systems is referred to as the class of convergent systems. In the next section we will address conditions for which (7) is uniformly convergent. The fact that the manufacturing system is uniformly convergent allows us to analyse the performance as discussed in the next section.

## 3. FREQUENCY DOMAIN ANALYSIS OF CONVERGENT SYSTEMS

As was stated earlier, certain choices for the controller parameters $k_P, k_I, k_A$ need to be made, based on performance criteria. For linear systems it is useful to investigate the closed loop behavior captured in the sensitivity and complementary sensitivity functions. These functions allow the quantification of the sensitivity of the closed-loop system to measurement noise and its tracking properties. A similar performance indicator for uniformly convergent nonlinear systems was introduced in Pavlov et al. [2007].

Based on the results in van den Berg et al. [2006], one can prove that for harmonic inputs $r(t) = b\sin(\omega t)$ the system (7) is uniformly convergent if $k_A k_P > 1$. The fact that system (7) is uniformly convergent means that for each input

$$r(t) = b\sin\omega t \tag{8}$$

there is one unique corresponding steady-state output response $\bar{z}_{b\omega}(t)$ and steady-state error $\bar{e}_{b\omega}(t)$. Details on the exact definition of (uniform) convergency can be found in van den Berg et al. [2006] and references therein.

*Definition 1.* (Pavlov et al. [2007]). For uniformly convergent nonlinear systems we can define the generalized sensitivity and generalized complementary sensitivity as follows:

$$\mathcal{S}(b,w) = \frac{\|\bar{e}_{b\omega}\|_2}{\|\bar{r}_{b\omega}\|_2},\ \mathcal{T}(b,w) = \frac{\|\bar{z}_{b\omega}\|_2}{\|\bar{r}_{b\omega}\|_2}. \tag{9}$$

Notice that due to the nonlinearity, $\mathcal{S}(b,w)$ and $\mathcal{T}(b,w)$ depend not only on the input frequency but also on the input amplitude, in contrast to the linear case, where the sensitivity functions are only a function of the frequency.

Simulations could now be used to obtain the steady-state responses for a range of input frequencies and amplitudes and calculate the generalized sensitivity functions. These simulations would, however, consume a considerable amount of time, especially if this has to be done for a range of control parameter values. In van den Berg et al. [2007] a computationally efficient method is presented, which finds bounds on the generalized sensitivity functions using harmonic linearization of system (7).

Bounds on $\|\bar{z}_{b\omega}\|_2$ can efficiently be calculated using harmonic linearization, such that bounds on the generalized complementary sensitivity are known.

First, using harmonic linearization the rms value of the linear approximation $\bar{\eta}(t)$ of the nonlinear output $\bar{z}(t)$ can be found as will be shown shortly hereafter. Next, an upper bound on the accuracy of the approximation $\|\bar{z}(t) - \bar{\eta}(t)\|_2$ is given. Finally, using the triangular inequality we know that

$$\|\bar{\eta}\|_2 - \|\bar{z} - \bar{\eta}\|_2 \le \|\bar{z}\|_2 \le \|\bar{\eta}\|_2 + \|\bar{z} - \bar{\eta}\|_2.$$

With the rms value of the input given by $\|\bar{r}_{b\omega}\|_2 = \frac{b}{\sqrt{2}}$, bounds on the generalized complementary sensitivity are known.

### 3.1 Harmonic linearization

Here we will briefly discuss how harmonic linearization – also referred to as the describing function method – can be used to approximate the nonlinear system (7). More details on well-posedness and accuracy of harmonic linearization of harmonically forced Lur'e systems can be found in van den Berg et al. [2007]. The nonlinear system (7) is approximated by the following linear system:

$$\dot{\xi} = A\xi - BK\left(a(b,\omega)\right)\phi(\zeta) + Fu \tag{10}$$
$$\zeta = C\xi + Du \tag{11}$$
$$\eta = H\xi + Eu, \tag{12}$$

where the gain $K$ is to be determined. In case matrix $A - BKC$ does not have eigenvalues on the imaginary axis, this system has a unique periodic limit solution with the corresponding approximated output equal to $\overline{\zeta} = a\sin(\omega t + \psi)$ for some amplitude $a = a(b,\omega) > 0$ and some phase $\psi$. For the saturation nonlinearity $\phi(\cdot) = \mathrm{sat}_k(\cdot)$ the equivalent gain $K$ is given by (see e.g. Khalil [1996]):

$$K(a) = \begin{cases} 1, & a \le k \\ \dfrac{2}{\pi}\left(\sin^{-1}\left(\dfrac{k}{a}\right) + \dfrac{k}{a}\sqrt{1 - \left(\dfrac{k}{a}\right)^2}\right), & a > k \end{cases} \tag{13}$$

The relation between the output amplitude $a$ and the input amplitude $b$ and frequency $\omega$ is given by the *harmonic balance equation*

$$|1 + K(a)G(i\omega)|^2 a^2 = |C(i\omega I_n - A)^{-1}F + D|^2 b^2. \tag{14}$$

If the amplitude $b$ and frequency $\omega$ of the input are given the right-hand side of this equation is known. Then we can (numerically) solve the left-hand side to obtain the amplitude $a$ of the output. We are only interested in positive and real solutions, because output amplitude $a$ can only be positive and real.

The left-hand side of (14) is a nonlinear function of $a$, due to which, in general, it is possible that there exist multiple solutions $a$ for one pair of $(b,\omega)$. If, however, $k_A k_P > 1$ there is one unique positive real solution $a(b,\omega)$ of (14) as shown in van den Berg et al. [2007].

The rms value of the approximated output $\overline{\eta}(t)$ is given by

$$\|\overline{\eta}\|_2 = \frac{b}{\sqrt{2}}\Big|H\left(i\omega I - \left(A - BK\left(a\left(b,\omega\right)\right)C\right)\right)^{-1} \left(F - BK\left(a\left(b,\omega\right)\right)D\right) + E\Big|, \tag{15}$$

which depends on both $b$ and $\omega$.

An upper bound on the accuracy $\|\bar{z} - \bar{\eta}\|_2$ of the approximation $\bar{\eta}$ is given in Theorem 7 of van den Berg et al. [2007]. The upper bound on the accuracy of the approximation can be used to obtain bounds on the generalized complementary sensitivity $\mathcal{T}(b,\omega)$.

To estimate the generalized sensitivity $\mathcal{S}(b,\omega)$ as given by (9) in a similar way, bounds on $\bar{e}(t)$ have to be known. In order to estimate the bounds where $\bar{e}(t)$ should lay between we need to choose matrices $H$ and $E$ such that $Hx + Eu$ represents the error $\bar{e}(t) = \bar{r}(t) - \bar{z}(t)$. Then, applying the same technique as discussed above for the generalized complementary sensitivity, bound on the generalized sensitivity $\mathcal{S}(b, \omega)$ can be calculated.

## 4. NUMERICAL EXAMPLE: PERFORMANCE OF MANUFACTURING SYSTEM

In this section it is demonstrated how the theory from Section 3 can be used to analyse the performance of a manufacturing system.

We consider the system given in Fig. 2, of which the closed loop dynamics are described by (7). We choose controller parameters $k_P = 10$, $k_I = 20$ and $k_A = 0.5$, such that the system is uniformly convergent.

The maximum production rate of the machine is $u_{\max} = 25.0$ and thus $k = 12.5$. The nominal desired production rate $u_d$ is chosen equal to $k$, i.e. the system has a utilization of 50% and $d = 0$. We consider different demand fluctuations $r(t) = b\sin(\omega t)$, with amplitudes $b \in \{2.5, 5.0, 12.5\}$ for a range of frequencies $\omega \in [0.1, 20]$ rad/sec. The initial condition ($y_{d0}$) does not matter, as the system is uniformly convergent and the limit solution is independent of the initial conditions.

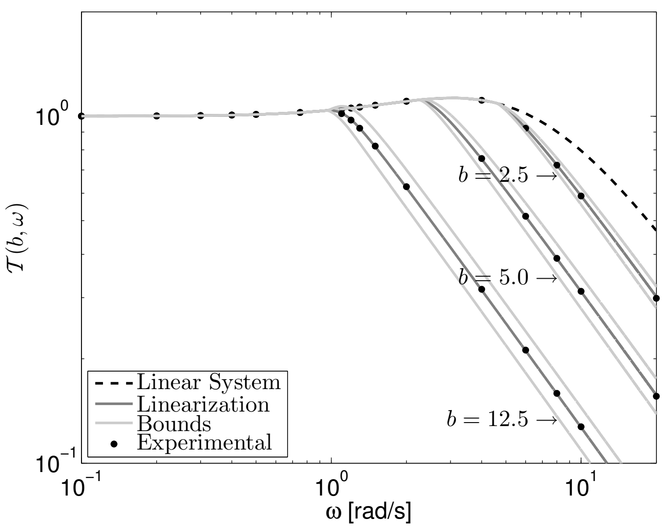


Fig. 4. Generalized complementary sensitivity function $\mathcal{T}(b, w) = \frac{||\bar{z}_{b\omega}||_2}{||\bar{r}_{b\omega}||_2}$, for $b \in \{2.5, 5.0, 12.5\}$.

The results of the approach described in Section 3 for the generalized complementary sensitivity function are depicted in Fig. 4. The results show the bounds on the generalized complementary sensitivity function for different fluctuation amplitudes $b$. For comparison, the complementary sensitivity of the linear system without saturation has also been added.

The calculated bounds are also compared with the experimentally determined generalized sensitivity functions. For this purpose simulations were performed for a range of frequencies to obtain $\bar{z}(t)$ and thus $\mathcal{T}(b, \omega)$. The simulations take a considerable time, which confirms the statement that the approach based on harmonic linearization is more time efficient.

First of all, it can be observed that the generalized complementary sensitivity depends on the reference amplitude $b$, unlike the linear complementary sensitivity. It is clear that for $b\omega > u_d$ there is a drop in the closed loop tracking performance. For demand fluctuations with $b\omega > u_d$ it is required that the machine produces beyond its capacity to follow the reference. This explains the sudden drop in the closed loop performance of the system with input saturation.

For $b\omega < u_d$ it can be observed that the closed loop behavior of the nonlinear system is equal to that of the linear system without input saturation. This is because at these frequencies, the anti-windup controller keeps the control signal within the saturation limits and thus the system acts in a linear way.

The results also show that the approach used here is accurate, as all experimental results lay well within the error bounds. In fact, for this case we observe that the experimental results are really close to the complementary sensitivity of the harmonic linearization.

For low frequencies (roughly below 1 rad/sec) we have good tracking performance for these demand amplitudes. Moreover, a resonant peak is visible at a frequency around 3 rad/sec. This observation might help to understand the bullwhip effect, where demand fluctuations in a supply chain move and grow upstream (see Lee et al. [1997]). A small demand fluctuation at the customer end of the line can result in a large fluctuation upstream at the supplier and manufacturing side.

Results like these, are valuable when control parameters need to be chosen. Throughout this paper we have worked with a continuous approximation of a discrete-event manufacturing machine. Next, we want to use the derived controller to control a discrete-event production system.

### 4.1 Discrete-event simulation

To implement the controller in the discrete-event domain, a discrete-event simulation of the single machine with the derived controller is used. This is an accurate and well-accepted modeling technique in the analysis of manufacturing systems. The discrete-event model was constructed using $\chi$, a specification language developed at Eindhoven University of Technology (see van Beek et al. [2006]).

In the discrete-event case, we do not control the production rate of the machine, but we instead control the rate at which unfinished products arrive at the buffer in front of the machine. This means that we can still control the production rate of the system as a whole. There is another important difference between the continuous approximation and the discrete-event system. The approximated continuous machine produces products instantly, without delay, whereas in the discrete-event case, it takes a processing time $t_0 = \frac{1}{u_{\max}}$ to finish a lot. This means that the $u_{\max}$ is a measure of how quick the system can respond to demand fluctuations.

As an illustration of the bad behavior without anti-windup, we consider the same machine and PI controller

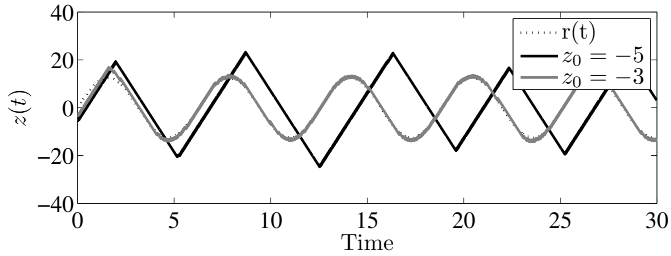


Fig. 5. Discrete-event simulation results for $k_A = 0.0$, and reference $b = 12.5, \omega = 1.0$.

as before, however *without* anti-windup, i.e. $k_A = 0.0$. We have the same desired production (2), with fluctuation amplitude $b = 12.5$ and frequency $\omega = 1.0$. The result of discrete-event simulations for different initial condition are shown in Fig. 5. The results demonstrate that without anti-windup $k_A > 1/k_P$ the steady-state solution depends on the initial conditions.

For the same machine and PI controller *with* anti-windup control action $k_A = 0.5$ the result of a discrete-event simulation is shown in Fig. 6 for a demand fluctuation parameters $b = 12.5$ and $\omega = 0.5$. This result demonstrates that the anti-windup controlled discrete-event manufacturing system is able to follow the demand fluctuation at low frequencies. This corresponds to the observations made before in Fig. 4, where we observed that the system has good tracking performance for these parameters $b$ and $\omega$.

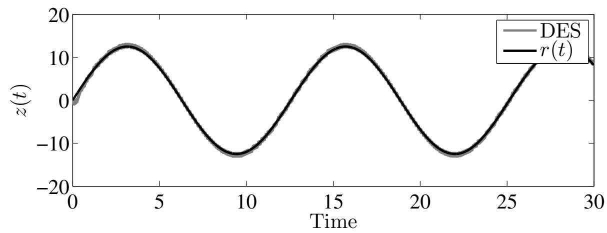


Fig. 6. Discrete-event simulation results for $k_A = 0.5$, and reference $b = 12.5, \omega = 0.5$.

Fig. 7 shows the result of a simulation for the same parameters, but now for a demand fluctuation with $b = 2.5$ and $\omega = 3.0$. It can be seen that in this case there is an overshoot of production. This confirms the observation of the resonant peak for these parameters in Fig. 4.

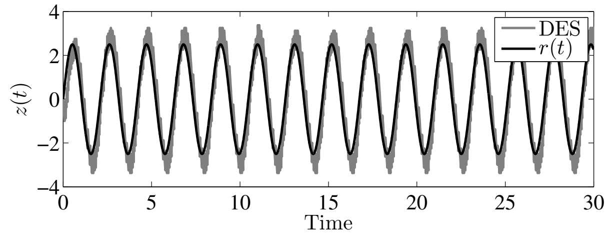


Fig. 7. Discrete-event simulation results for $k_A = 0.5$, and reference $b = 2.5, \omega = 3.0$.

## 5. CONCLUSION

In this paper we discussed the problem of controlling a discrete-event manufacturing system. Here we chose to use a continuous approximation model of a manufacturing machine and control this with a PI controller with anti-windup. For this system uniform convergency can be proven under certain conditions. For uniformly convergent systems it is possible to analyse the performance in the frequency domain by means of the generalized sensitivity functions. A method exists that efficiently calculates bounds on the generalized sensitivity functions. This method has been applied to the manufacturing system.

Finally the controller was implemented on the discrete-event system by means of discrete-event simulations. The simulation results confirmed that the application of a simple PI controller without anti-windup does not result in a unique steady-state solution. Discrete-event results with an anti-windup controller corresponded to the observations that were made during the performance analysis of the continuous approximation.

More research is needed to make the anti-windup control approach applicable to more complex manufacturing system, such as lines and networks with multiple machines. For such systems a comparison needs to be made between the anti-windup control approach and existing control strategies.

## REFERENCES

R. Alvarez-Vargaz, Y. Dallery, and R. David. A study of the continuous flow model of production lines with unreliable machines and finite buffers. *Journal of Manufacturing Systems*, 13(3):221–234, 1994.

E.K. Boukas. Manufacturing systems: LMI approach. *IEEE Transactions on Automatic Control*, 51(6):1014–1018, June 2006.

G.F. Franklin, D.J. Powell, and A. Emami-Naeini. *Feedback Control of Dynamic Systems.* Prentice Hall PTR, Upper Saddle River, NJ, USA, 2001.

W.J. Hopp and M.L. Spearman. *Factory Physics: Foundations of Manufacturing Management.* McGraw-Hill, New York, 2nd edition, 2000.

H.K. Khalil. *Nonlinear systems.* Prentice Hall, Upper Saddle River, 2nd edition, 1996.

Hau Lee, V. Padmanabhan, and Seungjin Whang. The bullwhip effect in supply chains. *Sloan Management Review*, 38(3):93–102, 1997.

A. Pavlov, N. v.d.Wouw, and H. Nijmeijer. Frequency response functions for nonlinear convergent systems. *IEEE Transactions on Automatic Control*, 52(6):1159–1165, 2007.

D.A. van Beek, K.L. Man, M.A. Reniers, J.E. Rooda, and R.R.H. Schiffelers. Syntax and consistent equation semantics of hybrid chi. *Journal of Logic and Algebraic Programming*, 68(1-2):129–210, 2006.

R.A. van den Berg, A.Y. Pogromsky, and J.E. Rooda. Convergent systems design: Anti-windup for marginally stable plants. In *Proceedings of 45th IEEE Conference on Decision and Control*, San Diego, 2006.

R.A. van den Berg, A.Y. Pogromsky, and J.E. Rooda. Well-posedness and accuracy of harmonic linearization for Lur'e systems. In *Proceedings of 46th IEEE Conference on Decision and Control*, New Orleans, 2007.

F. Vargas-Villamil, D. Rivera, and K. Kempf. A hierarchical approach to production control of reentrant semiconductor manufacturing lines. *IEEE Transactions on Control Systems Technology*, 11(4):578–587, 2003.