Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Measuring Optimization in Optimal Control of Flexible Aerospace Vehicles

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Abstract: The optimal control and parameters estimation of flexible vehicle motion and elastic displacements consist usually in solving the stabilization and trajectory tracking problems with many restrictions on dynamic properties. Sensors position defines influence of elastic oscillations on measured parameters of vehicle motion as solid body. The effective analytical approach and software for solving the problem of optimal choice of requirements for sensors number, type and positioning are suggested in this paper. Solution is based on linear programming method properties. The quadratic performance index for stochastic LTI systems and errors of measuring define inequalities-restrictions. The minimized goal function is related with number, type and accuracy of sensors.

1. INTRODUCTION[1]

State-space model of aeroelastic vehicle includes the dynamic equations of solid body motions, models of flexible relative displacements of construction, actuators dynamics from one side and from other side the cross relations defined by aerodynamic and trust forces and closed loop feedback control. Such effects as sloshing, stochastic models of non-stationary aerodynamic forces may be included also.

1.1 Solid-body equations

The rigid part of mathematical model of vehicle is described by the system of differential non-linear equations (1).

$$\frac{d\mathbf{v}_{xyz}}{dt} = \frac{\mathbf{f}_{xyz}}{M} - \omega_{xyz} \times \mathbf{v}_{xyz}, \qquad \frac{d\omega_{xyz}}{dt} = \mathbf{I}_{xyz}^{-1}\left(\mathbf{m}_{xyz} - \omega_{xyz} \times \left(\mathbf{I}_{xyz} \cdot \omega_{xyz}\right)\right). \tag{1}$$

Here $\mathbf{v}_{xyz}$ is velocity vector for center of gravity (c.g.), ω_{xyz} is angular velocity vector about the c.g, $\mathbf{f}_{xyz}$ is total external force vector, $\mathbf{m}_{xyz}$ is total external moment vector, M is total mass, $\mathbf{I}_{xyz}$ is inertia tensor of the rigid body. The solution $\{\omega_{xyz}, \mathbf{v}_{xyz}\}$ is vehicle motion in the body reference frame.

2.2 Elasticity equations

Discrete form of flexible forced oscillations in node displacements q at body axes frame as next

$$\Delta\mathbf{M}\ddot{\mathbf{q}} + \Delta\Xi\dot{\mathbf{q}} + \mathbf{q} = \Delta\mathbf{f}, \tag{2}$$

where $\mathbf{M}$ is diagonal mass matrix of lumped masses m_i, Δ is the inverse stiffness symmetrical matrix and Ξ is damping symmetrical matrix, $\mathbf{f}$ is vector of lumped loads in each node. Matrix Δ is calculated with tacking to account free boundaries and dynamic equilibrium conditions. It implies that the matrix Δ is singular, and pair of singular values corresponds to linear displacement and rotation of vehicle as solid body. In other words, the stiffness matrix ignores the part of distributed loads which do not cause the deformation. The solution of homogeneous part of ordinary differential equation (3)

$$\Delta\mathbf{M}\ddot{\mathbf{q}} + \mathbf{q} = \mathbf{0}, \tag{3}$$

without damping $\Xi = 0$, corresponds to free oscillations:

The dimension of equation is defined by the number of node points. If one multiplies equation on the left by $\mathbf{M}^{-\frac{1}{2}}$ and solves eigenvalues problem for symmetrical matrix

$$\mathbf{M}^{\frac{1}{2}}\Delta\mathbf{M}^{\frac{1}{2}}\left(\mathbf{M}^{\frac{1}{2}}\Phi\right) = \left(\mathbf{M}^{\frac{1}{2}}\Phi\right)\Lambda, \quad \left(\mathbf{M}^{\frac{1}{2}}\Phi\right)'\left(\mathbf{M}^{\frac{1}{2}}\Phi\right) = \mathbf{E}, \tag{4}$$

$$\Omega = diag[\omega_i] = \Lambda^{-\frac{1}{2}} = diag\left[\lambda_i^{-\frac{1}{2}}\right], \tag{5}$$

one obtains natural frequencies ω_i, as and mass normalized shapes of free oscillations $\varphi^{\langle j\rangle} = \{\Phi_{i,j}\}$, as columns of matrix Φ.

The displacements of forced flexible oscillations can be represented as linear combination of shapes of free oscillations. The components of vector ξ are known as modes of flexible oscillations (generalized coordinates).

$$\mathbf{q} = \Phi\xi. \tag{6}$$

By multiplying on the left by $\mathbf{M}^{-\frac{1}{2}}$ and substituting of $\mathbf{q}$, the equation (2) can be written as

[1] The work has been supported by the Russian Foundation for Basic Research under the project 07-08-00293-a

$$\mathbf{M}^{1/2}\Delta\mathbf{M}^{1/2}\mathbf{M}^{1/2}\Phi\ddot{\xi}+\mathbf{M}^{1/2}\Delta\mathbf{M}^{1/2}\mathbf{M}^{-1/2}\Xi\mathbf{M}^{-1/2}\mathbf{M}^{1/2}\Phi\dot{\xi}, \\ +\mathbf{M}^{1/2}\Phi\xi=\mathbf{M}^{1/2}\Delta\mathbf{f}. \quad (7)$$

It can be transformed by multiplying on the left by $\Phi'\mathbf{M}^{1/2}$ and taking into account trivial property of symmetrical matrix $\left(\Phi'\mathbf{M}^{1/2}\right)\mathbf{M}^{1/2}\,\Delta\,\mathbf{M}^{1/2}=\Lambda\left(\Phi'\mathbf{M}^{1/2}\right)$ to the following form

$$\Lambda\Phi'\mathbf{M}\Phi\ddot{\xi}+\Lambda\Phi'\Xi\Phi\dot{\xi}+\Phi'\mathbf{M}\Phi\xi=\Lambda\Phi'\mathbf{f}\,, \quad (8)$$

$$\Phi'\mathbf{M}\Phi\ddot{\xi}+\Phi'\Xi\Phi\dot{\xi}+\Omega^2\Phi'\mathbf{M}\Phi\xi=\Phi'\mathbf{f}\,. \quad (9)$$

The diagonal elements M_i of $\Phi'\mathbf{M}\Phi$ are known as general masses and the components of vector $\Phi'\mathbf{f}$ are known as general forces. In common case the matrix of general masses is diagonal, but for conditions (4) it is an identity matrix.

Theoretically the number of modes is equal to the number of local masses. Practically it is possible to decrease dimension of equation by eliminating the non-dominant harmonics. If one eliminates corresponding components of $\mathbf{q}$ and columns of matrix Φ, one obtains the reduced equation.

For analogy with pendulum equation the transformed damping matrix is approximately assumed as diagonal with elements equal to

$$\Phi'\Xi\Phi=diag(2\zeta_i M_i\omega_i)\,. \quad (10)$$

1.3 Aeroservoelasticity

Rigid body model of vehicle and model of elasticity are interconnected via distributed aerodynamic forces, which depend on parameters of body motion and local angle of attack at i nodes. For example for lateral elastic displacements in the pitching plane the local angle of attack is

$$a_i^{\,*}=a_i+\frac{x_{cg}-x_i}{V_i}\dot{\vartheta}-\frac{\dot{q}_i}{V_i}+\frac{\partial q_i}{\partial x_i}\,, \quad (11)$$

where $\partial q_i/\partial x_i$ is slope of node surface in current time, $\dot{q}_i$ is the lateral node velocity. Here V_i is local air velocity, a_i is local attack angle of node airfoil section for solid body. For mathematical simplicity the so-called strip theory as a first approximation is used. In this theory it is assumed that the local force is proportional to the local angle of attack.

The lumped loads $\mathbf{f}$ include aerodynamic and thrust forces, applied to points of body in various directions. These forces depend from flexible displacements of construction and control law $\mathbf{u}$, which defines value and direction of thrust and positions of aerodynamic control surfaces. The total forces **f**xyz and moment **m**xyz are formed from lumped loads.

1.4 LTI model

State-space model of object with consideration of all factors may be created by applying linearization procedure to system of all nonlinear and time-varying equations about points of calculated base trajectory. Approximately one can separate the motion of object to translation and rotation and research motion in one plane. For simplicity let us investigate the longitudinal motion of vehicle in pitch channel and use beam flexibility model of bending oscillations. It is reasonable to use minimal realization of system where all uncontrollable or unobservable modes have been removed.

2. MEASUREMENTS, ESTIMATION AND CONTROL

The output of sensors, measuring linear or angle parameters of motion, includes matched parameters of flexible displacements. Influence of oscillations depends on the positions of sensors. It is necessary to perform the estimation of state space vector and design the control law considering this information. The optimization of measuring and control systems for flexible aerospace vehicles is not separated from estimator and regulator optimization.

State-space model of aeroservoelastic object may be represented in the following matrix form:

$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{u}+\mathbf{w}\,, \quad (12)$$

$$\mathbf{y}=\mathbf{C}\,\mathbf{x}+\mathbf{D}\,\mathbf{u}+\mathbf{v}\,, \quad (13)$$

where $\mathbf{x}=col(\mathbf{x}_s,\xi)$ is a state vector includes the solid body and actuators state parameters $\mathbf{x}_s$ and modes of oscillations ξ. The input of system contains deterministic control **u**, process noise **w** and measurement noise **v**. The output of system is measurement vector **y**.

The matrix **A**nxn is called the dynamic coefficient matrix, and **B**nxm is the input coupling matrix. The matrix **C**kxn is the measurement sensitivity matrix, and $\mathbf{D}_{kxm}$ is the input-output coupling matrix.

The main feature of matrix **C** is that the rows of **C** contain information about sensors and their position. This is used to formalize and solve the problem of sensors choice and their accommodation.

2.1 Shapes based model of measurements

Let us define the measurement sensitivity matrix for sensors measuring angle, angular velocity and linear acceleration. For the pitching motion the state vector is given by $\mathbf{x}=col\left(\alpha,\theta,\dot{\theta},q_1,\dot{q},q_2,\dot{q}_2\right)$. If one defines matrix **C** for all available nodes n of elastic body, where it is possible to set the above types sensors, one obtains

$$\mathbf{C}_\theta=\begin{pmatrix}\mathbf{0} & \varphi_{00} & \mathbf{0} & \partial\varphi^{\langle 1\rangle}/\partial x & \mathbf{0} & \partial\varphi^{\langle 2\rangle}/\partial x & \mathbf{0}\end{pmatrix}, \\ \mathbf{C}_\omega=\begin{pmatrix}\mathbf{0} & \mathbf{0} & \varphi_{00} & \mathbf{0} & \partial\varphi^{\langle 1\rangle}/\partial x & \mathbf{0} & \partial\varphi^{\langle 2\rangle}/\partial x\end{pmatrix}, \quad (14) \\ \mathbf{C}_a=\begin{pmatrix}\mathbf{0} & \mathbf{0} & \varphi_{01} & \mathbf{0} & \varphi^{\langle 1\rangle} & \mathbf{0} & \varphi^{\langle 2\rangle}\end{pmatrix},$$

where measurement sensitivity matrix $\mathbf{C}_\theta$ corresponds to tangage, $\mathbf{C}_\omega$ corresponds to angular velocity , and $\mathbf{C}_a$ corresponds to acceleration. The vectors φ_{00} and φ_{01} are so-called solid body shapes for translation and rotation. The shapes of bending oscillations of homogenous beam $q(x,t)=\sum\phi^{\langle i\rangle}(x)\xi(t)$ and its derivatives $\partial\varphi^{\langle 1\rangle}/\partial x$, $\partial\varphi^{\langle 2\rangle}/\partial x$ are shown in Fig. 1.

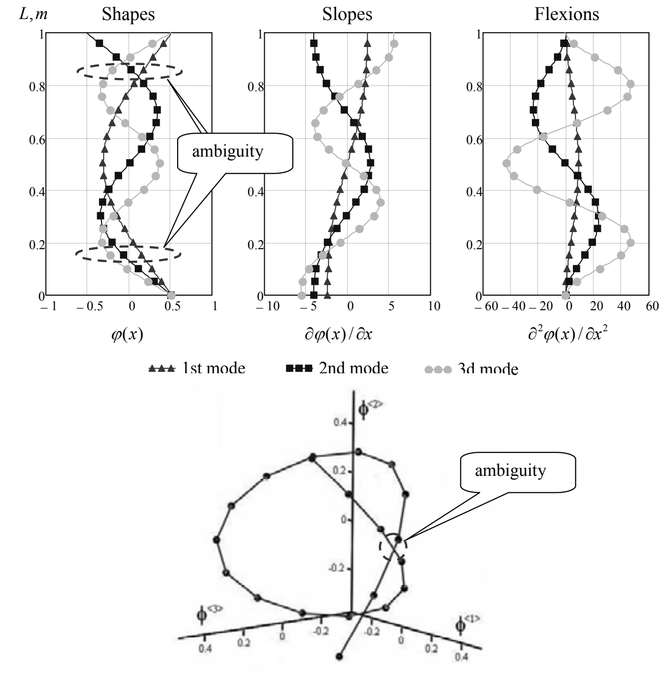

Fig. 1. Mass normalized shapes and their 1st and 2nd derivatives

The row-vectors $\mathbf{c}(_i)$ =[$C_{i,j}$] of matrix **C** correspond to sensors positions. The elimination of rows of matrix **C** in equation (13) is adequate to the elimination of sensors. It is reasonable to complete matrix **C** in consideration of $\| \mathbf{c}(_i) - \mathbf{c}(_j) \|$, thus *it is possible to change nodes partitions and exclude the possibility of ambiguity correspondence from rows of* **C** *to points of sensors location*. The elimination of a priori not suitable points decreases the dimension of measurements optimization problem.

The Linear Quadratic Gaussian (LQG) control system contains optimal linear-quadratic regulator or tracking controller and stationary Kalman filter for estimation of state vector. Let us investigate LQG control system purposely to optimize measurements satisfying requirements for control and estimation.

2.2 KALMAN ESTIMATION AND OPTIMAL LQ REGULATOR

The system (12),(13) must be completely controllable and observable. For the pitching motion with state vector $\mathbf{x} = col(\alpha, \theta, \dot{\theta}, q_1, \dot{q}, q_2, \dot{q}_2)$ let us expand the model of measurement.

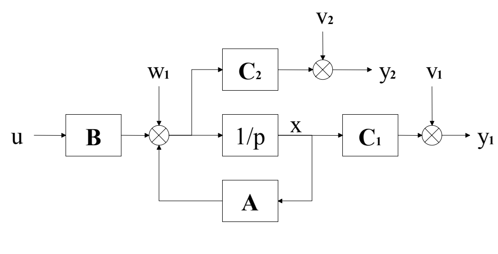

Fig. 2: Plant and measurements

The matrix $\mathbf{C_1} = col\{\mathbf{C}_\theta, \mathbf{C}_\omega\}$ corresponds to angle and angular velocity measurements, the matrix $\mathbf{C_2} = \mathbf{C}_a$ corresponds to accelerations.

$$\begin{aligned} &\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{w}, \\ &\mathbf{y_1} = \mathbf{C_1x} + \mathbf{v}_1, \\ &\mathbf{y_2} = \mathbf{C_2}(\mathbf{Ax} + \mathbf{Bu} + \mathbf{w}) + \mathbf{v}_2. \end{aligned} \tag{15}$$

Let us assign and substitute

$$\mathbf{y} = \begin{pmatrix} \mathbf{y_1} \\ \mathbf{y_2} \end{pmatrix}, \mathbf{v} = \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} \mathbf{C_1} \\ \mathbf{C_2A} \end{pmatrix}, \mathbf{D} = \begin{pmatrix} \mathbf{0} \\ \mathbf{C_2B} \end{pmatrix}, \mathbf{H} = \begin{pmatrix} \mathbf{0} \\ \mathbf{C_2} \end{pmatrix}, \tag{16}$$
$$\mathbf{y} = \mathbf{C\,x} + \mathbf{D\,u} + \mathbf{H\,w} + \mathbf{v}.$$

Let us determine stochastic properties of unbiased white noises

$$\begin{aligned} &E(\mathbf{w}) = E(\mathbf{v}) = \mathbf{0},\ E\left(\mathbf{ww}'\right) = \mathbf{Q}, E\left(\mathbf{v}_1\mathbf{v}_1{}'\right) = \mathbf{R}_1, \\ &E\left(\mathbf{v}_2\mathbf{v}_2{}'\right) = \mathbf{R}_2, E\left(\mathbf{wv}'\right) = \mathbf{0}, E\left(\mathbf{v}_1\mathbf{v}_2{}'\right) = \mathbf{0}. \end{aligned} \tag{17}$$

In this case, the optimal estimation of state-vector is

$$\dot{\mathbf{x}}_e = \mathbf{Ax}_e + \mathbf{Bu} + \mathbf{L}(\mathbf{y} - \mathbf{Cx}_e - \mathbf{Du}), \tag{18}$$

$$\mathbf{L} = (\mathbf{SC}' + \mathbf{N})\mathbf{R}^{-1}, \tag{19}$$

where **S**.>0 is solution of associated with estimator Riccati equation

$$\begin{aligned} &\mathbf{AS} + \mathbf{SA}' - (\mathbf{SC}' + \mathbf{N})\mathbf{R}^{-1}(\mathbf{CS} + \mathbf{N}') + \mathbf{Q} = \mathbf{0}, \\ &\mathbf{R} = \begin{pmatrix} \mathbf{R}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_2 \end{pmatrix} + \mathbf{HQH}', \quad \mathbf{N} = \mathbf{QH}'. \end{aligned} \tag{20}$$

The control is implemented using observer state variables

$$\mathbf{u} = -\mathbf{Fx}_e,\ \mathbf{F} = \mathbf{R_r}^{-1}\mathbf{B}'\mathbf{P}, \tag{21}$$

where P.>0 is solution of associated with regulator Riccati equation

$$\mathbf{A}'\mathbf{P} + \mathbf{PA} - \mathbf{PBR_r}^{-1}\mathbf{B}'\mathbf{P} + \mathbf{Q_r} = \mathbf{0}, \tag{22}$$

for the following quadratic performance index with weighting matrixes $\mathbf{Q_r}. \geq 0, \mathbf{R_r}. > 0$, which condense requirements for dynamic properties of closed-loop system

$$J = E\int_0^\infty (\mathbf{x}'\mathbf{Q_r x} + \mathbf{u}'\mathbf{R_r u})dt\,. \tag{23}$$

The time-averaged value of quadratic performance index is equal to

$$\begin{aligned} &\lim_{\substack{t_0 \to -\infty \\ t_1 \to \infty}} \frac{1}{t_1 - t_0} E\left\{ \int_{t_0}^{t_1} (\mathbf{x}'\mathbf{Q_r x} + \mathbf{u}'\mathbf{R_r u})dt \right\} = \\ &\mathrm{tr}(\mathbf{PLRL}' + \mathbf{SQ_r}) = \mathrm{tr}(\mathbf{PQ} + \mathbf{SF}'\mathbf{R_r F}), \end{aligned} \tag{24}$$

$$\overline{\sigma} = \mathrm{tr}(\mathbf{PQ} + \mathbf{SF}'\mathbf{R_r F})\,. \tag{25}$$

The value of $\overline{\sigma}$ linearly depends from covariance matrix of state error estimation **S**, which in one's turn linked with covariance matrix **R**, defining error dispersion of measurements. Let us assume that the measurement noises are not correlated variables and therefore the matrix **V** is diagonal matrix with $v_{i,j}$ elements corresponding to dispersions of noises of sensors in i nodes. The diagonal elements of inverse matrix $\mathbf{V}^{-1}$ equaling zero can be interpreted as absence of sensors in corresponding node.

Let us impose a responsibility for dynamical properties of closed-loop system with state-feedback law (21) to choice weight matrix **Q**r and **R**r and fix this by setting minimal value of time-averaged quadratic performance index $\overline{\sigma}$. The matrix of state error estimation **S** defines the accuracy of estimation.

3. MATHEMATICAL PROGRAMMING PROBLEM FORMULATION

The main question about measurement optimization is where, which and how many sensors one should use to provide a necessary accuracy of state estimation and to realize the desired control system.

3.1 Restrictions

Let us formulate the main requirements as

$$\overline{\sigma}(\mathbf{S}) \le \sigma^*, \tag{26}$$

$$\mathbf{S}(\mathbf{R})\ . \le \mathbf{S}^*. \tag{27}$$

The last inequality for solution of (20) defines that the difference is not a positive-definite matrix.

The restrictions may be not so stringent if the accuracy is declared only for some of components of state vector or their linear combination $\mathbf{v}'\mathbf{x}_e$

$$\mathbf{v}'\,\mathbf{S}(\mathbf{R})\,\mathbf{v} \le d^*. \tag{28}$$

Fulfilment of these inequalities for various weight matrixes of a functional (23) one shall use as restrictions. Performance of these restrictions by some composition of sensors provides permissible nonoptimal solution **R**o.

3.2 Goal function

Let us examine equation (20). All information about sensors condensed in diagonal matrix $\mathbf{R}^{-1}$. Let us define $x_i = R_{i,i}^{-1}$.
The goal function for **x** can be written as

$$f(\mathbf{x}) = \boldsymbol{\rho}'\mathbf{x}, \tag{29}$$

where ρ is weight vector. The physical meaning of this measurements cost function minimization can be explained by the following features:

$x_i \ge 0$	it is condition for dispersions,
$x_i = 0$	there are no sensor in *i* node,
$x_i = x_k + x_m$	there are two sensors in *i* node.

The last equality assumes that the signals from two sensors were processed as least squares solution $\mathbf{x_e}$ in the presence of known covariance diagonal matrix $\mathbf{R} = E\{\mathbf{ww}'\} = diag([x_k^{-1} x_m^{-1}])$ and $\mathbf{C} = [1\ \ 1]$. The descriptions of least squares solution is as following

$$\mathbf{y} = \mathbf{Cx} + \mathbf{w},$$
$$\mathbf{x_e} = \left(\mathbf{C}'\mathbf{R}^{-1}\mathbf{C}\right)^{-1}\mathbf{C}'\mathbf{R}^{-1}\mathbf{y}, \tag{30}$$
$$E\{(\mathbf{x}-\mathbf{x_e})(\mathbf{x}-\mathbf{x_e})'\} = \left(\mathbf{C}'\mathbf{R}^{-1}\mathbf{C}\right)^{-1}.$$

The coefficients of weight vector ρ are specified under the assumption about priority of applied sensors (cost of the sensor, its weight, reliability, etc.) and setting points which can differ by variance of noise of measurements.

3.2 Linear programming problem

Let matrix **S** satisfies restrictions (26),(27), then the equation (20) defines the restriction for **x**

$$(\mathbf{SC}' + \mathbf{N})diag(\mathbf{x})(\mathbf{CS} + \mathbf{N}') = \mathbf{AS} + \mathbf{SA}' + \mathbf{Q}. \tag{31}$$

With taking into account the requirement to minimize goal function (29) $\boldsymbol{\rho}'\mathbf{x} \to \min$ the problem can be represented in the following form

$$\min_{\mathbf{x}}\left\{\boldsymbol{\rho}'\mathbf{x} \Big/ \begin{matrix}\mathbf{x} \ge 0, \\ (\mathbf{SC}' + \mathbf{N})diag(\mathbf{x})(\mathbf{CS} + \mathbf{N}') = \mathbf{AS} + \mathbf{SA}' + \mathbf{Q}\end{matrix}\right\} \tag{32}$$

This is complete setting of linear programming problem. Reduction of the equations to a canonical form justifies an optimum amount of sensors, and the outcome of a solution determines a locations and parameters of sensors. In other words *the number of active restrictions defines a number of nonzero components of vector **x**, that equal to number of sensors*.
The solving problem with equalities-restrictions may exclude minimal solution for goal function, which increase precision of estimation. The next problem statement with extended vector of controlled variables $col(\mathbf{x}, \tilde{\mathbf{x}})$ formally compensates accuracy advantage

$$\min_{\mathbf{x},\tilde{\mathbf{x}}}\left\{\boldsymbol{\rho}'\mathbf{x} \Big/ \begin{matrix}\mathbf{x} \ge 0, \tilde{\mathbf{x}} \ge 0, \\ (\mathbf{SC}' + \mathbf{N})diag(\mathbf{x} - \tilde{\mathbf{x}})(\mathbf{CS} + \mathbf{N}') = \mathbf{AS} + \mathbf{SA}' + \mathbf{Q}\end{matrix}\right\} \tag{33}$$

4. CONCLUSIONS

The optimization of measuring system for flexible aerospace vehicle requires full information about the mathematical model of object motion and elastic oscillations and can not be solved separately from control optimization. The suggested approach can be included in specific software developed for these purposes.

The additionally developed methods and algorithms for solving this problem, such as choice of controllable variables and elimination of surplus inequalities, guarantee the convex programming conditions for goal function and restrictions. This implies uniqueness of solution and good performance and convergence.

The offered algorithm of optimization can be applied also to a problem, not linked with elastic vibrations - problems in which location of sensors determines parameters of linear combination of estimated parameters. The approach can be applied also to a problem of localization and choice devices for active damping of elastic vibrations, proceeding from the duality of problems of optimal control and estimation.

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