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New Dilated LMIs to Synthesize Controllers for a Class of Spatially Interconnected Systems

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Abstract: This paper presents sufficient conditions based on dilated LMIs to analyze and synthesize controllers that minimize the $\mathcal{L}_2$-norm of the the closed-loop system for spatially varying interconnected polytopic systems. The approach presented here searches for a parameter dependent Lyapunov function (PDLF) by dilating the original LMIs. This dilation not only helps in the search for a PDLF but also introduces extra degrees of freedom which may result in further reduction of conservatism. Approaches to synthesize full-order polytopic controllers and reduced-order, reduced-structure controllers are also presented here.

Keywords: Spatially interconnected Systems, Genetic Algorithms, Gain Scheduled Control Systems.

1. INTRODUCTION

The problem of finding suitable controllers for interconnected systems has attracted the attention of researchers for more than three decades. Earlier approaches to deal with such systems rely on designing first a precompensator to decouple the system into sub-systems. The next step is to design a controller for each of the subsystems (Siljak, 1978). However, the final controller will have a centralized structure as shown in Fig. 1, where G_0, G_1, … are subsystems of the interconnected system $\mathbf{G}$.

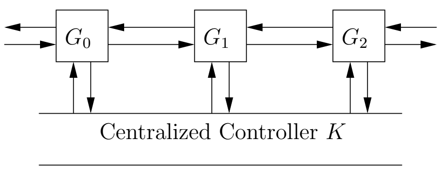

Fig. 1. Centralized Control Structure

The main problem associated with this approach is that modern MIMO controller design techniques may fail if the number of sub-systems becomes very large. In some cases interacting systems are distributed spatially and their interactions depends on the spatial location of one subsystem with respect to another. In (Curtain and Zwart, 1995) and (v. Keulen, 1993) a detailed mathematical analysis has been presented for systems having continuous time and space dynamics, while (D'Andrea and Dullerud, 2003) have considered continuous-time and discrete-space systems. In this paper the framework proposed in (D'Andrea and Dullerud, 2003) is used, which is briefly summarized here.

Consider the configuration shown in Fig. 2, consisting of identical subsystems G. The overall spatially interconnected system $\bar{\mathbf{G}}$ can be represented as

$$\begin{bmatrix} \mathbf{T}x^t(t,s) \\ \mathbf{S}x^s(t,s) \\ z \end{bmatrix} = \begin{bmatrix} \bar{A}^{tt} & \bar{A}^{ts} & \bar{B}_1^t \\ \bar{A}^{st} & \bar{A}^{ss} & \bar{B}_1^s \\ \bar{C}_1^t & \bar{C}_1^s & \bar{D}_{11} \end{bmatrix} \begin{bmatrix} x^t \\ x^s \\ w \end{bmatrix} \tag{1}$$

where x^t is the temporal state vector , z is the output vector and d is the input vector of one subsystem, $\mathbf{T}$ and $\mathbf{S}$ are the temporal differential and spatial shift operators defined as:

$$\begin{aligned} \mathbf{T}x(t,s) &= \frac{\partial x(t,s)}{\partial t}, \\ \mathbf{S_i}x(t,s) &= x(t, s_1, \ldots, s_i + 1, \ldots, s_L) \end{aligned} \tag{2}$$

and $s = [s_1, ..., s_L]$ represents the spatial co-ordinates. Physically, the spatial states x^s represent the interactions among the subsystems.

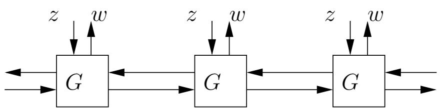

Fig. 2. Spatially Interconnected System

The assumption that the same system is replicated at each node of a fixed lattice, is relaxed in (Wu and Yildizoglu, 2005) by considering nearly identical subsystems present at the nodes of the spatial lattice, if these can be represented in Linear Fractional Transformation (LFT) form.The approach presented there then searches for a common Lyapunov function over a set of all allowed plants. In order to reduce the conservatism which may arise due to single Lyapunov function extra degree of freedom is introduced in the form of multipliers or scaling matrices.

In this paper the approach presented in (Chughtai and Munro, 2004) is extended to spatially varying interconnected polytopic (SVIP) systems. These are systems which can be represented as a convex combination of a number of vertex plants, whose dynamics vary only spatially. The approach presented here searches for a parameter-dependent Lyapunov function (PDLF) in a systematic way. This can been achieved by dilating the original LMIs. This dilation not only helps in search for PDLF but also introduces extra degree of freedom which may result in further reduction of conservatism. Two cases of controller synthesis are considered here:

1. Full-order interconnected polytopic controllers.
2. Reduced-order decentralized robust controller.

For the first case LMI conditions are presented while for the second a hybrid LMI-evolutionary approach is used.

The paper is organized as follows: Section 2 introduces the notation used in the paper along with a review of previous results. In section 3 sufficient conditions based on dilated LMIs for SVIP systems are presented to analyze robust performance. Section 4 deals with controller synthesis based on the analysis conditions in section 3. Both proposed approaches are applied to a simple example and results are compared with those achieved with previously presented LMI conditions. This is followed by concluding remarks in Section 5.

2. NOTATION AND PRELIMINARIES

For brevity let us define $x^T = \begin{bmatrix} x^{tT} & x^{sT} \end{bmatrix}$, where $x^t \in R^{n_t}$,$x^s \in R^{n_s}$, $n_s = \sum_{i=1}^{L} n_{s_i} + \sum_{i=1}^{L} n_{s_{-i}}$, $x^{s_1} \in R^n_{s_i}$, $w \in R^l$, $z \in R^k$. Also define,

$$\bar{A} = \begin{bmatrix} \bar{A}^{tt} & \bar{A}^{ts} \\ \bar{A}^{st} & \bar{A}^{ss} \end{bmatrix}, \qquad \bar{B}_1 = \begin{bmatrix} \bar{B}_1^t \\ \bar{B}_1^s \end{bmatrix} \qquad \bar{C}_1 = \begin{bmatrix} \bar{C}_1^t & \bar{C}_1^s \end{bmatrix} \tag{3}$$

If $(I - \bar{A}_{ss})$ is invertible then using a bilinear transformation the discrete (spatial) part of the system (1) can be converted to continuous as presented in (D'Andrea and Dullerud, 2003).

Let the transformed system be represented as $\mathbf{G}$, then (1) can be written as

$$\begin{bmatrix} \Xi x \\ z \end{bmatrix} = \begin{bmatrix} A & B_1 \\ C_1 & D_{11} \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} \tag{4}$$

where all matrices have compatible dimensions and $\Xi = \begin{bmatrix} \mathbf{T} & 0 \\ 0 & \mathbf{S} \end{bmatrix}$, where now $\mathbf{S}$ represents the spatial shift operator after a bilinear operation.

The sets $\mathcal{P}$ and $\mathcal{X}$ are defined as

$$\mathcal{P} = \{diag(P^t, P^s) : P^t \in R^{n_t \times n_t}, P^s \in \mathcal{P}^s\}$$

$$\mathcal{P}^s := \{diag(P^{s_1}, ..., P^{s_L}) : P^{s_i} \in R^{n_{s_i} \times n_{s_i}}\} \tag{5}$$

$$\mathcal{X} = \{diag(X^t, X^s) : X^t \in \mathcal{X}^t, X^s \in \mathcal{X}^s\}$$

$$\mathcal{X}^t := \{X^t \in R^{n_t \times n_t} : X^{tT} = X^t > 0\} \tag{6}$$

$$\mathcal{X}^s := \{diag(X^{s_1}, ..., X^{s_L}) : X^{s_i} \in R^{n_{s_i} \times n_{s_i}}, X^{sT} = X^s\} \tag{7}$$

Combining Theorem 1 and Theorem 2 of (D'Andrea and Dullerud, 2003), the distributed version of the Kalman-Yacubovich-Popov Lemma (KYP) can be presented as follows

Theorem 2.1. A system ($\mathbf{G}$) is well-posed, stable and has $\mathcal{L}_2$-norm $< \gamma$ if there exists $X \in \mathcal{X}$ such that the following inequality is satisfied:

$$\begin{bmatrix} A^T X + XA & XB_1 & C_1^T \\ B_1^T X & -\gamma^2 I & D_{11}^T \\ C_1 & D_{11} & -I \end{bmatrix} < 0 \tag{8}$$

The above theorem shows that after a bilinear transformation on spatial coordinates the overall system is stable, well-posed and has $\mathcal{L}_2$-norm $< \gamma$ if there exists $X \in \mathcal{X}$, such that

$$x^T X \Xi x + \Xi x^T X x + \frac{1}{\gamma} z^T z - \gamma w^T w < 0 \tag{9}$$

In the sequel we will use the notation $sym(X)$ to represent $X + X^T$ and in LMIs we will replace the symmetric terms in the upper matrix triangle by $*$.

3. PERFORMANCE ANALYSIS

Using the approach presented in (Chughtai and Munro, 2004) the following result can be proved for spatially interconnected systems, which may be referred to as extended KYP lemma.

Theorem 3.1. A system ($\mathbf{G}$) is well-posed, stable and has $\mathcal{L}_2$-norm $< \gamma$ if there exists $X \in \mathcal{X}$ and a matrix $F \in \mathcal{P}$ such that the following inequality is satisfied:

$$\begin{bmatrix} F_1 A + A^T F_1^T & X - F_1 + A^T F_2^T & F_1 B_1 & C_1^T \\ X - F_1^T + F_2 A & -F_2 - F_2^T & F_2 B_1 & 0 \\ B_1^T F_1^2 & B_1^T F_2^T & -\gamma I & D_{11}^T \\ C_1 & 0 & D_{11} & -\gamma I \end{bmatrix} < 0 \tag{10}$$

Proof: Let $y = \Xi x$, then we can write (4) in its "descriptor" form as

$$E \Xi \zeta = \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix} \zeta + \begin{bmatrix} 0 \\ B_1 \end{bmatrix} w$$

$$z = [C_1 \;\; 0]\, \zeta + D_{11} w \tag{11}$$

where $\zeta^T = \begin{bmatrix} x^T & y^T \end{bmatrix}$ and $E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$. Let a candidate Lyapunov function for the system (4) be

$$V = x^T X x = \zeta \begin{bmatrix} X & 0 \\ 0 & 0 \end{bmatrix} \zeta$$

$$= \begin{bmatrix} X & F_1 \\ 0 & F_2 \end{bmatrix} E = E^T \begin{bmatrix} X & 0 \\ F_1^T & F_2^T \end{bmatrix} \tag{12}$$

where $X \in \mathcal{X}$ and $F \in \mathcal{P}$. Now, we can write (9) for (11) as

$$\zeta^T \begin{bmatrix} X & F_1 \\ 0 & F_2 \end{bmatrix} E \Xi \zeta + \Xi \zeta^T E^T \begin{bmatrix} X & 0 \\ F_1^T & F_2^T \end{bmatrix} \zeta$$

$$+ \frac{1}{\gamma} z^T z - \gamma w^T w < 0 \tag{13}$$

Using (11) and applying the Schur complement, (13) holds if (10) holds. This completes the proof.

3.1 Robust Performance

The extended KYP lemma presented above can be used to analyze the $\mathcal{L}_2$-norm of a system. Note that in (10) there are no product terms between the Lyapunov matrix and state space model matrices. This observation helps in analyzing the robust performance of SVIP systems using PDLF.

To show this, let the interconnected system (4) be spatially varying, but be contained in a convex polyhedron Φ which is defined as

$$\Phi = \{M(\alpha) := \sum_{i=1}^{p} \alpha_i M, \alpha \geq 0, \sum_{i=1}^{p} \alpha_i = 1\} \qquad (14)$$

where, $M(\alpha) = \begin{bmatrix} A(\alpha) & B_1(\alpha) \\ C_1(\alpha) & D_{11}(\alpha) \end{bmatrix}$. That is, each state space model matrix may be written as a convex combination of p given vertices $M_1, \ldots, M_p$. Let us also define a parameter-dependent Lyapunov matrix as

$$X(\alpha) = diag(X^t, X^s(\alpha)), X^s(\alpha) = \sum_{i=1}^{p} \alpha_i X_i^s \qquad (15)$$

If the inequality (10) holds for each extreme system defined by M_i simultaneously, one can multiply each of these inequalities by $\alpha_i > 0$ and sum them up to obtain

$$\begin{bmatrix} A^T(\alpha)F_1^T + F_1 A(\alpha) & * & * & * \\ X(\alpha) - F_1^T + F_2 A(\alpha) & -F_2 - F_2^T & * & * \\ B_1(\alpha)^T F_1^T & B_1(\alpha)^T F_2^T & -\gamma I & * \\ C_1(\alpha) & 0 & D_{11}(\alpha) & -\gamma I \end{bmatrix} < 0 \qquad (16)$$

The above result can be formally presented as follows.

Theorem 3.2. A spatially varying interconnected polytopic system is stable, well posed and has worst case $\mathcal{L}_2$-norm less than γ if there exist $X_i \in \mathcal{X}$, $\forall i = 1, \ldots, p$ and $F \in \mathcal{P}$ such that (16) holds for all p vertex systems simultaneously.

4. CONTROLLER SYNTHESIS

For large-scale systems, the complexity of a centralized control structure makes it not only difficult to synthesize but also difficult to implement. A simpler control structure is a spatially varying interconnected polytopic controller (SVIPC) which is shown in Figure 3. The least complex control structure considered here is a spatially decentralized robust control (SDRC), which is as shown in Figure 4, where the same controller is used at each node point.

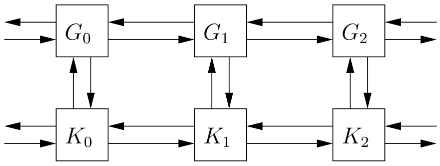

Fig. 3. Spatially Varying Interconnected Polytopic Control Structure

Let a state space representation of the plant be given as

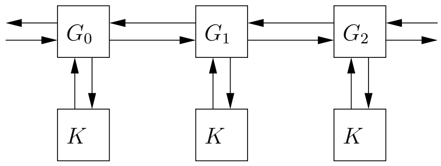

Fig. 4. Decentralized Robust Control Structure

$$\begin{bmatrix} x \\ z \\ y \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_y & D_{yw} & 0 \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix} \qquad (17)$$
$$e = r - y$$

For the synthesis problem, LMI (16) becomes a bilinear matrix inequality in X, γ and K. Next we will deal with how to solve this BMI problem for the SVIPC and SDRC cases.

4.1 SVIPC

Let the vertex SVIP controllers (K_i) after a bilinear transformation be given as

$$\begin{bmatrix} x_{K_i} \\ u \end{bmatrix} = \begin{bmatrix} A_{K_i} & B_{K_i} \\ C_{K_i} & D_{K_i} \end{bmatrix} \begin{bmatrix} x_{K_i} \\ y \end{bmatrix} \qquad (18)$$

where $x_{K_i} \in R^{n_t + n_s}$. In order to apply Theorem 3.1 define a permutation matrix P of (D'Andrea and Dullerud, 2003) as

$$P := \begin{bmatrix} P_T^G & 0 & P_T^K & 0 \\ 0 & P_s^G & 0 & P_s^K \end{bmatrix}$$
$$P_T^G = \begin{bmatrix} I_{n_t} \\ 0 \end{bmatrix}, \qquad P_T^K = \begin{bmatrix} 0 \\ I_{n_t} \end{bmatrix}, \qquad (19)$$

$$P_s^G = \begin{bmatrix} I_{n_{s_1}} & 0 & \ldots & 0 \\ 0 & \ldots & \ldots & 0 \\ 0 & I_{n_{s_{-1}}} & \ldots & 0 \\ & & \vdots & \\ 0 & 0 & \ldots & I_{n_{s_{-L}}} \\ 0 & \ldots & \ldots & 0 \end{bmatrix} \qquad (20)$$

$$P_s^K = \begin{bmatrix} 0 & \ldots & \ldots & 0 \\ I_{n_{s_1}} & 0 & \ldots & 0 \\ 0 & \ldots & \ldots & 0 \\ 0 & I_{n_{s_{-1}}} & \ldots & 0 \\ & & \vdots & \\ 0 & 0 & \ldots & I_{n_{s_{-L}}} \end{bmatrix} \qquad (21)$$

It can be seen that $P^T P = I$. After applying this transformation the closed loop system G_c becomes

$$\bar{A}_c = P A_c P^T, \qquad \bar{B}_c = P B_c,$$
$$\bar{C}_c = C_c P^T, \qquad \bar{D}_c = D_c. \qquad (22)$$

Furthermore, let $F_1 = F_2 = F$ will lead to the following BMI condition

$$\begin{bmatrix} sym(AJ+B_2W) & * & * & * & * & * \\ U+(A+B_2D_KC_y)^T & sym(NA+VC_y) & * & * & * & * \\ X_1-J^T+AJ+B_2W & X_2-S^T+A+B_2D_KC_y & sym(-J) & * & * & * \\ X_2^T-I+U & X_3-N^T+NA+VC_y & -I-S & sym(-N) & * & * \\ B_1^T+D_{yw}^TD_K^TB_2^T & B_w^TN^T+D_{yw}V^T & B_1^T+D_{yw}^TD_K^TB_2^T & B_w^TN^T+D_{yw}V^T & -\gamma I & * \\ C_1J+D_{12}W & C_1+D_{12}D_KC_y & 0 & 0 & D_{11}+D_{12}D_KD_{yw} & -\gamma I \end{bmatrix} < 0 \quad (27)$$

$$\begin{bmatrix} \bar{A}_cF+F^T\bar{A}_c^T & X-F+F^T\bar{A}_c^T & \bar{B}_c & F^T\bar{C}_c^T \\ X-F^T+\bar{A}_cF & -F-F^T & \bar{B}_c & 0 \\ \bar{B}_c^T & \bar{B}_c^T & -\gamma I & \bar{D}_c^T \\ \bar{C}_cF & 0 & \bar{D}_c & -\gamma I \end{bmatrix} < 0 \quad (23)$$

Remark 1: The assumption $F_1 = F_2 = F$ is found to be very restrictive for some examples. Thus, one may get either infeasibility or larger Γ.

Applying a congruence transformation on (23) with $T = diag(P, P, I, I)$, we obtain

$$\begin{bmatrix} A_c\bar{F}+\bar{F}^TA_c^T & \bar{X}-\bar{F}+\bar{F}^TA_c^T & B_c & \bar{F}^TC_c^T \\ \bar{X}-\bar{F}^T+A_c\bar{F} & -\bar{F}-\bar{F}^T & B_c & 0 \\ B_c^T & B_c^T & -\gamma I & D_c^T \\ C_c\bar{F} & 0 & D_c & -\gamma I \end{bmatrix} < 0 \quad (24)$$

where $\bar{X} = PXP^T$ and $\bar{F} = PFP^T$.

Remark 2: After this transformation $\bar{X}$ has the form (D'Andrea and Dullerud, 2003),

$$\bar{X} = \begin{bmatrix} X_1 & X_2 \\ X_2^T & X_3 \end{bmatrix} \quad (25)$$

where, X_1,$X_3 \in \mathcal{X}$ and $X_2 \in \mathcal{P}$

Now we can use the change of variable technique proposed in (Scherer et al., 1997) by defining new variables U, V, W as

$$\begin{bmatrix} U & V \\ W & D_K \end{bmatrix} = \begin{bmatrix} R & NB_2 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} \begin{bmatrix} L & 0 \\ C_yJ & I \end{bmatrix} + \begin{bmatrix} N \\ 0 \end{bmatrix} A \, [J \;\; 0]$$
$$S = NJ + RL \quad (26)$$

where $J, N, R, L \in \mathcal{P}$. This transformation leads to linearity in the variables, and the resulting LMI condition is given as (27).

Now, let us consider a SVIP system (G) which resides in a polyhedron Φ defined by (14) where $M_i = \begin{bmatrix} A_i & B_{1i} & B_2 \\ C_{1i} & D_{11i} & D_{12} \\ C_y & D_{yw} & 0 \end{bmatrix}$ of appropriate dimensions, such that

A1. B_2, C_y, D_{12} and D_{yw}.

It can be seen that under this assumption we can guarantee stability and performance using a PDLF as discussed in Theorem 3.2. Then we can synthesize an SVIP controller by following approach.

1. Solve the LMI problem

$$\min_{U_i,V_i,W_i,D_{Ki},(J,N,X_{3i})\in\mathcal{P},(X_{1i},X_{2i})\in\mathcal{X}} \gamma$$
$$\text{s.t (27) hold, } \forall i = 1, \ldots, p \quad (28)$$

2. Find $M_{Ki} = \begin{bmatrix} A_{K_i} & B_{K_i} \\ C_{K_i} & D_{K_i} \end{bmatrix}$ by inverting (26).
3. Find a controller for each node point as convex combination of p extreme points.
4. Apply an inverse bi-linear transformation, as discussed in (D'Andrea and Dullerud, 2003), to convert the controller to discrete space.

4.2 SDRC

The problem of designing SDRC for SVIP systems is a non-convex problem. The approach presented here is based on a hybrid evolutionary-algebraic approach proposed in (Farag and Werner, 2004) for non-convex synthesis problems involving lumped LTI systems, where the fitness of controllers was evaluated by solving algebraic Riccati equations. Here LMIs will be used to assess fitness. A genetic algorithm (GA) is used to construct K and the LMI solver is applied to calculate P, Θ and γ. An algorithm to find a controller that minimizes the worst case performance γ over the complete operational envelope of an SVIP system, can be summarized as follows:

- **Generate** an initial random population of controllers $\{K_1, \ldots, K_\mu\}$,
  where $K_i = \{{A_k}^i, {B_k}^i, {C_k}^i, {D_k}^i\}$ can have any structure.
- **Evaluate** the objective function:
  $$f(K_i) = \begin{cases} \gamma & \text{if } A_{cl} \text{ is stable} \\ \kappa(A_{cl}) + \beta & \text{if } A_{cl} \text{ is unstable} \end{cases} \quad (29)$$
  where A_{cl} is the A-matrix of the closed-loop system, $\kappa(A_{cl})$ is the maximum real part of the eigenvalues of A_{cl}, β is a penalty on destabilizing controllers, and γ is the worst-case performance obtained by solving
  $$\min_{F\in\mathcal{P},X_i\in\mathcal{X}} \gamma$$
  $$\text{s.t (16) holds, } \forall i = 1, \ldots, p \quad (30)$$
  using standard LMI solvers
- **Evolve** the current generation using evolutionary operators to produce the next generation
- **Repeat** evaluate and evolve steps until a stopping criterion is met.

4.3 Example

To illustrate the above algorithm, consider the problem of temperature control of a nonuniform two-dimensional plate. This example is a modified version of an example given in (D'Andrea and Dullerud, 2003) (to test the above algorithm, a spatial variation of the thermal conductivity has been introduced). Let Q be a heat source, then the multi-dimensional heat transfer in the absence of any convective heat loss, is given by

$$\rho c \frac{\partial T}{\partial t} = \nabla(K\nabla T) + Q \tag{31}$$

where T, ρ, c, K are the temperature, density, specific heat and thermal conductivity of the material and $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$. It is assumed that the thermal conductivity of the plate varies linearly along both the dimensions according to the following relation.

$$K(x,y) = K_0(1 - \epsilon\frac{x}{L_1} - \epsilon\frac{y}{L_2}) \tag{32}$$

Using the finite difference approximation of the two spatial partial derivatives results in the following continuous-time, discrete-space approximation,

$$\begin{aligned}\frac{\partial T}{\partial t} &= (K' - \alpha\delta_1 - \alpha\delta_2)(S_1 + S_1^{-1} - 2)T \\ &+(K' - \alpha\delta_1 - \alpha\delta_2)(S_2 + S_2^{-1} - 2)T \\ &-\frac{2\alpha}{L_1}(S_1 - S_1^{-1})T - \frac{2\alpha}{L_2}(S_2 - S_2^{-1})T \\ &+\frac{1}{\rho c}Q,\end{aligned} \tag{33}$$

where $K' = K_0(1-\epsilon)$, $\alpha = \epsilon\frac{K_0}{2\rho c}$, $\delta_1 = 2x/L_1 - 1$ and $\delta_2 = 2y/L_2 - 1$. Thus, the extreme plants are obtained for $\delta_{1,2} = \pm 1$ The boundary conditions are taken to be $T(t,0,y) = T(t,L_1,y) = T(t,x,0) = T(t,x,L_2) = 0$. Let d_1 be the input disturbance and r be the reference temperature. The control objective is disturbance rejection with minimum control effort.

For comparison, first the approach presented in (Wu and Yildizoglu, 2005) (the controller shown in Figure 3) is applied to the system for different values of ϵ; the achieved performance γ_w is shown in Table 1. It can be seen that as the variation in the system increases the worst-case $\mathcal{L}_2$-norm also increases. Next, SVIP controllers are designed using the LMI conditions 27. The worst case $\mathcal{L}_2$-norm achieved by these controllers are represented as (γ_d). It can be seen that better performance is achieved using the dilated LMIs, which is due to the parameter-dependent Lyapunov function. To complete the comparison, fixed-structured controllers are also designed using the hybrid algorithm presented above. The worst-case $\mathcal{L}_2$-norm achieved by this controller, after 100 generations with 20 individuals, is γ_{ro}. Note that the worst case performance index has increased, which indicates a deterioration in achieved control objectives. However, this is the price one has to pay in order to achieve simply structured controllers. The main advantage of which is that now the communication burden among the controllers of subsystems has reduced which makes these controllers easier to implement.

Table 1. Worst-case $\mathcal{L}_2$-norm (γ_w) as a function of ϵ.

ϵ	γ_w	γ_d	γ_{ro}
0.1	1.549	1.253	1.98
0.3	2.332	1.542	4.79
0.7	3.436	2.984	9.53

5. CONCLUSIONS

This paper presents sufficient LMI conditions to analyze the performance of SVIP systems in terms of its worst case induced $\mathcal{L}_2$-norm. These LMIs search for PDLF which makes the approach less conservative than previously presented LMI conditions. Furthermore, these LMIs have an extra degree of freedom due to presence of slack variables which further reduce the conservativeness. These slack variables also help in the synthesis of less conservative controllers by multi objective optimization. This, however, is the topic of our future research.

The analysis condition proposed here is extended to synthesize distributed controllers. Two cases of controller synthesis were considered,

1. Full order interconnected polytopic controllers.
2. Reduced order decentralized robust controller.

Full order polytopic controllers are designed by minimizing worst case induced $\mathcal{L}_2$-norm of the closed loop system. It has been shown that this problem is convex after a change of variables and can be solved by standard LMI solvers.

To design reduced-order, fixed-structure controllers, analysis LMIs are used in a combined GA-LMI algorithm . The proposed algorithm involves genetic operators to span the solution space and LMI solvers to find the worst case performance.

The efficiency of the proposed algorithms is demonstrated by applying it to the problem of controlling the temperature profile of a large non-uniform plate. The result shows that using the proposed LMI conditions, less conservative controllers can be synthesized compared with existing approaches.

REFERENCES

S. Chughtai and N. Munro. Lmi based gain-scheduled control. In *Proc. of UKACC Control 2004*, 2004.

R. Curtain and H. Zwart. *An introduction to infinite-dimensional linear systems theory.* Springer-Verlag, New York, 1995. ISBN 0-387-94475-3.

R. D'Andrea and G. E. Dullerud. Distributed control design for spatially interconnected systems. *IEEE Transactions on Automatic Control*, 48(9), 2003.

A. Farag and H. Werner. A Riccati - genetic algorithms approach to fixed-structure controller synthesis. In *Proc. of the American Control Conference*, 2004.

C. Scherer, P. Gahinet, and M. Chilali. Multi-objective Output-Feedback Control Via LMI Optimisation. *IEEE Transactions on Automatic Control*, 42(7):896–911, 1997.

D. Siljak. *Large Scale Dynamic Systems.* system science and engineering. North-Holland, NewYork, USA, 1978.

B. v. Keulen. *H_∞ control for distributed parameter systems: a state-space appraoch.* Birkhauser, 1993. ISBN 3-7643-3709-5, 0-8176-3709-5.

F. Wu and S. E. Yildizoglu. Distributed paramter dependent modelling and control of flexible structures. *Transactions of the AMSE*, 127, 2005.